

ON OPERATORS RELATED TO p -STABLE MEASURES IN BANACH SPACES

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1. Introduction and notations. Let E be a Banach space with the dual space E' and p be a real number such that $0 < p \leq 2$. We say that E is of stable type p if for each sequence $\{x_n\}$ in E , $\sum_n \|x_n\|^p < \infty$ implies the series $\sum_n x_n \theta_n^{\rho_i}$ converges almost surely (a.s.); and E is of stable cotype p if for each sequence $\{x_n\}$ in E such that the series $\sum_n x_n \theta_n^{\rho_i}$ converges a.s., there holds $\sum_n \|x_n\|^p < \infty$. Here $\{\theta_n^{\rho_i}\}$ denotes the sequence of independent identically distributed real random variables with the characteristic function (ch. f.) $\exp(-|t|^p)$, $t \in R$. Let us denote by $L(E, F)$ the set of all continuous linear operators from E into a Banach space F . For an operator T in $L(E, F)$, we say that T is S_p -factorizable (resp. SQ_p -factorizable) if it is factorizable through a subspace (resp. a subspace of a quotient) of some L_p . Let us recall that a sequence $\{x_n\}$ in E is weakly p -summable if $\sum_n |\langle x_n, x' \rangle|^p < \infty$ for all $x' \in E'$. For an operator T in $L(E, F)$, we say that T is of stable type p if for each sequence $\{x_n\}$ in E , $\sum_n \|x_n\|^p < \infty$ implies the series $\sum_n T(x_n) \theta_n^{\rho_i}$ converges a.s. in F ; T is γ_p -summing if for each weakly p -summable sequence $\{x_n\}$ in E , the series $\sum_n T(x_n) \theta_n^{\rho_i}$ converges a.s. in F ; and T is p -summing if for each weakly p -summable sequence $\{x_n\}$ in E , $\sum_n \|T(x_n)\|^p < \infty$. We denote by $\Pi_{\gamma_p}(E, F)$ (resp. $\Pi_p(E, F)$), the set of all γ_p -summing operators (resp. p -summing operators) from E into F . Let X be a Banach space and $1 < p \leq 2$. In the following we shall write with $X' \subset L_p$ if X' is linearly isometric to a subspace of L_p . For such a space X , we say that an operator T in $L(X, E)$ is γ_p -Radonifying if $\exp(-\|T'(x')\|^p)$, $x' \in E'$ is the ch. f. of a Radon measure on E , where T' denotes the adjoint of T . The set of all γ_p -Radonifying operators from X into E will be denoted by $R_p(X, E)$. It is known that a symmetric Radon probability measure μ on E is p -stable if and only if there exist a Banach space X with $X' \subset L_p$ and an operator T in $R_p(X, E)$ such that $\exp(-\|T'(x')\|^p)$, $x' \in E'$ is the ch. f. of μ (see [4, Prop. 3]).

Then the main results of this paper are the following :

(1) Let $1 < p \leq 2$. Then the following properties of a Banach space E are equivalent.

(1a) For each Banach space X with $X' \subset l_p$, we have

$$R_\rho(X, E) \subset \Pi_\rho(X, E).$$

(1b) If $\{x_n\}$ and $\{y_n\}$ are two sequences in E such that

$$\sum_n |\langle y_n, x' \rangle|^p \leq \sum_n |\langle x_n, x' \rangle|^p \text{ for all } x' \in E',$$

and the series $\sum_n x_n \theta_n^{(p)}$ converges a.s. in E , then $\sum_n \|y_n\|^p < \infty$.

(2) Let $1 < p \leq 2$ and suppose that a Banach space E has the property (1a). Then for each Banach space F , every operator of stable type p from F into E is S_ρ -factorizable. In particular, if E is of type (B_ρ) , $1 < p < 2$, in the sense of [1], then every operator of stable type p from F into E is S_ρ -factorizable; and if E is of stable cotype 2 and F is of stable type 2, then every continuous linear operator from F into E is Hilbertian.

(3) Let $1 < p \leq 2$ and let E be a Banach space satisfying the condition $R_\rho(l_{p'}, E) \subset \Pi_\rho(l_{p'}, E)$, where $1/p + 1/p' = 1$. Then for each reflexive Banach space F , every operator of stable type p from F into E is SQ_ρ -factorizable. In particular, if E belongs to the class $V_\rho(i)$ in the sense of [5], then every operator of stable type p , $1 < p < 2$, from a reflexive Banach space F into E is SQ_ρ -factorizable.

