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INDIVIDUAL ERGODIC THEOREMS FOR PSEUDO-RESOLVENTS

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1. Introduction. This paper deals with individual ergodic theorems for a pseudo-resolvent of linear operators acting on the space of functions which take their values in a reflexive Banach space.

Let $(\Omega, \mathscr{F}, \mu)$ be a σ-finite measure space and $(X, |\cdot|)$ a reflexive Banach space. Denote by D the set of all complex numbers λ with $\mathrm{Re}(\lambda)>0$. Then the family $\mathbf{J}=(J_\lambda : \lambda\in D)$ of linear operators on $L_1(\Omega, X)=L_1(\Omega, \mathscr{F}, \mu, X)$ will be called a pseudo-resolvent if $\mathbf{J}$ satisfies the resolvent equation

$$J_\lambda - J_\nu = (\nu-\lambda)J_\lambda J_\nu \quad (\lambda, \nu\in D).$$

Denote by D_+ the set of all positive reals. In this paper we shall assume that $\|tJ_t\|_1\leq 1$ for all $t\in D_+$ and that for some constant $M\geq 1$, $\|tJ_t f\|_\infty \leq M\|f\|_\infty$ for all $f\in L_1(\Omega, X)\cap L_\infty(\Omega, X)$ and all $t\in D_+$. By the Riesz convexity theorem we see that for each $t\in D_+$ and each $1<p<\infty$, tJ_t may be regarded as a linear operator on $L_p(\Omega, X)$ such that $\|tJ_t\|_p\leq M$. We shall prove below that for each $1\leq p<\infty$ and each $f\in L_p(\Omega, X)$ there exists an X-valued function $g(\lambda, \omega)$, defined on $D\times\Omega$ and strongly measurable with respect to the product of the Lebesgue measure on D and μ, such that for each fixed $\lambda\in D$, $g(\lambda, \omega)$ as a function of ω belongs to the equivalence class of $J_\lambda f$ and also such that for each fixed $\omega\in\Omega$, $g(\lambda, \omega)$ as a function of λ is continuous on D. Thus we may agree to take, for all $\lambda\in D$,

$$(J_\lambda f)(\omega)=g(\lambda, \omega).$$

The main purpose of this paper is to prove that, putting

$$D(K)=\{\lambda\in D: \left|\frac{\mathrm{Im}(\lambda)}{\mathrm{Re}(\lambda)}\right|<K\}$$

for each constant $K>0$, the following two individual ergodic limits

$$\lim_{\substack{|\lambda|\to\infty \\ \lambda\in D(K)}} \lambda J_\lambda f(\omega) \quad\text{and}\quad \lim_{\substack{|\lambda|\to 0 \\ \lambda\in D(K)}} \lambda J_\lambda f(\omega)$$

exist almost everywhere on Ω. To do this we use abelian limit theorems, which will be prepared in the next section.

2. Abelian limit theorems. In this section the following two theorems are proved. (Cf. [4].)

Theorem 1. *Let f be a strongly Lebesgue measurable function from the interval* $(0, \infty)$ *to a Banach space* $(Y, |\cdot|)$ *such that for some* $\lambda_0 \in D_+$, $\int_0^\infty e^{-\lambda_0 t}|f(t)|\,dt < \infty$. *If the limit*

$$\lim_{b \to 0} \frac{1}{b}\int_0^b f(t)\,dt = x(\in Y)$$

exists, then we have

$$\lim_{\substack{|\lambda| \to \infty \\ \lambda \in D(K)}} \lambda \int_0^\infty e^{-\lambda t} f(t)\,dt = x.$$

Theorem 2. *Let f be a strongly Lebesgue measurable function from the interval* $(0, \infty)$ *to a Banach space* $(Y, |\cdot|)$ *such that for all* $\lambda \in D_+$, $\int_0^\infty e^{-\lambda t}|f(t)|\,dt < \infty$. *If the limit*

$$\lim_{b \to \infty} \frac{1}{b}\int_0^b f(t)\,dt = x(\in Y)$$

exists, then we have

$$\lim_{\substack{|\lambda| \to 0 \\ \lambda \in D(K)}} \lambda \int_0^\infty e^{-\lambda t} f(t)\,dt = x.$$

