A CLASSIFICATION OF FREE EXTENSIONS OF RINGS OF AUTOMORPHISM TYPE AND DERIVATION TYPE

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Introduction. Let B be a ring with the identity 1. In the previous paper [3], the author has studied on a semigroup and a group of some B-ring isomorphism classes of free quadratic extensions of B. In the present paper, as a natural sequel of [3], we shall continue our study on a semigroup and a group of some isomorphism classes of free extensions on which a cyclic group G of order n acts as a group of automorphisms. In case H is the dual Hopf B-algebra of the group algebra BG over a commutative ring B, A. Nakajima [4] has proved that the isomorphism classes of strongly Galois H-objects and those of p-cyclic Galois objects (in the category of commutative B algebras) form abelian groups. More precisely, the former is isomorphic to $U(B)/U(B)^n$ and the latter is isomorphic so $B/B^{\mathfrak{p}}$, where $B^{\mathfrak{p}} = \{b^{\mathfrak{p}}(=b^p - b) \mid b \in B\}$.

In this paper, §0 is devoted to notations and terminologies for the subsequent study. In §1, we shall show that some isomorphism classes of free extensions of ρ-automorphism type on which G acts form an abelian semigroup with the identity, and determine the structure of the semigroup under the assumption that n is invertible in B and the center Z of B contains a primitive n-th root ζ of 1. As a consequence, we can see that if B is commutative then the semigroup is isomorphic to $B'U(B)$. In §2, we assume that B is an algebra over a prime field $GF(p)$. In this case, we shall show that some isomorphism classes of p-cyclic extensions of D-derivation type on which G acts form an abelian group, and determine the structure of the group. Especially, if B is commutative then the group is isomorphic to $(B, +)/B^{\mathfrak{p}}$.

0. Notations and terminologies. Let ρ and D be an automorphism and a derivation of B, respectively. We use the following conventions:

$Z =$ the center of B.

$B_1 = B^\rho = \{b \in B \mid \rho(b) = b\}$, $Z_1 = Z \cap B_1$.

$B(\rho^n) = \{b \in B \mid cb = b\rho^n(c)$ for all $c \in B\}$, $B_1(\rho^n) = B_1 \cap B(\rho^n)$.

$LN_\rho(B\,;\,n) = \{LN_\rho(b\,;\,n) = \rho^{n-1}(b)\rho^{n-2}(b)\cdots\rho(b)b \mid b \in B\}$.

$\widetilde{b} = b_l b_r^{-1}$, the inner automorphism effected by $b \in U(B)$.

$I_b = b_r - b_l$, the inner derivation effected by $b \in B$.

$B_0 = B^D = \{b \in B \mid D(b) = 0\}$, $Z_0 = Z \cap B_0$.

$B(D^{\mathfrak{p}}) = \{b \in B \mid I_b = D^{\mathfrak{p}}(= D^p - D)\}$.

If B contains a primitive n-th root ζ of 1, $\Gamma(n) = \{n, \zeta, 1 - \zeta^i$ $(i = 1, 2, \cdots, n-1)\}$.

Now, let $B[X; \rho]$ (resp. $B[X; D]$) will represent the ring of all polynomials $\sum_i X^i b_i (b_i \in B)$ in an indeterminate X whose multiplication is defined by $bX = X\rho(b)$ (resp. $bX = Xb + D(b)$) for any $b \in B$. If $b \in B_1(\rho^n)$ then $(X^n - b) B[X; \rho]$ is a two-sided ideal of $B[X; \rho]$, and conversely. When this is the case, the ring extension $B[X; \rho]/(X^n - b) B[X; \rho]$ of B is called an *n-binomial extension of ρ-automorphism type*. While, in case B is an algebra over $\mathrm{GF}(p)$, $(X^p - X - b) B[X; D]$ is a two-sided ideal of $B[X; D]$ if and only if $b \in B_0(D^{\mathfrak{p}})$. When this is the case, the ring extension $B[X; D]/(X^p - X - b) B[X; D]$ of B becomes a G-cyclic extension with respect to a cyclic group G of order p (see [1]), and we shall call it a *G-cyclic extension of D-derivation type.* For future reference, we put

$$\Omega_\rho(B) = \{B[X; \rho]/(X^n - b)B[X; \rho] \mid b \in B_1(\rho^n)\},$$
$$\Omega_D(B) = \{B[X; D]/(X^p - X - b) B[X; D] \mid b \in B_0(D^{\mathfrak{p}})\}.$$