(4) Let E be a Banach space and $1 < p < 2$. Then E is of stable type p if and only if for each Banach space F , every p -summing operator from F into E is γ_ρ -summing; and E is of finite dimension if and only if every operator of stable type p from l_ρ into E is γ_ρ -Radonifying.

(5) Let $1 < p < \infty$ and let T be a continuous linear operator from a Banach space F into a Banach space E . Then T is S_ρ -factorizable if and only if for each Banach space X with $X' \subset l_\rho$ and each $S \in L(X, F)$ with $\sum_n \|S(f_n)\|^p < \infty$, TS is p -summing. Here $f_n = J'(e_n)$, where J is an isometric imbedding from X' into l_ρ and e_n is the n -th unit vector of l_ρ . Furthermore, if we assume that F is reflexive, then T is SQ_ρ -factorizable if and only if for each $S \in L(l_{p'}, F)$ with $\sum_n \|S(e_n)\|^p < \infty$, TS is p -summing.

Remark. In Section 2, the equivalence of (1a) and (1b) is proved, and some examples of Banach spaces E having the property (1a) are given. For the case $p = 2$, it is well-known that E has the property (1a) if and only if it is of stable cotype 2, and on the other hand, every Banach space is of stable cotype p with $p < 2$ (see [6]). We also prove that a Banach space E belongs to the class $V_\rho(i)$, $1 < p \leq 2$, if and only if $R_\rho(l_{p'}, E) \subset \Pi_{\gamma_\rho}(l_{p'}, E)$. It is easy to see that every Banach space belongs to the class $V_2(i)$; and a

Banach space of stable type p , $1 < p < 2$, belongs to the class $V_p(i)$ if and only if it is of SQ_p type in the sense of [2], i.e. it is isomorphic to a subspace of a quotient of some L_p . This extends a result of [5].

In Section 3, (4) is proved. We also prove that every r -summing operator is of stable type p , where $0 < p \leq 2$ and $0 < r < \infty$.

In Section 4, the results (2), (3) and (5) are proved. We note that (2) extends the results of [1], [5] and [7]; (3) extends the results of [4] and [5]; and (5) is an analogue of the results of [2] and [7]. Let us recall that a Banach space E is of S_p type if it is isomorphic to a subspace of some L_p . As a consequence of (2), we obtain that a Banach space E is of stable type p and of S_p type, $1 < p < 2$, if and only if for each Banach space X with $X' \subset l_p$, there holds $R_p(X, E) = \Pi_p(X, E)$. This extends a result of [1].

The paper is motivated from the works of [1], [2], [4], [5] and [7].

2. Banach spaces having the property (1a). We first prove the equivalence of (1a) and (1b) mentioned in Section 1.

Theorem 1. *Let $1 < p \leq 2$. Then the following properties of a Banach space E are equivalent.*

(1) *For each Banach space X with $X' \subset l_p$, we have*

$$R_p(X, E) \subset \Pi_p(X, E).$$

(2) *If $\{x_n\}$ and $\{y_n\}$ are two sequences in E such that*

$$\sum_n |\langle y_n, x' \rangle|^p \leq \sum_n |\langle x_n, x' \rangle|^p \text{ for all } x' \in E',$$

and the series $\sum_n x_n \theta_n^{p'}$ converges a.s. in E , then $\sum_n \|y_n\|^p < \infty$.

Proof. For the case $p = 2$, the equivalence of (1) and (2) easily follows from the fact that E is of stable cotype 2 if and only if $R_2(l_2, E) \subset \Pi_2(l_2, E)$ (see [6]). Hence we may prove only the case $1 < p < 2$.

