ON OPERATORS RELATED TO $p$-STABLE MEASURES IN BANACH SPACES

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1. Introduction and notations. Let $E$ be a Banach space with the dual space $E'$ and $p$ be a real number such that $0 < p \leq 2$. We say that $E$ is of stable type $p$ if for each sequence $|x_n|$ in $E$, $\sum_n ||x_n||_p < \infty$ implies the series $\sum_n x_n \theta_n^p$ converges almost surely (a.s.) ; and $E$ is of stable cotype $p$ if for each sequence $|x_n|$ in $E$ such that the series $\sum_n x_n \theta_n^p$ converges a.s., there holds $\sum_n ||x_n||_p < \infty$. Here $|\theta_n^p|$ denotes the sequence of independent identically distributed real random variables with the characteristic function (ch. f.) \(\exp(-|t|^p)\). \(t \in \mathbb{R}\). Let us denote by $L(E, F)$ the set of all continuous linear operators from $E$ into a Banach space $F$. For an operator $T$ in $L(E, F)$, we say that $T$ is $S_p$-factorizable (resp. $SQ_p$-factorizable) if it is factorizable through a subspace (resp. a subspace of a quotient) of some $L_p$. Let us recall that a sequence $|x_n|$ in $E$ is weakly $p$-summable if $\sum_n |<x_n, x'||^p < \infty$ for all $x' \in E'$. For an operator $T$ in $L(E, F)$, we say that $T$ is of stable type $p$ if for each sequence $|x_n|$ in $E$, $\sum_n \sum_{x} ||x||_T < \infty$ implies the series $\sum_n T(x_n) \theta_n^p$ converges a.s. in $F$; $T$ is $\gamma_p$-summing if for each weakly $p$-summable sequence $|x_n|$ in $E$, the series $\sum_n T(x_n) \theta_n^p$ converges a.s. in $F$; and $T$ is $p$-summing if for each weakly $p$-summed sequence $|x_n|$ in $E$, $\sum_n \sum_{x} ||x||_T < \infty$. We denote by $\mathcal{P}_p(E, F)$ (resp. $\mathcal{P}_p(E, F)$), the set of all $\gamma_p$-summing operators (resp. $p$-summing operators) from $E$ into $F$. Let $X$ be a Banach space and $1 < p \leq 2$. In the following we shall write with $X' \subset L_p$ if $X$ is linearly isometric to a subspace of $L_p$. For such a space $X$, we say that an operator $T$ in $L(X, E)$ is $\gamma_p$-Radonifying if $\exp(-||T'(x')||^p)$, $x' \in E'$, is the ch. f. of a Radon measure on $E$, where $T'$ denotes the adjoint of $T$. The set of all $\gamma_p$-Radonifying operators from $X$ into $E$ will be denoted by $R_p(X, E)$. It is known that a symmetric Radon probability measure $\mu$ on $E$ is $p$-stable if and only if there exist a Banach space $X$ with $X' \subset L_p$ and an operator $T$ in $R_p(X, E)$ such that $\exp(-||T'(x')||^p)$, $x' \in E'$, is the ch. f. of $\mu$ (see [4, Prop. 3]).

Then the main results of this paper are the following:

(1) Let $1 < p \leq 2$. Then the following properties of a Banach space $E$ are equivalent.

(1a) For each Banach space $X$ with $X' \subset l_p$, we have
(1b) If \(|x_n|\) and \(|y_n|\) are two sequences in \(E\) such that

\[
\sum_n |\langle y_n, x' \rangle|^p \leq \sum_n |\langle x_n, x' \rangle|^p \text{ for all } x' \in E',
\]

and the series \(\sum_n x_n \theta_n^p\) converges a.s. in \(E\). then \(\sum_n \|y_n\|^p < \infty\).

(2) Let \(1 < p \leq 2\) and suppose that a Banach space \(E\) has the property (1a). Then for each Banach space \(F\), every operator of stable type \(p\) from \(F\) into \(E\) is \(S_p\)-factorizable. In particular, if \(E\) is of type \((B_p)\), \(1 < p < 2\), in the sense of [1], then every operator of stable type \(p\) from \(F\) into \(E\) is \(S_{\sigma'}\)-factorizable; and if \(E\) is of stable cotype 2 and \(F\) is of stable type 2, then every continuous linear operator from \(F\) into \(E\) is Hilbertian.