Proof of Theorem 1. Using Fubini's theorem and Tonelli's theorem (see e. g. [1], Theorems III. 11. 9 and III. 11. 14), for $\lambda \in D(K)$ with Re $(\lambda) > \lambda_0$ we have

$$\begin{aligned}\lambda \int_0^\infty e^{-\lambda t} f(t)\,dt &= \lambda^2 \int_0^\infty e^{-\lambda t}\int_0^t f(s)\,ds\,dt \\ &= \left(\int_0^\infty e^{-\lambda t}\int_0^t f(s)\,ds\,dt\right)\Big/\left(\int_0^\infty te^{-\lambda t}\,dt\right).\end{aligned}$$

By hypothesis, given an $\varepsilon > 0$, we can choose $B > 0$ so that

$$0 < b < B \quad \text{implies} \quad \left|\frac{1}{b}\int_0^b f(t)\,dt - x\right| < \varepsilon.$$

Then

$$\left|\int_B^\infty e^{-\lambda t}\int_0^t f(s)\,ds\,dt\right| \leq e^{-[\mathrm{Re}(\lambda)-\lambda_0]B}\int_B^\infty e^{-\lambda_0 t}\int_0^t |f(s)|\,ds\,dt$$

and further

$$|\lambda^2 e^{-\mathrm{Re}(\lambda)B}| = \left|\frac{\lambda}{\mathrm{Re}(\lambda)}\right|^2 [\mathrm{Re}(\lambda)]^2 e^{-\mathrm{Re}(\lambda)B}$$
$$<(1+K^2)[\mathrm{Re}(\lambda)]^2 e^{-\mathrm{Re}(\lambda)B} \qquad (\lambda\in D(K)),$$

therefore, since $\lambda\in D(K)$ and $|\lambda|\longrightarrow\infty$ imply Re $(\lambda)\longrightarrow\infty$, we get

$$\lim_{\substack{|\lambda|\to\infty\\ \lambda\in D(K)}} \lambda^2\int_B^\infty e^{-\lambda t}\int_0^t f(s)\,ds\,dt=0.$$

Similarly (or directly)

$$\lim_{\substack{|\lambda|\to\infty\\ \lambda\in D(K)}} \left(\int_B^\infty te^{-\lambda t}dt\right)\Big/\left(\int_0^B te^{-\lambda t}dt\right)=0.$$

Thus to prove the theorem it suffices to show that

$$\limsup_{\substack{|\lambda|\to\infty\\ \lambda\in D(K)}} \left|\left(\int_0^B e^{-\lambda t}\int_0^t f(s)\,ds\,dt\right)\Big/\left(\int_0^B te^{-\lambda t}dt\right)-x\right|$$

can be arbitrarily small. To see this, put $\xi(t)=\frac{1}{t}\int_0^t f(s)\,ds-x$ for $0<t<B$. Then we have

$$\left(\int_0^B e^{-\lambda t}\int_0^t f(s)\,ds\,dt\right)\Big/\left(\int_0^B te^{-\lambda t}\,dt\right)$$
$$=x+\left(\int_0^B te^{-\lambda t}\,\xi(t)\,dt\right)\Big/\left(\int_0^B te^{-\lambda t}\,dt\right)$$

and

$$\limsup_{\substack{|\lambda|\to\infty\\ \lambda\in D(K)}} \left|\left(\int_0^B te^{-\lambda t}\,\xi(t)\,dt\right)\Big/\left(\int_0^B te^{-\lambda t}\,dt\right)\right|$$
$$\leq \varepsilon\cdot\limsup_{\substack{|\lambda|\to\infty\\ \lambda\in D(K)}} \left|\frac{\lambda}{\mathrm{Re}\ (\lambda)}\right|^2 \frac{|1-e^{-\mathrm{Re}(\lambda)B}-\mathrm{Re}(\lambda)Be^{-\mathrm{Re}(\lambda)B}|}{|1-e^{-\lambda B}-\lambda Be^{-\lambda B}|}$$
$$\leq \varepsilon(1+K^2).$$

Therefore the proof is complete.