(1) \Rightarrow (2) : Let us assume that (1) is satisfied and let $\{x_n\}$ and $\{y_n\}$ be two sequences in E such that

$$\sum_n |\langle y_n, x' \rangle|^p \leq \sum_n |\langle x_n, x' \rangle|^p \text{ for all } x' \in E',$$

and the series $\sum_n x_n \theta_n^{p'}$ converges a.s. in E . Since every Banach space is of stable cotype p with $p < 2$, we have $\sum_n \|x_n\|^p < \infty$. Then there is a continuous linear operator S from l_p into E such that $S(e_n) = x_n$ for all n , where e_n is the n -th unit vector of l_p ($1/p + 1/p' = 1$). Evidently, S is

γ_p -Radonifying and there holds $S'(x') = (\langle x_n, x' \rangle)_{n=1}^\infty$ for all $x' \in E'$. Let $X = Y'$, where $Y = S'(E')$. Obviously, X is a Banach space whose dual X' is a closed subspace of l_p , and the operator S can be factorized as follows;

$$l_p \xrightarrow{J'} X \xrightarrow{T} E,$$

where J denotes the natural injection from X' into l_p . Then T is clearly γ_p -Radonifying since S is so. From the assumption (1) it follows that T is p -summing. We note here that $S'(E')$ is a dense subspace of X' . Define the operator $V: X' \rightarrow l_p$ by

$$V: (\langle x_n, x' \rangle)_{n=1}^\infty \rightarrow (\langle y_n, x' \rangle)_{n=1}^\infty \text{ for all } x' \in E'.$$

Then V is a continuous linear operator and there holds $TV'(e_n) = y_n$ for all n . Since TV' is p -summing, we have $\sum_n \|y_n\| < \infty$.

(2) \Leftrightarrow (1): Let us assume that (2) is satisfied and let X be a Banach space with $X' \subset l_p$ and $T \in R_p(X, E)$. To prove that T is p -summing, let $\{x_n\}$ be an weakly p -summable sequence in X . Then there is a continuous linear operator S from l_p into X such that $S(e_n) = x_n$ for all n . Evidently, we have

$$\|S'T'(x')\|^p = \sum_n |\langle S'T'(x'), e_n \rangle|^p = \sum_n |\langle T(x_n), x' \rangle|^p, \quad x' \in E',$$

and

$$\|T'(x')\|^p = \|JT'(x')\|^p = \sum_n |\langle TJ'(e_n), x' \rangle|^p, \quad x' \in E',$$

where J is an isometric imbedding from X' into l_p . Hence

$$\sum_n |\langle T(x_n), x' \rangle|^p \leq \|S'\|^p \sum_n |\langle TJ'(e_n), x' \rangle|^p, \quad x' \in E'.$$

Since $TJ': l_p \rightarrow E$ is clearly γ_p -Radonifying, the series $\sum_n TJ'(e_n) \theta_n^{(p)}$ converges a.s. in E . From the assumption (2) it follows that $\sum_n \|T(x_n)\|^p < \infty$ proving $T \in \Pi_p(X, E)$. Thus the proof is completed.

Now we give some examples of Banach spaces E having the property (1a). Let us recall that for $p = 2$, E has the property (1a) if and only if it is of stable cotype 2.

Following [1], we say that a Banach space E is of type (B_p) , $1 < p \leq 2$, if for each Banach space X with $X' \subset l_p$, there holds $R_p(X, E) = \Pi_{\tau_p}(X, E)$. Of course every Banach space is of type (B_2) (see [1]). For $1 < p < 2$, every Banach space of type (B_p) has the property (1a) since γ_p -summing operators are p -summing.

Following [8], we say that a Banach space E is of type (S) if there exists an S -topology on E' . It is well-known that if E has the approximation property, then E is of type (S) if and only if it is isomorphic to a subspace of some L_0 . Obviously, every Banach space of type (S) has the property (1a) for $1 < p \leq 2$.

Following [12], we say that a Banach space E belongs to the class (V_p) , $1 \leq p \leq 2$, if for each p -stable Radon probability measure μ and each stable cylindrical measure ν on E , the inequality

$$|1 - \hat{\nu}(x')| \leq |1 - \hat{\mu}(x')|, \quad x' \in E',$$

implies ν is a Radon measure on E . Here $\hat{\mu}$ (resp. $\hat{\nu}$) denotes the ch. f. of μ (resp. ν). It is easy to see that every Banach space belonging to the class (V_p) has the property (1a) for $1 < p < 2$.