(3) Let \(1 < p \leq 2\) and let \(E\) be a Banach space satisfying the condition \(R_p(l_p, E) \subset II_p(l_p, E)\), where \(1/p + 1/p' = 1\). Then for each reflexive Banach space \(F\), every operator of stable type \(p\) from \(F\) into \(E\) is \(SQ_{\sigma'}\)-factorizable. In particular, if \(E\) belongs to the class \(V_p(i)\) in the sense of [5], then every operator of stable type \(p\), \(1 < p < 2\), from a reflexive Banach space \(F\) into \(E\) is \(SQ_{\sigma'}\)-factorizable.

(4) Let \(E\) be a Banach space and \(1 < p < 2\). Then \(E\) is of stable type \(p\) if and only if for each Banach space \(F\), every \(p\)-summing operator from \(F\) into \(E\) is \(\gamma_p\)-summing; and \(E\) is of finite dimension if and only if every operator of stable type \(p\) from \(l_p\) into \(E\) is \(\gamma_p\)-Radonifying.

(5) Let \(1 < p < \infty\) and let \(T\) be a continuous linear operator from a Banach space \(F\) into a Banach space \(E\). Then \(T\) is \(S_p\)-factorizable if and only if for each Banach space \(X\) with \(X' \subset l_p\) and each \(S \in L(X, F)\) with \(\sum_n \|S(f_n)\|^p < \infty\). \(TS\) is \(p\)-summing. Here \(f_n = J'(e_n)\), where \(J\) is an isometric imbedding from \(X'\) into \(l_p\) and \(e_n\) is the \(n\)-th unit vector of \(l_p\). Furthermore, if we assume that \(F\) is reflexive, then \(T\) is \(SQ_{\sigma'}\)-factorizable if and only if for each \(S \in L(l_p, F)\) with \(\sum_n \|S(e_n)\|^p < \infty\). \(TS\) is \(p\)-summing.

Remark. In Section 2, the equivalence of (1a) and (1b) is proved, and some examples of Banach spaces \(E\) having the property (1a) are given. For the case \(p = 2\), it is well-known that \(E\) has the property (1a) if and only if it is of stable cotype 2, and on the other hand, every Banach space is of stable cotype \(p\) with \(p < 2\) (see [6]). We also prove that a Banach space \(E\) belongs to the class \(V_p(i)\), \(1 < p \leq 2\), if and only if \(R_p(l_p, E) \subset II_p(l_p, E)\). It is easy to see that every Banach space belongs to the class \(V_2(i)\); and a
Banach space of stable type $p$, $1 < p < 2$, belongs to the class $V_p(i)$ if and only if it is of $SQ_p$ type in the sense of [2], i.e. it is isomorphic to a subspace of a quotient of some $L_p$. This extends a result of [5].

In Section 3, (4) is proved. We also prove that every $r$-summing operator is of stable type $p$, where $0 < p \leq 2$ and $0 < r < \infty$.

In Section 4, the results (2), (3) and (5) are proved. We note that (2) extends the results of [1], [5] and [7]; (3) extends the results of [4] and [5]; and (5) is an analogue of the results of [2] and [7]. Let us recall that a Banach space $E$ is of $S_p$ type if it is isomorphic to a subspace of some $L_p$. As a consequence of (2), we obtain that a Banach space $E$ is of stable type $p$ and of $S_p$ type, $1 < p < 2$, if and only if for each Banach space $X$ with $X \subset l_p$, there holds $R_p(X, E) = \Pi_p(X, E)$. This extends a result of [1].

The paper is motivated from the works of [1], [2], [4], [5] and [7].

2. Banach spaces having the property (1a). We first prove the equivalence of (1a) and (1b) mentioned in Section 1.

Theorem 1. Let $1 < p \leq 2$. Then the following properties of a Banach space $E$ are equivalent.

