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A SIMPLE PROOF OF A THEOREM ON ABELIAN REGULAR RIGHT IDEALS

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We give a simple proof of a theorem of A.V. Andrunakievič and V.A. Andrunakievič [1] which states that every abelian regular right ideal is a (two-sided) ideal, whose proof makes use of modular maximal right ideals and is somewhat roundabout.

A right ideal P of a ring R is called *abelian regular*, if for every $a \in R$ there exists $e \in aR$ such that $a-ea \in P$ and $ex-xe \in P$ for all $x \in R$. In this case, there holds that $e-e^2 \in P$ and $eP \subseteq P$. If $a \notin P$ then $ae = a-(a-ea)-(ea-ae) \notin P$ shows that $a^2 \notin P$. Needless to say, every right ideal containing an abelian regular right ideal is also abelian regular. These facts will be used freely in our proof.

Theorem. *Every abelian regular right ideal P of a ring R is an ideal.*

Proof. It suffices to prove that given $a \in R \setminus P$, there exists an ideal $T \supseteq P$ excluding a. Choose $e \in aR$ such that $a-ea \in P$ and $ex-xe \in P$ for all $x \in R$. Obviously, $Q=\{x \in R \mid ex \in P\}$ is a right ideal of R containing P but excluding e and $x-ex \in Q$ for all $x \in R$. Given a right ideal $I \supseteq Q$, it is easy to see that if e is in I then $I=R$. Now, by Zorn's lemma, there exists a right ideal $T \supseteq Q$ which is maximal with respect to excluding e (or a). We prove that T is an ideal. Suppose, to the contrary, that there exist $t \in T$ and $b \in R$ such that $bt \notin T$. Then $e=btb'+t'$ with some $b' \in R$ and $t' \in T$, and $b=(b-tb'b^2)b''+t''$ with some $b'' \in R$ and $t'' \in T$. But, $b-t'' \notin T$ and

$$(b-t'')^2=b(b-tb'b^2)b''-t''(b-t'')=\{t'b^2+(b^2-eb^2)\}b''-t''(b-t'') \in T.$$

This contradiction shows that T is an ideal.

REFERENCES

[1] A.V. ANDRUNAKIEVIČ and V.A. ANDRUNAKIEVIČ: Abelian regular ideals of a ring, Soviet Math. Dokl. 25 (1982), 462—465.

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