Finally, we give some examples of Banach space E satisfying the condition $R_p(l_{p'}, E) \subset \Pi_p(l_{p'}, E)$, $1 < p \leq 2$. Evidently, every Banach space having the property (1a) satisfies this condition, but the converse is not true. It is known that every Banach space of SQ_p type satisfies this condition (see [4]). Note that for $p = 2$, E satisfies this condition if and only if it is of stable cotype 2. In the following we give another example of Banach spaces E satisfying this condition.

Let E be a Banach space and $1 < p \leq 2$. Denote by $\Lambda_p(E', l_p)$ the set of all continuous linear operators T from E' into l_p such that $\exp(-\|T(x')\|^p)$, $x' \in E'$, is the ch. f. of a Radon measure on E . It is known that for an operator T in $L(E', l_p)$, $T \in \Lambda_p(E', l_p)$ if and only if there exists an operator S in $R_p(l_{p'}, E)$ such that $T = S'$ (see [3, Th. 5]). Following [5], we say that E belongs to the class $V_p(i)$ if $T \in \Lambda_p(E', l_p)$ implies $ST \in \Lambda_p(E', l_p)$ for all $S \in L(l_{p'}, l_p)$.

Proposition 1. *A Banach space E belongs to the class $V_p(i)$, $1 < p \leq 2$, if and only if the inclusion $R_p(l_{p'}, E) \subset \Pi_{\gamma_p}(l_{p'}, E)$ holds.*

Proof. Let us first assume that E belongs to the class $V_p(i)$ and let $T \in R_p(l_{p'}, E)$. In order to prove that T is γ_p -summing, take an weakly p -summable sequence $\{x_n\}$ in $l_{p'}$. Then there is an operator S in $L(l_{p'}, l_{p'})$ such that $S(e_n) = x_n$ for all n . By the assumption, $T' \in \Lambda_p(E', l_p)$ implies $S'T' \in \Lambda_p(E', l_p)$. But this means that TS is γ_p -Radonifying, and so the series $\sum_n TS(e_n) \theta_n^{(p)} = \sum_n T(x_n) \theta_n^{(p)}$ converges a. s. in E (see [1] or [4]). Hence we get $T \in \Pi_{\gamma_p}(l_{p'}, E)$. On the other hand, suppose that the inclusion

$R_p(l_{p'}, E) \subset \Pi_{\gamma_p}(l_{p'}, E)$ holds. Let $T \in \Lambda_p(E', l_p)$. Then there is an operator V in $R_p(l_{p'}, E)$ such that $T = V'$. By the assumption, V is γ_p -summing. Let S be any operator in $L(l_p, l_p)$. Since $\{S'(e_n)\}$ is an weakly p -summable sequence in l_p , the series $\sum_n VS'(e_n)\theta_n^{(p)}$ converges a. s. in E . But this means that VS' is γ_p -Radonifying, and so we get $ST = (VS')' \in \Lambda_p(E', l_p)$. Thus E belongs to the class $V_p(i)$, and the proof is completed.

Corollary 1. *Suppose that a Banach space E is of stable type p , $1 < p < 2$. Then E belongs to the class $V_p(i)$ if and only if it is of SQ_p type.*

Proof. The assertion follows from Proposition 1 and [4, Th. 3].

3. Operators of stable type p and γ_p -summing operators. Let us first remark that every operator factorizable through a Banach space of stable type p is always of stable type p . It is well known that for $2 \leq r < \infty$, L_r is of stable type 2, and in particular, it is of stable type p for all $p \in (0, 2]$.

Proposition 2. *For $0 < p \leq 2$ and $0 < r < \infty$, every r -summing operator is of stable type p .*

Proof. The assertion easily follows from the facts that every r -summing operator is s -summing for $r < s$, and every r -summing operator is factorizable through a subspace of some L_r (see [10]).

Remark. Every γ_p -summing operator, $0 < p < 2$, is always p -summing, but in general, the converse is not true. It is known that if a Banach space E is of stable type p , $0 < p \leq 2$, then for each Banach space F , every p -summing operator from F into E is γ_p -summing (see [1]). The following result shows that the converse is true for $1 < p < 2$.

Proposition 3. *Let $1 < p < 2$. Then the following properties of a Banach space E are equivalent.*

- (1) E is of stable type p .
- (2) $\Pi_p(l_{p'}, E) = \Pi_{\gamma_p}(l_{p'}, E)$.
- (3) For each Banach space F , we have $\Pi_p(F, E) = \Pi_{\gamma_p}(F, E)$.