(1) For each Banach space $X$ with $X' \subset l_p$, we have

$$R_p(X, E) \subset \Pi_p(X, E).$$

(2) If $|x_n|$ and $|y_n|$ are two sequences in $E$ such that

$$\sum_n |< y_n, x'^n >|^p \leq \sum_n |< x_n, x'^n >|^p \text{ for all } x' \in E',$$

and the series $\sum_n x_n \theta_{n}^{x}$ converges a.s. in $E$, then $\sum_n \|y_n\|^p < \infty$.

Proof. For the case $p = 2$, the equivalence of (1) and (2) easily follows from the fact that $E$ is of stable cotype 2 if and only if $R_2(l_2, E) \subset \Pi_2(l_2, E)$ (see [6]). Hence we may prove only the case $1 < p < 2$.

(1) $\Rightarrow$ (2): Let us assume that (1) is satisfied and let $|x_n|$ and $|y_n|$ be two sequences in $E$ such that

$$\sum_n |< y_n, x'^n >|^p \leq \sum_n |< x_n, x'^n >|^p \text{ for all } x' \in E',$$

and the series $\sum_n x_n \theta_{n}^{x}$ converges a.s. in $E$. Since every Banach space is of stable cotype $p$ with $p < 2$, we have $\sum_n \|x_n\|^p < \infty$. Then there is a continuous linear operator $S$ from $l_p$ into $E$ such that $S(e_n) = x_n$ for all $n$, where $e_n$ is the $n$-th unit vector of $l_p$ $(1/p + 1/p' = 1)$. Evidently, $S$ is
\( \gamma_p \)-Radonifying and there holds \( S'(x') = (\langle x_n', x' \rangle)_{n=1}^{\infty} \) for all \( x' \in E' \). Let \( X = Y' \), where \( Y = S'(E) \). Obviously, \( X \) is a Banach space whose dual \( X' \) is a closed subspace of \( l_p \), and the operator \( S \) can be factorized as follows;

\[
l_p \overset{J'}{\rightarrow} X' \overset{T}{\rightarrow} E,
\]

where \( J \) denotes the natural injection from \( X' \) into \( l_p \). Then \( T \) is clearly \( \gamma_p \)-Radonifying since \( S \) is so. From the assumption (1) it follows that \( T \) is \( p \)-summing. We note here that \( S'(E') \) is a dense subspace of \( X' \). Define the operator \( V : X' \rightarrow l_p \) by

\[
V : (\langle x_n, x' \rangle)_{n=1}^{\infty} \rightarrow (\langle y_n, x' \rangle)_{n=1}^{\infty} \text{ for all } x' \in E'.
\]

Then \( V \) is a continuous linear operator and there holds \( TV'(e_n) = y_n \) for all \( n \). Since \( TV' \) is \( p \)-summing, we have \( \sum_n \| y_n \| < \infty \).

(2) \( \Rightarrow \) (1) : Let us assume that (2) is satisfied and let \( X \) be a Banach space with \( X' \subset l_p \) and \( T \in R_p(X, E) \). To prove that \( T \) is \( p \)-summing, let \( \{x_n\} \) be an weakly \( p \)-summable sequence in \( X \). Then there is a continuous linear operator \( S \) from \( l_p \) into \( X \) such that \( S(e_n) = x_n \) for all \( n \). Evidently, we have

\[
\| S'T'(x') \|^p = \sum_n |\langle S'T'(x'), e_n \rangle|^p = \sum_n |\langle T(x_n), x' \rangle|^p, x' \in E',
\]

and

\[
\| T'(x') \|^p = \| JT'(x') \|^p = \sum_n |\langle TJ'(e_n), x' \rangle|^p, x' \in E',
\]

where \( J \) is an isometric imbedding from \( X' \) into \( l_p \). Hence

\[
\sum_n |\langle T(x_n), x' \rangle|^p \leq || S'||^p \sum_n |\langle TJ'(e_n), x' \rangle|^p, x' \in E'.
\]

Since \( TJ' : l_p \rightarrow E \) is clearly \( \gamma_p \)-Radonifying, the series \( \sum_n TJ'(e_n) \theta_n^p \) converges a.s. in \( E \). From the assumption (2) it follows that \( \sum_n \| T(x_n) \|^p < \infty \) proving \( T \in \Pi_p(X, E) \). Thus the proof is completed.