Proof of Theorem 2. Since for any constant $B>0$

$$\lim_{\substack{|\lambda|\to 0\\ \lambda\in D(K)}} \lambda^2\int_0^B e^{-\lambda t}\int_0^t f(s)\,ds\,dt=0$$

and

$$\lim_{\substack{|\lambda|\to 0\\ \lambda\in D(K)}} \left(\int_0^B te^{-\lambda t}dt\right)/\left(\int_B^\infty te^{-\lambda t}dt\right)=0,$$

Theorem 2 follows from an easy modification of the proof of Theorem 1, and so we omit the details.

3. Ergodic theorems for pseudo-resolvents. In this section $\mathbf{J}=(J_\lambda: \lambda\in D)$ will denote a pseudo-resolvent of linear operators on $L_1(\Omega, X)$, where $(X, |\cdot|)$ is a reflexive Banach space, such that $\|tJ_t\|_1\leq 1$ for all $t\in D_+$ and also such that for some $M\geq 1$, $\|tJ_t f\|_\infty \leq M\|f\|_\infty$ for all $f\in L_1(\Omega, X)\cap L_\infty(\Omega, X)$ and all $t\in D_+$.

Lemma 1. (a) *For every* $1\leq p<\infty$ *and every* $f\in L_p(\Omega, X)$, $tJ_t f$ *converges in* L_p*-norm as* $t\longrightarrow\infty$, *when* t *is restricted to be in* D_+.

(b) *For every* $1<p<\infty$ *and every* $f\in L_p(\Omega, X)$, $tJ_t f$ *converges in* L_p*-norm as* $t\longrightarrow 0$, *when* t *is restricted to be in* D_+.

Proof. Since $L_p(\Omega, X)$, with $1<p<\infty$, is reflexive (because X is reflexive) and $\|tJ_t\|_p\leq M$ for all $t\in D_+$, it follows from Yosida's theory (cf. [5], VIII) that for every $1<p<\infty$ and every $f\in L_p(\Omega, X)$, tJ_t converges in L_p-norm as $t\longrightarrow\infty$ and also does as $t\longrightarrow 0$, when t is restricted to be in D_+. Thus we have proved (b). To complete the proof of (a), it suffices to show that for each $f\in L_1(\Omega, X)\cap L_\infty(\Omega, X)$, $tJ_t f$ converges in L_1-norm as $t\longrightarrow\infty$, $t\in D_+$, since $\|tJ_t\|_1\leq 1$ for all $t\in D_+$. To do this, choose $f_\infty\in L_2(\Omega, X)$ so that

$$\lim_{\substack{t\to\infty\\ t\in D_+}} \|tJ_t f-f_\infty\|_2=0.$$

Then using Fatou's lemma we have $\|f_\infty\|_1\leq\|f\|_1$. On the other hand, the resolvent equation shows that

$$J_\lambda(tJ_t)=(\lambda-t)^{-1}[tJ_t-tJ_\lambda],$$

and hence $J_\lambda f_\infty=J_\lambda f$ for all $\lambda\in D$. Therefore $\|f_\infty\|_1=\lim_{\substack{t\to\infty\\ t\in D_+}}\|tJ_t f_\infty\|_1$, and since $tJ_t f$ converges in measure to f_∞ as $t\longrightarrow\infty$, $t\in D_+$, we conclude that

$$\lim_{\substack{t\to\infty\\ t\in D_+}}\|tJ_t f-f_\infty\|_1=\lim_{\substack{t\to\infty\\ t\in D_+}}\|tJ_t f_\infty-f_\infty\|_1=0,$$

which completes the proof.