Proof. Since every γ_p -summing operator from $l_{p'}$ into E is γ_p -Radonifying, the assertion follows from [4, Th. 2].

Remark. Proposition 3 becomes false in the case $p = 2$. In this case, one of the properties (2) and (3) is equivalent to the fact that E is of stable cotype 2 (see [1], [6]).

Finally, we investigate the relationship among operators of stable type p , γ_p -summing and γ_p -Radonifying operators.

Theorem 2. *Let $1 < p < 2$. Then the following properties of a Banach space E are equivalent.*

- (1) E is of finite dimension.
- (2) Every operator of stable type p from l_p into E is γ_p -summing.
- (3) Every operator of stable type p from l_p into E is γ_p -Radonifying.

Proof. Of course, we only have to prove (3) \Leftrightarrow (1). Let us assume that (3) is satisfied. Then E is of stable type p (see Prop. 2 and [4, Th. 2]). Let $\{x_n\}$ be an weakly p -summable sequence in E . Then there is an operator T in $L(l_p, E)$ such that $T(e_n) = x_n$ for all n . Since E is of stable type p , T is of stable type p . From the assumption (3) it follows that T is γ_p -Radonifying, and so the series $\sum_n T(e_n) \theta_n^{(p)} = \sum_n x_n \theta_n^{(p)}$ converges a.s. in E . Since every Banach space is of stable cotype p with $p < 2$ (see [6]), we have $\sum_n \|x_n\|^p < \infty$. But this means that the identity map on E is p -summing, and so E is a nuclear Banach space (see [10]). Thus E is of finite dimension, and the proof is completed.

4. S_p -factorizable operators and SQ_p -factorizable operators. In this section, we prove the results (2), (3) and (5) stated in Section 1. The following two propositions are analogues of the results of [2] and [7].

Proposition 4. *Let $1 < p < \infty$ and let T be a continuous linear operator from a Banach space F into a Banach space E . Then the following are equivalent.*

- (1) T is S_p -factorizable.
- (2) For each Banach space X with $X' \subset l_p$ and each $S \in L(X, E)$ with $\sum_n \|S(f_n)\|^p < \infty$, TS is p -summing. Here $f_n = J'(e_n)$, where J is an isometric imbedding from X' into l_p .

Proof. (1) \Leftrightarrow (2) easily follows from [11, Th. 3. 1]. On the other hand, let us assume that (2) is satisfied. In order to prove (1), we use the Maurey criterion [7] for the factorizability through a subspace of L_p . Let

$\{x_n\}$ and $\{y_n\}$ be two sequences in F such that

$$\sum_n |\langle y_n, x' \rangle|^p \leq \sum_n |\langle x_n, x' \rangle|^p \text{ for all } x' \in F'$$

and

$$\sum_n \|x_n\|^p < \infty.$$

Then by the same way as in the proof of Theorem 1, we can find an operator S in $L(X, F)$ such that $S(f_n) = x_n$ for all n , where X is a Banach space with $X' \subset l_p$ and $\{(\langle x_n, x' \rangle)_{n=1}^\infty; x' \in F'\}$ is a dense subspace of X' . Define the operator $V: X' \rightarrow l_p$ by

$$V: (\langle x_n, x' \rangle)_{n=1}^\infty \rightarrow (\langle y_n, x' \rangle)_{n=1}^\infty \text{ for all } x' \in F'.$$

Then V is a continuous linear operator and there holds $SV'(e_n) = y_n$ for all n . From the assumption (2) it follows that TS is p -summing, and so is TSV' . Thus we get

$$\sum_n \|T(y_n)\|^p = \sum_n \|TSV'(e_n)\|^p < \infty.$$

By the Maurey criterion [7], T is S_p -factorizable, and the proof is completed.

Proposition 5. *Let $1 < p < \infty$ and let T be a continuous linear operator from a reflexive Banach space F into a Banach space E . Then the following are equivalent.*

- (1) T is SQ_p -factorizable.
- (2) For each $S \in L(l_p, F)$ with $\sum_n \|S(e_n)\|^p < \infty$, TS is p -summing.

