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ON REAL HYPERSURFACES IN A COMPLEX SPACE FORM

HIROYUKI KURIHARA

0. INTRODUCTION

A complex n-dimensional Kähler manifold of constant holomorphic sectional curvature $4c$ is called a complex space form, which denoted by $M_n(c)$. The complete and simply connected complex space form is a complex projective space $P_n(\mathbb{C})$, a complex Euclidean space $\mathbb{C}$, or a complex hyperbolic space $H_n(\mathbb{C})$, according as $c > 0, c = 0, c < 0$.

The induced almost contact metric structure of a real hypersurface M in $M_n(c)$ is denoted by $(\phi, \xi, \eta, <, >)$.

Typical examples of real hypersurfaces in $P_n(\mathbb{C})$ are homogeneous one. R.Takagi ([18]) classified all homogeneous real hypersurfaces in $P_n(\mathbb{C})$ into six types. Namely he proved

Theorem A ([18]). *Every homogeneous real hypersurfaces in $P_n(\mathbb{C})$ is locally congruent to one of the following;*

(A_1) *a tube over a hyperplane $P_{n-1}(\mathbb{C})$,*
(A_2) *a tube over a totally geodesic $P_k(\mathbb{C})(1 \leq k \leq n-2)$,*
(B) *a tube over a complex quadric Q_{n-1},*
(C) *a tube over a $P_1(\mathbb{C}) \times P_{(n-1)/2}(\mathbb{C})$ and n is odd,*
(D) *a tube over a complex Grassmann $G_{2,5}(\mathbb{C})$ and $n = 9$,*
(E) *a tube over a Hermitian symmetric space $SO(10)/U(5)$ and $n = 15$.*

On the other hand, J.Berndt([2]) classified all real hypersurfaces in $H_n(\mathbb{C})$ with constant principal curvatures under the condition such that ξ is principal. Namely he proved

Theorem B ([2]). *Let M be real hypersurfaces in $H_n(\mathbb{C})$. Then M has constant principal curvature and ξ is principal if and only if M is locally congruent to one of the following;*

(A_0) *a horosphere,*
(A_1) *a geodesic hypershere or a tube over a hyperplane $H_{n-1}(\mathbb{C})$,*
(A_2) *a tube over a totally geodesic $H_k(\mathbb{C})(1 \leq k \leq n-2)$,*
(B) *a tube over a totally real hyperbolic space $H_n(\mathbb{R})$.*

Let M be a real hypersurface of type A_1 or A_2 in $P_n(\mathbb{C})$ or that of A_0, A_1 or A_2 in $H_n(\mathbb{C})$. Then M is said to be of type A for simplicity.

Moveover, M.Kimura and S.Maeda([9]) and S.S.Ahn, S.B.Lee and Y.J.Suh ([1]) found non-homogeneous real hypersurfaces in $P_n(\mathbb{C})$ and $H_n(\mathbb{C})$, respectively which is called ruled real hypersurfaces.

The purpose of this paper is to give some characterizations of real hypersurfaces in $M_n(c), c \neq 0$. In section 1 we investigate M by using the action ϕ on the curvature tensor R. The action of the derivation $R(X,Y)$ on the algebra of tensor fields of a real hypersurfaces on $M_n(c), c \neq 0$ have been studied by many authors([4],[5],[6],[9]~[11] and so on). In particular,we investigate M by using the action $R(X,Y)$ on the tensor field ϕ in section 2.

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1. Preliminaries

Let M be a real hypersurface of $M_n(c), c \neq 0$. In a neighborhood of each point, we choose a unite normal vector field N in $M_n(c)$. The Levi-Civita connection D in $M_n(c)$ and ∇ in M are related by the following formulas for any $X, Y \in \mathfrak{X}(M)$, where $\mathfrak{X}(M)$ is the set of all vector fields on M:

$$(1.1) \qquad \begin{aligned} D_X Y &= \nabla_X Y + < AX, Y > N, \\ D_X N &= -AX, \end{aligned}$$

where $<,>$ denotes the Riemannian metric of M induced from the metric g on $M_n(c)$ and A is the shape operator of M. An eigenvector field X of the shape operator A is called a principal curvature vector field. Also an eigenvalue λ of A called a principal curvature. In what follows, we denote V_λ the eigenspace of A with eigenvalue λ.