Lemma 2. *There exists a strongly continuous one-parameter semigroup*

$\Gamma=(T_t : t\geq 0)$ of linear operators on $L_1(\Omega, X)$ such that

(i) $\|T_t\|_1\leq 1$ *for all* $t\geq 0$,

(ii) $\|T_t f\|_\infty\leq M^2\|f\|_\infty$ *for all* $f\in L_1(\Omega, X)\cap L_\infty(\Omega. X)$ *and all* $t\geq 0$,

(iii) tJ_t *converges strongly to* T_0 *as* $t\longrightarrow\infty$, *when* t *is restriced to be in* D_+,

(iv) *for all* $\lambda\in D$ *and all* $f\in L_1(\Omega, X)$

$$J_\lambda f=\int_0^\infty e^{-\lambda t}T_t f\, dt.$$

Proof. For $f\in L_1(\Omega, X)$ define Ef to be the function in $L_1(\Omega, X)$ such that

$$\lim_{\substack{t\longrightarrow\infty\\ t\in D_+}}\|tJ_t f-Ef\|_1=0.$$

Then E is a linear operator on $L_1(\Omega, X)$ such that

$$E=E^2,\ J_\lambda E=EJ_\lambda=J_\lambda\ (\lambda\in D),\ \|E\|_1\leq 1$$

and

$$\|Ef\|_\infty\leq M\|f\|_\infty\qquad (f\in L_1(\Omega, X)\cap L_\infty(\Omega, X)).$$

Let us put $R=EL_1(\Omega, X)$. Then $\mathbf{J}=(J_\lambda:\ \lambda\in D)$ may be regarded as a pseudo-resolvent of linear operators on the Banach space R. To see that J_λ, $\lambda\in D$, are one to one operators on R, let $f\in R$ and $J_\lambda f=0$ for some $\lambda\in D$. Then we have

$$f=Ef=\lim_{\substack{t\longrightarrow\infty\\ t\in D_+}} tJ_t f=0,$$

because the resolvent equation implies that $J_\lambda f=0$ if and only if $J_\nu f=0$ for all $\nu\in D$. Thus we may apply Theorem VIII. 4. 1 in [5] to infer that

$$J_\lambda=(\lambda-A)^{-1}\text{ on } R\ \ (\lambda\in D)$$

for some closed linear operator A with dense domain in R. Since $\|tJ_t\|_1\leq 1$ for all $t\in D_+$, it then follows from the Hille-Yosida theorem ([5], p. 248) that there exists a strongly continuous one-parameter semigroup $\mathcal{J}=(S_t:$ $t\geq 0)$ of linear operators on R, with $S_0=I$ (the identity operator) and $\|S_t\|_1\leq 1$ (on R) for all $t\geq 0$, such that A is the infinitisimal generator of $\mathcal{J}$. Since $\|tJ_t f\|_\infty\leq M\|f\|_\infty$ for all $t\in D_+$ and all $f\in L_1(\Omega, X)\cap L_\infty(\Omega, X)$, it also follows that

$$\|S_t f\|_\infty\leq M\|f\|_\infty\ \ (t\geq 0\text{ and } f\in R\cap L_\infty(\Omega, X)).$$

Define $T_t=S_tE$ $(t\geq 0)$. Then it is easily seen that $\Gamma=(T_t:\ t\geq 0)$ is a strongly continuous one-parameter semigroup of linear operators on $L_1(\Omega, X)$ such that $\|T_t\|_1 \leq 1$ for all $t \geq 0$, $\text{strong-}\lim_{t \to 0} T_t = T_0$, and for every $\lambda \in D$ and every $f \in L_1(\Omega, X)$

$$J_\lambda f = J_\lambda E f = \int_0^\infty e^{-\lambda t} S_t E f \, dt = \int_0^\infty e^{-\lambda t} T_t f \, dt.$$

Since $\|T_t f\|_\infty \leq M \|Ef\|_\infty \leq M^2 \|f\|_\infty$ for all $t \geq 0$ and all $f \in L_1(\Omega, X) \cap L_\infty(\Omega, X)$, the proof is completed.

Corollary 1. (a) *For every* $1 \leq p < \infty$ *and every* $f \in L_p(\Omega, X)$, $\lambda J_\lambda f$ *converges in* L_p*-norm as* $|\lambda| \longrightarrow \infty$, *when* λ *is restricted to be in* $D(K)$.

(b) *For every* $1 < p < \infty$ *and every* $f \in L_p(\Omega, X)$, $\lambda J_\lambda f$ *converges in* L_p*-norm as* $|\lambda| \longrightarrow 0$, *when* λ *is restricted to be in* $D(K)$.