Proof. (1) \Rightarrow (2) easily follows from [11, Th. 3. 1]. On the other hand, let us assume that (2) is satisfied. In order to prove (1), it is enough to show that T' is SQ_p -factorizable (see [11, Theorem 3. 1]). For the proof, we use the Kwapien criterion [2] for the factorizability through a subspace of a quotient of L_p . Let V be a p -integral operator from F' into a Banach space G . Since F is reflexive, by [9, Cor. 1], V is p -nuclear, and so it is factorized by the bounded linear operators $U: F' \rightarrow l_\infty$, $D: l_\infty \rightarrow l_p$ and $W: l_p \rightarrow G$, where D is a diagonal operator. Evidently, $U'D'$ is a continuous linear operator from l_p into F , and there holds $\sum_n \|U'D'(e_n)\|^p < \infty$. From the assumption (2) it follows that $TU'D'$ is p -summing, and so we get $(VT')' \in \Pi_p(G', E'')$. Thus by Kwapien [2, Cor. 7], T' is SQ_p -factorizable, and the proof is completed.

Now we prove the following main theorem extending the results of [1] and [7].

Theorem 3. *Let $1 < p \leq 2$ and suppose that a Banach space E has the property (1a). Then for each Banach space F , every operator of stable type p from F into E is S_p -factorizable.*

Proof. Let T be an operator of stable type p from F into E . Then for each Banach space X with $X' \subset l_p$ and each $S \in L(X, F)$ such that $\sum_n \|S(f_n)\|^p < \infty$, the series $\sum_n TS(f_n) \theta_n^{(p)}$ converges a.s. in E , where f_n is the same as in (2) of Proposition 4. But this means that TS is γ_p -Radonifying (see [1]), and so TS must be p -summing because E has the property (1a). By Proposition 4, it follows that T is S_p -factorizable, and the proof is completed.

Corollary 2. *Let E be a Banach space having the property (1a) and F be a Banach space of stable type p , $1 < p \leq 2$. Then every continuous linear operator from F into E is S_p -factorizable.*

Corollary 3. *Let E be a Banach space of type (B_p) and F be a Banach space of stable type p , $1 < p < 2$. Then every continuous linear operator from F into E is S_p -factorizable.*

Corollary 4 (MAUREY [7]). *Let E be a Banach space of stable cotype 2 and F be a Banach space of stable type 2. Then every continuous linear operator from F into E is Hilbertian. In particular, if E is both of stable type 2 and of stable cotype 2, then E is isomorphic to a Hilbert space.*

Corollary 5. *Let E be a Banach space of stable type p with $1 < p < 2$. Then the following are equivalent.*

- (1) E is of S_p type.
- (2) E has the property (1a).
- (3) E is of type (S) .
- (4) E is of type (B_p) .
- (5) E belongs to the class (V_p) .

Theorem 4. *Let $1 < p < 2$. Then the following properties of a Banach space E are equivalent.*

- (1) E is of stable type p and of S_p type.

(2) For each Banach space X with $X' \subset l_p$, we have

$$R_p(X, E) = \Pi_p(X, E).$$

Proof. The assertion follows from Corollary 5 and [4, Th. 2].

Remark. Corollaries 3, 5 and Theorem 4 become false in the case $p = 2$. It is known that E has the property (2) of Theorem 4 for $p = 2$ if and only if E is of stable cotype 2 (see [6]).

Finally, we prove the following theorem extending the results of [4] and [5].

Theorem 5. *Let $1 < p \leq 2$ and suppose that a Banach space E satisfies the condition $R_p(l_p, E) \subset \Pi_p(l_p, E)$. Then for each reflexive Banach space F , every operator of stable type p from F into E is SQ_p -factorizable.*

Proof. Let T be an operator of stable type p from F into E . Then for each $S \in L(l_p, F)$ with $\sum_n \|S(e_n)\|^p < \infty$, the series $\sum_n TS(e_n) \theta_n^{(p)}$ converges a.s. in E . But this means that TS is γ_p -Radonifying, and so by the assumption, TS is p -summing. By Proposition 5, it follows that T is SQ_p -factorizable, and the proof is completed.

Corollary 6. *Let $1 < p < 2$ and let E be a Banach space belonging to the class $V_p(i)$. Then for each reflexive Banach space F , every operator of stable type p from F into E is SQ_p -factorizable.*

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