It is known that M has an almost contact metric structure induced from the complex structure J on $M_n(c)$, i.e., we define a tensor ϕ of type (1,1), a vector field ξ and a 1-form η on M by the following,

$$(1.2) \qquad < \phi X, Y >= g(JX, Y), \quad < \xi, X >= \eta(X) = g(JX, N).$$

Then we have

$$(1.3) \qquad \phi^2 X = -X + \eta(X)\xi, \quad < \xi, \xi >= 1, \quad \phi\xi = 0.$$

From (1.1), we have easily

$$(1.4) \qquad (\nabla_X \phi)Y = \eta(Y)AX - < AX, Y > \xi,$$

$$(1.5) \qquad \nabla_X \xi = \phi AX.$$

Let R be the curvature tensor of M. Since the curvature tensor of $M_n(c)$ has a nice form, we have the following Gauss and Codazzi equations:

$$(1.6) \qquad \begin{aligned} R(X,Y)Z =& c(< Y, Z > X - < X, Z > Y \\ & + < \phi Y, Z > \phi X - < \phi X, Z > \phi Y - 2 < \phi X, Y > \phi Z) \\ & + < AY, Z > AX - < AX, Z > AY, \end{aligned}$$

$$(\nabla_X A)Y - (\nabla_Y A)X = \eta(X)\phi Y - \eta(Y)\phi X - 2 < \phi X, Y > \xi. \tag{1.7}$$

2. THE ACTION ϕ ON THE CURVATURE TENSOR OF M IN $M_n(c), c \neq 0$.

First of all, for a ruled real hypersurface in $M_n(c), c \neq 0$, it is known that following Lemma

Lemma 2.1 ([1],[9]). *Let M be a real hypersurface in $M_n(c), c \neq 0$. The shape operator A of M is written by*

$$\begin{aligned} &A\xi = \alpha\xi + \beta W \quad (\beta \neq 0) \\ &AW = \beta\xi \\ &AX = 0 \quad (\textit{for any } X \perp \xi, W) \end{aligned} \tag{2.1}$$

where W is a unit vector field orthogonal to ξ, $\alpha, \beta \in C^\infty(M)$ ($C^\infty(M)$ is the set of all smooth functions on M) if and only if M is locally congruent to ruled real hypersurfaces.

Let M be a real hypersurface in $M_n(c)(c \neq 0, n \geq 3)$ and T_0 the distribution defined by $T_0(x) = \{X(x) \in T_xM | X(x) \perp \xi(x)\}$ for $x \in M$.
Using (1.3) and (1.6), we get

$$\begin{aligned} (\phi R)(X,Y)Z &= \phi R(X,Y)Z - R(X,Y)\phi Z - R(\phi X, Y)Z - R(X, \phi Y)Z \\ &= < AY, Z > (\phi A - A\phi)X - < AX, Z > (\phi A - A\phi)Y \\ &\quad + < (\phi A - A\phi)Y, Z > AX - < (\phi A - A\phi)X, Z > AY, \end{aligned} \tag{2.2}$$

for any $X, Y, Z \in T_0$.

The purpose of this section is to prove the following

Theorem 1. *Let M be a real hypersurface in $M_n(c), (c \neq 0, n \geq 3)$. Suppose*

$$(\phi R)(X,Y)Z = 0, \tag{2.3}$$

for any $X, Y, Z \in T_0$, then M is locally congruent to be of type A or ruled real hypersurfaces.

Let us prove Theorem 1.