Now we give some examples of Banach spaces \( E \) having the property (1a). Let us recall that for \( p = 2 \), \( E \) has the property (1a) if and only if it is of stable cotype 2.

Following [1], we say that a Banach space \( E \) is of type \( (B_p) \), \( 1 < p \leq 2 \), if for each Banach space \( X \) with \( X' \subset l_p \), there holds \( R_p(X, E) = \Pi_r(X, E) \). Of course every Banach space is of type \( (B_2) \) (see [1]). For \( 1 < p < 2 \), every Banach space of type \( (B_p) \) has the property (1a) since \( \gamma_p \)-summing operators are \( p \)-summing.
Following [8], we say that a Banach space $E$ is of type $(S)$ if there exists an $S$-topology on $E'$. It is well-known that if $E$ has the approximation property, then $E$ is of type $(S)$ if and only if it is isomorphic to a subspace of some $L_0$. Obviously, every Banach space of type $(S)$ has the property (1a) for $1 < p \leq 2$.

Following [12], we say that a Banach space $E$ belongs to the class $(V_p)$, $1 \leq p \leq 2$, if for each $p$-stable Radon probability measure $\mu$ and each stable cylindrical measure $\nu$ on $E$, the inequality

$$|1 - \hat{\nu}(x')| \leq |1 - \hat{\mu}(x')|, \; x' \in E',$$

implies $\nu$ is a Radon measure on $E$. Here $\hat{\mu}$ (resp. $\hat{\nu}$) denotes the ch. f. of $\mu$ (resp. $\nu$). It is easy to see that every Banach space belonging to the class $(V_p)$ has the property (1a) for $1 < p < 2$.

Finally, we give some examples of Banach space $E$ satisfying the condition $R_p(l_{p'}, E) \subset \Pi_p(l_{p'}, E)$, $1 < p \leq 2$. Evidently, every Banach space having the property (1a) satisfies this condition, but the converse is not true. It is known that every Banach space of $SQ_p$ type satisfies this condition (see [4]). Note that for $p = 2$, $E$ satisfies this condition if and only if it is of stable cotype 2. In the following we give another example of Banach spaces $E$ satisfying this condition.

Let $E$ be a Banach space and $1 < p \leq 2$. Denote by $\Lambda_p(E'; l_p)$ the set of all continuous linear operators $T$ from $E'$ into $l_p$ such that $\exp(-\|T(x')\|^p)$, $x' \in E'$, is the ch. f. of a Radon measure on $E$. It is known that for an operator $T$ in $L(E', l_p)$, $T \in \Lambda_p(E', l_p)$ if and only if there exists an operator $S$ in $R_p(l_{p'}, E)$ such that $T = S'$ (see [3, Th. 5]). Following [5], we say that $E$ belongs to the class $V_p(i)$ if $T \in \Lambda_p(E', l_p)$ implies $ST \in \Lambda_p(E', l_p)$ for all $S \in L(l_p, l_p)$.

**Proposition 1.** A Banach space $E$ belongs to the class $V_p(i)$, $1 < p \leq 2$, if and only if the inclusion $R_p(l_{p'}, E) \subset \Pi_{\gamma_p}(l_{p'}, E)$ holds.

