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CORRECTION TO "ON RIGHT P.P. RINGS"

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Lemma 2 in [2] is false. In fact, let R be a right Ore domain which is not left Ore (see e.g., [3, Example 1.3.7]). Let Q denote the skew field of right fractions of R. (Obviously Q is a maximal right quotient ring of R.) Then R is a right Utumi ring, but Q is not a left quotient ring of R. Lemma 2 was used in the proof of Theorem 3. We present a revised version of Theorem 3. The other results of the paper remain true without change.

Theorem 3′. *Let R be a right Utumi ring with maximal right quotient ring Q. Then the following are equivalent:*

1) *R is a Baer ring.*

2) *For every $e \in E(Q)$ there exists $f \in E(R)$ such that $Qe \cap R = Rf$.*

If moreover R is normal, then 2) is equivalent to the following condition:

2′) $E(Q) = E(R)$.

Proof. 1) ⇒ 2). We claim that $Qe = l_Q(R \cap (1-e)Q)$ for every $e \in E(Q)$. Indeed, let a be an arbitrary element of $l_Q(R \cap (1-e)Q)$. If we take an essential right ideal I of R such that $(1-e)I \subseteq R$, then $a(1-e)I = 0$. Hence $a = ae$, which shows the above equality. Now for any non-zero $e \in E(Q)$, $R \cap (1-e)Q$ is a non-essential right ideal of R. Hence by hypothesis $l_R(R \cap (1-e)Q) = Rf$ for some non-zero $f \in E(R)$. Combining this with what we have shown above, we have $Qe \cap R = l_R(R \cap (1-e)R) = Rf$.

2) ⇒ 1). Since every annihilator right ideal of Q is a closed right ideal [1, Proposition 8.5 (3)], it is generated by an idempotent [1, Theorem 8.4 (3)]. Therefore Q is a Baer ring. Hence for any non-empty subset X of R there exists $e \in E(Q)$ such that $l_Q(X) = Qe$. By hypothesis there exists $f \in E(R)$ such that $Qe \cap R = Rf$, and so $l_R(X) = l_Q(X) \cap R = Qe \cap R = Rf$. This proves that R is a Baer ring.

Thus we have proved the equivalence of 1) and 2). Trivially 2′)

implies 2). Now suppose R is normal and 2) holds. Since the centralizer of R in Q coincides with the center of Q, every idempotent of R is central in Q. Let $e \in E(Q)$ and take $f \in E(R)$ such that $Qe \cap R = Rf$. Clearly we have $f = fe$ and $Q(1-f)e \cap R = 0$. Since R is right Utumi, the latter implies $(1-f)e = 0$. Therefore we have $f = fe = e$. This completes the proof.

References

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