First we assume that the structure vector field ξ is not principal. Then we can put $A\xi = \alpha\xi + \beta W$, where $W \in T_0, \|W\| = 1$ and $\alpha, \beta (\neq 0) \in C^\infty(M)$. From (1.2),(1.3) and (2.2), the equation (2.3) shows

$$< (\phi R)(\phi W, X)W - (\phi R)(W, X)\phi W, \xi >= \beta < \phi X, A\phi W >= 0,$$

for any $X \in T_0$. Therefore, since (1.2) and $\beta \neq 0$, we obtain $< \phi A \phi W, X >= 0$. Together with $< \phi A\phi W, \xi >= 0$, we find $\phi A \phi W = 0$. Applying ϕ to this equality and using (1.3), we get

$$A\phi W = 0. \tag{2.4}$$

Again putting $X = \phi W$ and $Z = W$ in (2.3), from (1.3),(2.1) and (2.4) we get

$$< AY, W > AW - < AW, W > AY = 0. \tag{2.5}$$

Taking the inner product with ξ, by $\beta \neq 0$ we have

$$< AY, W > = < AW, W >< Y, W > .$$

Thus we observe $< AW, X > = 0$ for any $X \in T_0, X \perp W$. From this equation we can write

$$AW = \beta\xi + \gamma W, \tag{2.6}$$

where $\gamma = < AW, W > \in C^\infty(M)$.

Putting $Y = W$ and $Z = X$ in (2.3) and taking the inner product with ξ, together with (1.2),(1.3),(2.2),(2.4) and $\beta \neq 0$, we observe

$$< A\phi X, X > = 0.$$

Replacing X with $Y + Z$, we obtain

$$< A\phi Y - \phi AY, Z > = 0, \tag{2.7}$$

for any $Y, Z \in T_0$. Again replacing Y with ϕY in above equation, we get

$$< AY + \phi A\phi Y, Z > = 0. \tag{2.8}$$

Using (2.2) and (2.7), for any $X, Y, Z \in T_0$ and $X, Y, Z \perp W$, we rewrite (2.3) as

$$< AY, Z > (\phi A - A\phi)X - < AX, Z > (\phi A - A\phi)Y = 0.$$

Putting $X = \phi W$ in above equation, by (2.4) and $\beta \neq 0$, we obtain

$$< AY, Z > = 0.$$

Above equation implies

$$\gamma = 0 \ \ \textit{and} \ \ AY = 0, \tag{2.9}$$

for any $Y \perp \xi, W$. Thus because of (2.6),(2.9) and Lemma 2.1, M is locally congruent to a ruled real hypersurface.

Next we assume that the structure vector field ξ is principal with corresponding principal curvature α. Then the following three propositions are known.

Proposition A ([6],[12]). *If ξ is a principal curvature vector field, then the corresponding principal curvature α is locally constant.*

Proposition B ([12]). *If ξ is a principal curvature vector field with corresponding principal curvature α. Suppose $X_\lambda \in V_{\lambda(\neq\alpha)}$ and $\alpha \neq 2\lambda$, then $\phi X_\lambda \in V_{\mu(=(\alpha\lambda+2c)/(2\lambda-\alpha))}$.*

Proposition C ([12]). *If ξ is a principal curvature vector field with corresponding principal curvature α. Suppose $\phi X_\lambda \in V_\lambda$ for any vector fields $X_\lambda \in V_{\lambda(\neq\alpha)}$. Then ϕ and A are commutative.*

Furthermore, M.Okumura([14]) and S.Montiel and A.Romero([13]) proved

Theorem C ([13],[14]). *Let M be a real hypersurface in $M_n(c), c \neq 0$. Then ϕ and A are commutative if and only if M is locally congruent to be of type A real hypersurfaces.*

We can consider the following two cases.

First we suppose that $\alpha^2 - 4c = 0$. In this case M is of type A_0 in Theorem B (for detail see ([2])).

Second we suppose that $\alpha^2 - 4c \neq 0$.

Lemma 2.2. *Let M be a real hypersurface in $M_n(c)(c \neq 0, n \geq 3)$ satisfying (2.3) and ξ is a principal vector field with corresponding principal curvature α. Suppose that $\alpha^2 - 4c \neq 0$. Then there exists a principal curvature vector field $X_\lambda \in V_{\lambda(\neq\alpha)}$ such that $\phi X_\lambda \in V_\lambda$.*