**Proof.** Let us first assume that $E$ belongs to the class $V_p(i)$ and let $T \in R_p(l_{p'}, E)$. In order to prove that $T$ is $\gamma_p$-summing, take an weakly $p$-summable sequence $(x_n)$ in $l_{p'}$. Then there is an operator $S$ in $L(l_{p'}, l_{p'})$ such that $S(e_n) = x_n$ for all $n$. By the assumption, $T' \in \Lambda_p(E', l_p)$ implies $S'T' \in \Lambda_p(E', l_p)$. But this means that $TS$ is $\gamma_p$-Radonifying, and so the series $\sum \theta_n^p \converges \sum T(x_n) \theta_n^p$ converges a. s. in $E$ (see [1] or [4]). Hence we get $T \in \Pi_{\gamma_p}(l_{p'}, E)$. On the other hand, suppose that the inclusion
$R_\rho(l_\rho', E) \subseteq \Pi_{\gamma_\rho}(l_\rho', E)$ holds. Let $T \in A_\rho(E', l_\rho)$. Then there is an operator $V$ in $R_\rho(l_\rho', E)$ such that $T = V'$. By the assumption, $V$ is $\gamma_\rho$-summing. Let $S$ be any operator in $L(l_p, l_p)$. Since $|S'(e_n)|$ is an weakly $p$-summable sequence in $l_\rho'$, the series $\sum_n V S'(e_n) \vartheta_n^p$ converges a. s. in $E$. But this means that $VS'$ is $\gamma_\rho$-Radonifying, and so we get $ST = (VS')' \in A_\rho(E', l_\rho)$. Thus $E$ belongs to the class $V_\rho(i)$, and the proof is completed.

**Corollary 1.** Suppose that a Banach space $E$ is of stable type $p$, $1 < p < 2$. Then $E$ belongs to the class $V_\rho(i)$ if and only if it is of $SQ_\rho$ type.

**Proof.** The assertion follows from Proposition 1 and [4, Th. 3].

3. **Operators of stable type $p$ and $\gamma_\rho$-summing operators.** Let us first remark that every operator factorizable through a Banach space of stable type $p$ is always of stable type $p$. It is well known that for $2 \leq r < \infty$, $L_r$ is of stable type 2, and in particular, it is of stable type $p$ for all $p \in (0, 2]$.

**Proposition 2.** For $0 < p \leq 2$ and $0 < r < \infty$, every $r$-summing operator is of stable type $p$.

**Proof.** The assertion easily follows from the facts that every $r$-summing operator is $s$-summing for $r < s$, and every $r$-summing operator is factorizable through a subspace of some $L_r$ (see [10]).

**Remark.** Every $\gamma_\rho$-summing operator, $0 < p < 2$, is always $p$-summing, but in general, the converse is not true. It is known that if a Banach space $E$ is of stable type $p$, $0 < p \leq 2$, then for each Banach space $F$, every $p$-summing operator from $F$ into $E$ is $\gamma_\rho$-summing (see [1]). The following result shows that the converse is true for $1 < p < 2$.

**Proposition 3.** Let $1 < p < 2$. Then the following properties of a Banach space $E$ are equivalent.

1. $E$ is of stable type $p$.
2. $\Pi_\rho(l_\rho', E) = \Pi_{\gamma_\rho}(l_\rho', E)$.
3. For each Banach space $F$, we have $\Pi_\rho(F, E) = \Pi_{\gamma_\rho}(F, E)$.

**Proof.** Since every $\gamma_\rho$-summing operator from $l_\rho'$ into $E$ is $\gamma_\rho$-Radonifying, the assertion follows from [4, Th. 2].
Remark. Proposition 3 becomes false in the case $p = 2$. In this case, one of the properties (2) and (3) is equivalent to the fact that $E$ is of stable cotype 2 (see [1], [6]).

Finally, we investigate the relationship among operators of stable type $p$, $\gamma_p$-summing and $\gamma_p$-Radonifying operators.

**Theorem 2.** Let $1 < p < 2$. Then the following properties of a Banach space $E$ are equivalent.

1. $E$ is of finite dimension.
2. Every operator of stable type $p$ from $l_p$ into $E$ is $\gamma_p$-summing.
3. Every operator of stable type $p$ from $l_p$ into $E$ is $\gamma_p$-Radonifying.

**Proof.** Of course, we only have to prove (3) $\Rightarrow$ (1). Let us assume that (3) is satisfied. Then $E$ is of stable type $p$ (see Prop. 2 and [4, Th. 2]). Let $\{x_n\}$ be an weakly $p$-summable sequence in $E$. Then there is an operator $T$ in $L(l_p, E)$ such that $T(e_n) = x_n$ for all $n$. Since $E$ is of stable type $p$, $T$ is of stable type $p$. From the assumption (3) it follows that $T$ is $\gamma_p$-Radonifying, and so the series $\sum_n T(e_n) \theta_n^p = \sum_n x_n \theta_n^p$ converges a.s. in $E$. Since every Banach space is of stable cotype $p$ with $p < 2$ (see [6]), we have $\sum_n \|x_n\|^p < \infty$. But this means that the identity map on $E$ is $p$-summing, and so $E$ is a nuclear Banach space (see [10]). Thus $E$ is of finite dimension, and the proof is completed.