Proof. Since the proof of Lemma 2 implies that $T_0 = \text{strong-}\lim_{t \to 0} T_t$ on $L_1(\Omega, X)$ and hence also on each $L_p(\Omega, X)$ with $1 \leq p < \infty$, we have

$$\lim_{b \to 0} \left\| \frac{1}{b} \int_0^b T_t f \, dt - T_0 f \right\|_p = 0$$

for every $1 \leq p < \infty$ and every $f \in L_p(\Omega, X)$. Thus by Theorem 1, (a) follows. Similarly, (b) follows from Theorem 2, since the reflexivity of $L_p(\Omega, X)$, $1 < p < \infty$, implies (see e. g. [3]) that if $1 < p < \infty$ and $f \in L_p(\Omega, X)$ then the averages

$$\frac{1}{b} \int_0^b T_t f \, dt$$

converge in L_p-norm as $b \longrightarrow \infty$.

Corollary 2. *For every* $1 \leq p < \infty$ *and every* $f \in L_p(\Omega, X)$ *there exists an X-valued function* $g(\lambda, \omega)$, *defined on* $D \times \Omega$ *and strongly measurable with respect to the product of the Lebesgue measure on* D *and* μ, *such that for each fixed* $\lambda \in D$, $g(\lambda, \omega)$ *as a function of* ω *belongs to the equivalence class of* $J_\lambda f$, *and also such that for each fixed* $\omega \in \Omega$, $g(\lambda, \omega)$ *as a function of* λ *is continuous on* D.

Proof. By an approximation argument it is known that there exists an X-valued function $T_t f(\omega)$, defined on $(0, \infty) \times \Omega$ and strongly measurable with respect to the product of the Lebesgue measure on $(0, \infty)$ and μ, such that for each fixed $t > 0$, $T_t f(\omega)$ as a function of ω belongs to the equivalence class of $T_t f \in L_p(\Omega, X)$. Then we see that there exists a μ-null set $N(f)$, dependent on f but independent of $\lambda \in D$, such that if $\omega \notin N(f)$ then the X-valued function $t \longrightarrow e^{-\lambda t} T_t f(\omega)$ is Bochner integrable with respect to the Lebesgue measure on the interval $(0, \infty)$, and the integral $\int_0^\infty e^{-\lambda t} T_t f(\omega)\, dt$ as a function of ω belongs to the equivelence class of $\int_0^\infty e^{-\lambda t} T_t f\, dt = J_\lambda f$ for every $\lambda \in D$. Let us put

$$g(\lambda, \omega) = \begin{cases} \int_0^\infty e^{-\lambda t} T_t f(\omega)\, dt & (\omega \notin N(f)) \\ 0 & \text{otherwise.} \end{cases}$$

It is clear that g satisfies the required properties.

In what follows, $g(\lambda, \omega)$ will be denoted by $J_\lambda f(\omega)$. Now we are in a position to prove the main theorem in this paper.

Theorem 3. *For every $1 \leq p < \infty$ and every $f \in L_p(\Omega, X)$ the following limits*

$$\lim_{\substack{|\lambda| \to \infty \\ \lambda \in D(K)}} \lambda J_\lambda f(\omega) \quad \text{and} \quad \lim_{\substack{|\lambda| \to 0 \\ \lambda \in D(K)}} \lambda J_\lambda f(\omega)$$

exist almost everywhere on Ω.

Proof. By Theorems 1 and 2, it suffices to notice that the limits

$$\lim_{b \to 0} \frac{1}{b} \int_0^b T_t f(\omega)\, dt \quad \text{and} \quad \lim_{b \to \infty} \frac{1}{b} \int_0^b T_t f(\omega)\, dt$$

exist almost everywhere on Ω, and the almost everywhere existence of these limits follows from [3]. Thus the proof is completed.