Proof. Take an orthonormal frame field $\{\xi, X_{\lambda_i}, \phi X_{\lambda_i} (i = 1, \ldots, n-1)\}$ consisting of principal curvatures by α, λ_i, μ_i, respectively because of Proposition B. Such a frame field is said to be a *local CR-frame field* on M. Suppose that $\lambda_i \neq \mu_i$ for all $i = 1, \ldots, n-1$. In (2.3) setting $Y = \phi X_{\lambda_i}$ and $Z = X_{\lambda_i}$, using (1.2),(1.3),(2.2) and Proposition B we have

$$AX = \lambda_i < X_{\lambda_i}, X > X_{\lambda_i} + \mu_i < \phi X_{\lambda_i}, X > \phi X_{\lambda_i}.$$

It follows that $\lambda_i = \mu_i = 0$, which is a contradiction. □

Lemma 2.3. *Under the assumptions of Lemma 2.2, the principal curvature of ϕX_{λ_i} is equal to that of $X_{\lambda_i} (i = 1, \ldots, n-1)$.*

Proof. There exists $X_\nu \in \{X_{\lambda_i} (i = 1, \ldots, n-1)\}$ such that $\nu^2 = 1$ because of Lemma 2.2 and Proposition B. Then from (2.2) and Proposition B, we get

$$(\phi R)(X_{\lambda_i}, X_\nu)Z = \nu(\lambda_i - \mu_i)(- < X_\nu, Z > \phi X_{\lambda_i} + < \phi X_{\lambda_i}, Z > X_\nu) = 0,$$

for any $Z \in T_0$. It follows that $\lambda_i = \mu_i$. □

Hence from Proposition C and Theorem C, M is locally congruent to be of type A_1 or A_2 in Theorem A. Conversely by Lemma 2.1, the shape operator A of ruled real hypersurfaces M in $M_n(c)$ is written by (2.1). From this and Theorem C, it is easily checked that ruled real hypersurfaces and real hypersurfaces of type A satisfy condition (2.3).
It completes the proof of Theorem 1.

Corollary 2. *Let M be a real hypersurface in $M_n(c), (c \neq 0, n \geq 3)$. If $(\phi R)(X, Y)\ Z = 0$ for one of $X, Y, Z \in \mathfrak{X}(M)$ and the rest in T_0, then M is locally congruent to be of type A real hypersurfaces.*

In fact, by (2.1) it is easily checked that ruled real hypersurfaces don't satisfy above condition.

Remark 2.1. *Y.Maeda([12]) classified real hypersurfaces in $P_n(\mathbb{C})(n \geq 3)$ which satisfy the condition $\phi R \equiv 0$ and ξ is a principal curvature vector field.*

3. Curvature operator of M in $M_n(c), c \neq 0$.

In this section, we consider the action of the derivation $R(X,Y)$ on the algebras of tensor fields of a real hypersurface in $M_n(c), c \neq 0$. We recall that if T is a tensor field of type (r,s), then $R(X,Y) \cdot T = \nabla_X \nabla_Y T - \nabla_Y \nabla_X T - \nabla_{[X,Y]} T$ for $X, Y \in \mathfrak{X}(M)$.

For brevity of notation, we denote by RT the tensor of type $(r, s+2)$ defined by

$$(RT)(X_1, X_2, \ldots, X_s, X, Y) = (R(X,Y) \cdot T)(X_1, X_2, \ldots, X_s).$$

First of all we shall study real hypersurfaces M in $M_n(c), c \neq 0$ which satisfy $\mathfrak{S}(R\phi)(X,Y)Z = 0$ for any $X, Y, Z \in \mathfrak{X}(M)$, where $\mathfrak{S}$ denotes the cyclic sum with respect to X, Y and Z.

Using (1.2),(1.3) and (1.6), we get

$$\begin{aligned} (R\phi)(X,Y)Z &= R(X,Y)\phi Z - \phi R(X,Y)Z \\ &= c(< \phi X, Z > \eta(Y)\xi - < \phi Y, Z > \eta(X)\xi + \eta(X)\eta(Z)\phi Y \\ &\quad - \eta(Y)\eta(Z)\phi X) - < AY, Z > \phi AX + < AX, Z > \phi AY \\ &\quad + < AY, \phi Z > AX - < AX, \phi Z > AY, \end{aligned} \tag{3.1}$$

for any $X, Y, Z \in \mathfrak{X}(M)$.