4. $S_p$-factorizable operators and $SQ_p$-factorizable operators. In this section, we prove the results (2), (3) and (5) stated in Section 1. The following two propositions are analogues of the results of [2] and [7].

**Proposition 4.** Let $1 < p < \infty$ and let $T$ be a continuous linear operator from a Banach space $F$ into a Banach space $E$. Then the following are equivalent.

1. $T$ is $S_p$-factorizable.
2. For each Banach space $X$ with $X' \subset l_p$ and each $S \in L(X, E)$ with $\sum_n \|S(f_n)\|^p < \infty$, $TS$ is $p$-summing. Here $f_n = J(e_n)$, where $J$ is an isometric imbedding from $X'$ into $l_p$.

**Proof.** (1) $\Rightarrow$ (2) easily follows from [11, Th. 3. 1]. On the other hand, let us assume that (2) is satisfied. In order to prove (1), we use the Maurey criterion [7] for the factorizability through a subspace of $L_p$. Let
$|x_n|$ and $|y_n|$ be two sequences in $F$ such that

$$\sum_n |<y_n, x'>|^p \leq \sum_n |<x_n, x'>|^p \text{ for all } x' \in F'$$

and

$$\sum_n \|x_n\|^p < \infty.$$ 

Then by the same way as in the proof of Theorem 1, we can find an operator $S$ in $L(X, F')$ such that $S(f_n) = x_n$ for all $n$, where $X$ is a Banach space with $X' \subset l_p$ and $\{(<x_n, x'>)_{n=1}^\infty; x' \in F'\}$ is a dense subspace of $X'$. Define the operator $V : X' \to l_p$ by

$$V : (x_n, x'_n)_{n=1}^\infty \to (y_n, x'_n)_{n=1}^\infty \text{ for all } x' \in F'.$$

Then $V$ is a continuous linear operator and there holds $SV'(e_n) = y_n$ for all $n$. From the assumption (2) it follows that $TS$ is $p$-summing, and so is $TSV'$. Thus we get

$$\sum_n \|T(y_n)\|^p = \sum_n \|TSV'(e_n)\|^p < \infty.$$ 

By the Maurey criterion [7], $T$ is $S_p$-factorizable, and the proof is completed.

**Proposition 5.** Let $1 < p < \infty$ and let $T$ be a continuous linear operator from a reflexive Banach space $F$ into a Banach space $E$. Then the following are equivalent.

1. $T$ is $SQ_p$-factorizable.
2. For each $S \in L(l_p, F)$ with $\sum_n \|S(e_n)\|^p < \infty$, $TS$ is $p$-summing.

**Proof.** (1) $\Rightarrow$ (2) easily follows from [11, Th. 3.1]. On the other hand, let us assume that (2) is satisfied. In order to prove (1), it is enough to show that $T'$ is $SQ_p$-factorizable (see [11, Theorem 3.1]). For the proof, we use the Kwapien criterion [2] for the factorizability through a subspace of a quotient of $L_p$. Let $V$ be a $p$-integral operator from $F'$ into a Banach space $G$. Since $F$ is reflexive, by [9, Cor. 1], $V$ is $p$-nuclear, and so it is factorized by the bounded linear operators $U : F' \to l_w$, $D : l_w \to l_p$ and $W : l_p \to G$, where $D$ is a diagonal operator. Evidently, $U'D'$ is a continuous linear operator from $l_p$ into $F$, and there holds $\sum_n \|U'D'(e_n)\|^p < \infty$. From the assumption (2) it follows that $TU'D'$ is $p$-summing, and so we get $(VT')' \in \Pi_p(G', E')$. Thus by Kwapien [2, Cor. 7], $T'$ is $SQ_p$-factorizable, and the proof is completed.
Now we prove the following main theorem extending the results of [1] and [7].