4. An extension of Theorem 3. In this section we shall prove that Theorem 3 holds for functions f in $L_1(\Omega, X) + L_\infty(\Omega, X)$ such that

$$\int_{\{|f| > t\}} |f|\, d\mu < \infty \text{ for all } t > 0.$$

Following Fava [2], the class of such functions f will be denoted by $R_0(\Omega, X)$. It is known that $R_0(\Omega, X)$ is a linear manifold of $L_1(\Omega, X) + L_\infty(\Omega, X)$ including $\bigcup_{1 \leq p < \infty} L_p(\Omega, X)$. A linear operator T on $L_1(\Omega, X)$ such that $\|T\|_1 \leq 1$ and $\|Tf\|_\infty \leq M \|f\|_\infty$ for all $f \in L_1(\Omega, X) \cap L_\infty(\Omega, X)$ may be extended to a linear operator on $R_0(\Omega, X)$ as follows. Let $f \in R_0(\Omega, X)$, and choose f_n, $n = 1, 2, \cdots$, in $L_1(\Omega, X)$ so that

$$\lim_n f_n(\omega) = f(\omega) \quad \text{almost everywhere on } \Omega$$

and

$$\lim_{n, m \to \infty} \|f_n - f_m\|_\infty = 0.$$

Then $\lim_{n, m \to \infty} \|Tf_n - Tf_m\|_\infty = 0$, and hence the limit

$$g(\omega) = \lim_n Tf_n(\omega)$$

exists almost everywhere on Ω. It is a routine matter to see that $g \in R_0(\Omega, X)$, and thus if we set $g = Tf$ then T is well-defined on $R_0(\Omega, X)$ and linear.

Next let $\Gamma = (T_t : t \geq 0)$ be as in Lemma 2. Put

$$T_t f(\omega) = \lim_n T_t f_n(\omega)$$

and

$$J_\lambda f(\omega) = \int_0^\infty e^{-\lambda t} T_t f(\omega)\, dt \quad (\lambda \in D).$$

From the preceding section and the above argument we observe that for each fixed $\lambda \in D$, $J_\lambda f(\omega)$ as a function of ω belongs to the equivalence class of $J_\lambda f \in R_0(\Omega, X)$. Here we may assume without loss of generality that for each fixed $\omega \in \Omega$, $J_\lambda f(\omega)$ as a function of λ is continuous on D. It then follows from [3] together with an easy approximation argument that for every $f \in R_0(\Omega, X)$ the limits

$$\lim_{b \to 0} \frac{1}{b} \int_0^b T_t f(\omega)\, dt \quad \text{and} \quad \lim_{b \to \infty} \frac{1}{b} \int_0^b T_t f(\omega)\, dt$$

exist almost everywhere on Ω. (In fact, if $f \in R_0(\Omega, X)$, we can choose $f_n \in L_1(\Omega, X)$, $n = 1, 2, \cdots$, so that $\lim_n \|f - f_n\|_\infty = 0$. Then we have

$$\left| \frac{1}{b} \int_0^b T_t f(\omega)\, dt - \frac{1}{b} \int_0^b T_t f_n(\omega)\, dt \right| \leq M^2 \|f - f_n\|_\infty$$

almost everywhere on Ω for all $b > 0$ (cf. Lemma 2), and further the limits $\lim_{b \to 0} \frac{1}{b} \int_0^b T_t f_n(\omega)\, dt$ and $\lim_{b \to \infty} \frac{1}{b} \int_0^b T_t f_n(\omega)\, dt$ exist almost everywhere on Ω. Hence for almost all $\omega \in \Omega$ we have

$$\limsup_{b, b' \to 0} \left| \frac{1}{b} \int_0^b T_t f(\omega)\, dt - \frac{1}{b'} \int_0^{b'} T_t f(\omega)\, dt \right| = 0$$

and

$$\limsup_{b, b' \to \infty} \left| \frac{1}{b} \int_0^b T_t f(\omega)\, dt - \frac{1}{b'} \int_0^{b'} T_t f(\omega)\, dt \right| = 0.$$

This establishes the desired conclusion.) Thus we may apply Theorems 1 and 2 to obtain the following extension of Theorem 3.

Theorem 4. *For every* $f \in R_0(\Omega, X)$ *the ergodic limits*

$$\lim_{\substack{|\lambda| \to \infty \\ \lambda \in D(K)}} \lambda J_\lambda f(\omega) \quad \text{and} \quad \lim_{\substack{|\lambda| \to 0 \\ \lambda \in D(K)}} \lambda J_\lambda f(\omega)$$

exist almost everywhere on Ω.

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