Furthermore by (3.1) we have

$$\begin{aligned} \mathfrak{S}(R\phi)(X,Y)Z &= (R\phi)(X,Y)Z + (R\phi)(Y,Z)X + (R\phi)(Z,X)Y \\ &= 2c(< X, \phi Y > \eta(Z) + < Y, \phi Z > \eta(X) + < Z, \phi X > \eta(Y)) \\ &\quad + < (A\phi + \phi A)X, Y > AZ + < (A\phi + \phi A)Y, Z > AX \\ &\quad + < (A\phi + \phi A)Z, X > AY. \end{aligned} \tag{3.2}$$

Lemma 3.1. *Let M be a real hypersurface in $M_n(c), c \neq 0$. If*

$$\mathfrak{S}(R\phi)(X,Y)Z = 0, \tag{3.3}$$

for any $X, Y, Z \in \mathfrak{X}(M)$, then the structure vector field ξ is principal.

Proof. We assume that the structure vector field ξ is not principal. Then we can put $A\xi = \alpha\xi + \beta W$, where $W \in T_0, \|W\| = 1$ and $\alpha, \beta(\neq 0) \in C^\infty(M)$.

Putting $X = \xi, Y = W$ and $Z = \phi W$ in (3.3), by (1.2),(1.3) and (3.2), we have

$$-2c\xi + < (A\phi + \phi A)\phi W, W > A\xi + \beta AW = 0. \tag{3.4}$$

From (3.4), we can write

$$AW = \beta\xi + \gamma W, \tag{3.5}$$

where $\gamma =< AW, W >\in C^{\infty}(M)$. From (3.4) and (3.5), we obtain $< A\phi W, \phi W > = 0$ and therefore

$$\beta^2 = \alpha\gamma - 2c. \tag{3.6}$$

On the other hand, let $e_1, \ldots, e_{2n-1}$ be a local orthonormal flame field on M. Let $X = e_i$ and $Y = \phi e_i$ in (3.3). Then contraction yields that

$$-4c(n-1)\eta(Z)\xi + 2A^2Z - 2A\phi A\phi Z + (1 - \alpha^2 trA)AZ = 0. \tag{3.7}$$

Putting $Z = \xi$ in (3.7), we find

$$-4c(n-1) + 2\beta^2 - 2\alpha\gamma = 0,$$

together with (3.6), we get $-4cn = 0$, which is a contradiction. □

Suppose M satisfies (3.3). Then by Lemma 3.1, the structure vector field ξ is principal with corresponding principal curvature α.
If $\alpha^2 - 4c = 0$, then by the same discussion as in section 2, M is of type A_0 in Theorem B.
Hence we suppose that $\alpha^2 - 4c \neq 0$. Then putting $X = \xi$ and $Z = X_\lambda \in V_\lambda$ in (3.2), from (1.2) and (1.3), we find

$$2c + \alpha(\lambda + \mu) = 0.$$

This equation tells us that $\alpha \neq 0$. Together with Proposition B, we have

$$\lambda(\alpha\lambda + 2c) = 0.$$

Thus M has three distinct constant principal curvatures

$$0,\ \alpha,\ -\frac{2c}{\alpha}. \tag{3.8}$$

On the other hand, M.Kimura([8]) proved

Theorem D ([8]). *Let M be a real hypersurface in $P_n(\mathbb{C})$. Then M has constant principal curvature and ξ is principal if and only if M is locally congruent to a homogeneous real hypersurface.*

Together with Theorem B, M is of type $A_1 \sim E$ in Theorem A or type $A_0 \sim B$ in Theorem B.