**Theorem 3.** Let $1 < p \leq 2$ and suppose that a Banach space $E$ has the property (1a). Then for each Banach space $F$, every operator of stable type $p$ from $F$ into $E$ is $S_p$-factorizable.

**Proof.** Let $T$ be an operator of stable type $p$ from $F$ into $E$. Then for each Banach space $X$ with $X' \subset l_p$ and each $S \in L(X,F)$ such that $\sum_n \|S(f_n)\|_p < \infty$, the series $\sum_n TS(f_n) \theta_n^{\omega}$ converges a.s. in $E$, where $f_n$ is the same as in (2) of Proposition 4. But this means that $TS$ is $\gamma_p$-Radonifying (see [1]), and so $TS$ must be $p$-summing because $E$ has the property (1a). By Proposition 4, it follows that $T$ is $S_p$-factorizable, and the proof is completed.

**Corollary 2.** Let $E$ be a Banach space having the property (1a) and $F$ be a Banach space of stable type $p$, $1 < p \leq 2$. Then every continuous linear operator from $F$ into $E$ is $S_p$-factorizable.

**Corollary 3.** Let $E$ be a Banach space of type $(B_p)$ and $F$ be a Banach space of stable type $p$, $1 < p < 2$. Then every continuous linear operator from $F$ into $E$ is $S_p$-factorizable.

**Corollary 4 (Maurey [7]).** Let $E$ be a Banach space of stable cotype 2 and $F$ be a Banach space of stable type 2. Then every continuous linear operator from $F$ into $E$ is Hilbertian. In particular, if $E$ is both of stable type 2 and of stable cotype 2, then $E$ is isomorphic to a Hilbert space.

**Corollary 5.** Let $E$ be a Banach space of stable type $p$ with $1 < p < 2$. Then the following are equivalent.

1. $E$ is of $S_p$ type.
2. $E$ has the property (1a).
3. $E$ is of type $(S)$.
4. $E$ is of type $(B_p)$.
5. $E$ belongs to the class $(V_p)$.

**Theorem 4.** Let $1 < p < 2$. Then the following properties of a Banach space $E$ are equivalent.

1. $E$ is of stable type $p$ and of $S_p$ type.
(2) For each Banach space $X$ with $X' \subset l_p$, we have

$$R_p(X, E) = \Pi_p(X, E).$$

Proof. The assertion follows from Corollary 5 and [4, Th. 2].

Remark. Corollaries 3, 5 and Theorem 4 become false in the case $p = 2$. It is known that $E$ has the property (2) of Theorem 4 for $p = 2$ if and only if $E$ is of stable cotype 2 (see [6]).

Finally, we prove the following theorem extending the results of [4] and [5].

Theorem 5. Let $1 < p \leq 2$ and suppose that a Banach space $E$ satisfies the condition $R_p(l_{p'}, E) \subset \Pi_p(l_{p'}, E)$. Then for each reflexive Banach space $F$, every operator of stable type $p$ from $F$ into $E$ is SQ$_p$-factorizable.

Proof. Let $T$ be an operator of stable type $p$ from $F$ into $E$. Then for each $S \in L(l_{p'}, F)$ with $\sum_{n} \|S(e_n)\|^{p} < \infty$, the series $\sum_{n} TS(e_n) \theta_{n}^{m}$ converges a.s. in $E$. But this means that $TS$ is $\gamma_{p}$-Radonifying, and so by the assumption, $TS$ is $p$-summing. By Proposition 5, it follows that $T$ is SQ$_p$-factorizable, and the proof is completed.

Corollary 6. Let $1 < p < 2$ and let $E$ be a Banach space belonging to the class $V_p(i)$. Then for each reflexive Banach space $F$, every operator of stable type $p$ from $F$ into $E$ is SQ$_p$-factorizable.

References


[6] B. Maurey; Espaces de cotype $p$, 0 < $p$ ≤ 2, Seminaire Maurey-Schwartz 1972–73, Expose VII.


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