Finally a ring extension A/B is called a *separable extension* if the A-A-homomorphism $a \otimes a' \rightarrow aa'$ of $A \otimes_B A$ onto A splits. As is well known, every G-Galois extension is separable.

1. A classification of n-binomial extensions of ρ-automorphism type with respect to a cyclic group G of order n. In this section, we assume that $U(Z_1) \supseteqq \Gamma(n)$, $\rho^n = \tilde{u}^{-1}$ with some $u \in U(B_1)$, and G is a cyclic group of order n with a generator σ. If $A = B[X; \rho]/(X^n - b)B[X; \rho] \in \Omega_\rho(B)$ then we write $A = A_b$ and $X + (X^n - b) B[X; \rho] = x_b$. Noting that $x_b^n = b \in B$, A_b can be regarded as a left BG-module via $\sigma(\sum_{i=0}^{n-1} x_b^i b_i) = \sum_{i=0}^{n-1} (x_b \zeta)^i b_i$. In all that follows, we understand each $A_b \in \Omega_\rho(B)$ is a BG-module in the above sense. Given $A \in \Omega_\rho(B)$, the BG-ring isomorphism class of A in $\Omega_\rho(B)$ will be denoted by $\langle A \rangle$. It is obvious that any BG-ring isomorphism $A_b \longrightarrow A_c$ is right B-linear. We set here

$$P_\rho(B) = \{\langle A \rangle \mid A \in \Omega_\rho(B)\}.$$

Now, we shall begin our study with the following

Lemma 1.1. *If A_b, $A_c \in \Omega_\rho(B)$, then the following are equivalent:*

(1) $\langle A_b \rangle = \langle A_c \rangle$.

(2) *There exists a BG-ring isomorphism* ϕ *of* A_b *into* A_c *such that* $\phi(x_b) = x_c\alpha$ *with* $\alpha \in U(Z)$.

(3) $b = cLN_\rho(\beta\,;\,n)$ *for some* $\beta \in U(Z)$.

Proof. (1) → (2). Let ϕ be a BG-ring isomorphism of A_b into A_c, and $\phi(x_b) = \sum_{i=0}^{n-1} x_c^i b_i\,(b_i \in B)$. Then $\sum_{i=0}^{n-1} x_c^i \zeta b_i = \phi\sigma(x_b) = \sigma\phi(x_b) = \sum_{i=0}^{n-1} x_c^i \zeta^i b_i$ implies $\phi(x_b) = x_c b_1$. Noting that $x_c\rho(d)b_1 = d(x_c b_1) = d\phi(x_b) = \phi(dx_b) = \phi(x_b\rho(d)) = x_c b_1\rho(d)$ and $\phi^{-1}(x_c) = x_b b_1'$ with some $b_1' \in B$, we can easily see $b_1 \in U(Z)$.

(2) → (3). This is obvious by $b = (x_b)^n = \phi(x_b^n) = (x_c\alpha)^n = x_c^n LN_\rho(\alpha\,;\,n) = cLN_\rho(\alpha\,;\,n)$.

(3) → (1). Let ϕ be the mapping of A_b into A_c defined by $\sum_{i=0}^{n-1} x_c^i b_i \to \sum_{i=0}^{n-1}(x_c\beta)^i b_i$. Then $\phi(b) = \phi(x_b^n) = (x_c\beta)^n = cLN_\rho(\beta\,;\,n) = b$ and ϕ is a B-ring isomorphism. Morever, $\sigma\phi(x_b) = \sigma(x_c\beta) = x_c\zeta\beta = \phi\sigma(x_b)$ shows that ϕ is a BG-ring isomorphism.