Conversely it is easily checked that these real hypersurfaces don't satisfy (3.8) and therefore also condition (3.3). Thus we obtain following Proposition

Proposition 3. *Let M be a real hypersurface in $M_n(c), c \neq 0$. Then M cannot satisfies $\mathfrak{S}(R\phi)(X,Y)Z = 0$ for any $X, Y, Z \in \mathfrak{X}(M)$.*

Remark 3.1. *R.Takagi([19]) and J.Saito([16]) classified real hypersurfaces in $P_n(\mathbb{C})$,$n \geq 3$ and $H_n(\mathbb{C}), n \geq 3$, with three distinct principal curvature, respectively.*

Corollary 4. *Let M be a real hypersurface in $M_n(c), c \neq 0$. Then M cannot satisfies $R\phi \equiv 0$.*

Next, we shall study real hypersurfaces M in $M_n(c), c \neq 0$ which satisfy $(R\phi)(X, Y)Z = 0$ for any $X, Y, Z \in T_0$.
Using (3.1), we get

$$(3.9) \qquad (R\phi)(X,Y)Z = -< AY, Z > \phi AX + < AX, Z > \phi AY + < AY, \phi Z > AX - < AX, \phi Z > AY,$$

for any $X, Y, Z \in T_0$. Assume

$$(3.10) \qquad (R\phi)(X,Y)Z = 0.$$

First we assume that the structure vector field ξ is not principal. Then we can put $A\xi = \alpha\xi + \beta W$, where $W \in T_0, \|W\| = 1$ and $\alpha, \beta(\neq 0) \in C^\infty(M)$.

Taking the inner product of (3.9) with ξ, from (1.2),(1.3) and (3.10), we have

$$(3.11) \qquad << Y, W > \phi AX - < X, W > \phi AY, Z >= 0.$$

Putting $X = W$ and $Z = \phi W$ in (3.11), we observe

$$(3.12) \qquad AW = \beta\xi + \gamma W,$$

where $\gamma =< AW, W >\in C^\infty(M)$. The equation (3.11) also implies

$$< Y, W > AX =< X, W > AY.$$

This equation tells us that

$$(3.13) \qquad AX = 0,$$

for all $X \in T_0, X \perp W$. In this case, three equations (3.12),(3.13) and $A\xi = \alpha\xi + \beta W$ imply the type number of M is smaller than 3, where the *type number* of M is defined as the rank of A. For the probrem with respect to the type number t, Y.J.Suh([17]) and M.Ortega and J.D.Perez([15]) showed that

Theorem E ([15],[17]). *Let M be a real hypersurface in $M_n(c)(c \neq 0, n \geq 3)$ satisfying $t(p) \leq 2$ for any point p in M. Then M is a ruled real hypersurface.*

Thus in this case, M is locally congruent to a ruled real hypersurface.

Second we assume that the structure vector field ξ is principal with corresponding principal curvature α. If $\alpha^2 - 4c = 0$, then M is of type A_0 in Theorem B. Thus we suppose that $\alpha^2 - 4c \neq 0$. Take a local CR-frame field $\{\xi, X_{\lambda_i}, \phi X_{\lambda_i} (i = 1, \ldots, n-1)\}$ consisting of principal curvatures by α, λ_i, μ_i, respectively. Putting $X = Z = X_{\lambda_j}$ and $Y = X_{\lambda_j}$ in (3.10), by (1.2),(1.3) and (3.9), we obtain $\lambda_i = 0$ for some i. Then from Proposition B, we get $\mu_i = -2c/\alpha$. Furthermore putting $X = Z = \phi X_{\lambda_j}$ and $Y = \phi X_{\lambda_j}$ in (3.10), we have $\mu_j = 0$ for any $j \neq i$. Thus we get $\lambda_j = -2c/\alpha$ for any $j \neq i$. Therefore M is of type $A_1 \sim E$ in Theorem A or type $A_0 \sim B$ in Theorem B.

Conversely it is trivial that these real hypersurfaces don't satisfy (3.8) and from (2.1) ruled real hypersurfaces satisfy (3.10).

Consequently, we obtain the following theorem.

Theorem 5. *Let M be a real hypersurface in $M_n(c), c \neq 0, n \geq 3$. Suppose $(R\phi)(X,Y)Z = 0$ for any $X, Y, Z \in T_0$, then M is locally congruent to a ruled real hypersurface.*

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Hiroyuki Kurihara
Department of Mathematics and Informatics
Graduate School of Science and Technology
Chiba University, Chiba-Shi, 263-8522, Japan
e-mail address: hkurihar@math.s.chiba-u.ac.jp

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