Lemma 1.2. $B_1(\rho^n)$ *coincides with* uZ_1.

Proof. Obviously $uZ_1 \subseteq B_1(\rho^n)$. Conversely, if b is an element of $B_1(\rho^n)$ then $cb = b\rho^n(c) = bu^{-1}cu$ for all $c \in B$. Hence $bu^{-1} \in Z \cap B_1 = Z_1$.

Theorem 1.1. (1) $P_\rho(B)$ *is an abelian semigroup with the identity* $\langle A_u \rangle$ *under the composition* $*$ *defined by* $\langle A_b \rangle * \langle A_c \rangle = \langle A_{bcu^{-1}} \rangle$. *Moreover,* $\langle A_b \rangle$ *is an element of* $U(P_\rho(B))$ *if and only if* $b \in U(B)$.

(2) $P_\rho(B)$ *is isomorphic to the factor semigroup* $Z_1/LN_\rho(U(Z)\,;\,n)$. *In particular,* $U(P_\rho(B))$ *is isomorphic to* $U(Z_1)/LN_\rho(U(Z)\,;\,n)$.

Proof. (1) Since $\langle A_b \rangle = \langle A_c \rangle$ if and only if $b = cLN_\rho(\alpha\,;\,n)$ with some $\alpha \in U(Z)$ (Lemma 1.1), the assertion is evident by Lemma 1.2.

(2) By Lemma 1.2, the mapping $f: z \to \langle A_{uz} \rangle\ (z \in Z_1)$ is a semigroup epimorphism of Z_1 onto $P_\rho(B)$. Then $Z_1/LN_\rho(U(Z)\,;\,n)$ is isomorphic to $P_\rho(B)$ by Lemma 1.1, and the rest is obvious.

Proposition 1.1. *If* $A_b \in \Omega_\rho(B)$ *then the following are equivalent:*

(1) A_b/B *is a separable extension.*

(2) A_b/B *is a strongly G-cyclic extension.*

(3) b *is invertible in* B.

Proof. Since (2)→(1) is known and (3)→(2) is evident by $\sigma(x_b) = x_b\zeta$ (see [2]), it remains only to prove (1)→(3). Now, we write $A = A_b$

and $x = x_b$, and consider a separable coordinate system for A/B: $\{\sum_{i=0}^{n-1} x^i a_{ik}\ ;\ \sum_{i=0}^{n-1} x^i b_{ik} \mid k = 1, 2, \cdots, m\}$. Then,

$$(1) \qquad 1 = \sum_{k=1}^{m} ((\sum_{i=0}^{n-1} x^i a_{ik}) (\sum_{j=0}^{n-1} x^j b_{jk}))$$

and for any $x \in A$

$$(2) \qquad x (\sum_{k=1}^{m} ((\sum_{i=0}^{n-1} x^i a_{ik}) \otimes (\sum_{j=0}^{n-1} x^j b_{jk})) = (\sum_{k=1}^{m} ((\sum_{i=0}^{n-1} x^i a_{ik}) \otimes (\sum_{j=0}^{n-1} x^j b_{jk}) x.$$

By a direct computation, (1) yields

$$1 = \sum_{k=1}^{m} (\sum_{l=0}^{2n-2} x^l (\sum_{i+j=n} \rho^j (a_{ik}) b_{jk})).$$

Hence, we have

$$(3) \qquad 1 = \sum_{k=1}^{m} a_{0k} b_{0k} + \sum_{k=1}^{m} b (\sum_{i+j=n} \rho^j (a_{ik}) b_{jk}).$$

We put $A \otimes_B A = (x \otimes 1) B \oplus M$ as a B-module. Then, by (2) we have $x (\sum_{k=1}^{m} ((\sum_{i=0}^{n-1} x^i a_{ik}) \otimes (\sum_{j=0}^{n-1} x^j b_{jk}))) = \sum_{k=1}^{m} ((\sum_{i=0}^{n-1} x^{i+1} a_{ik}) \otimes (\sum_{j=0}^{n-1} x^j b_{jk})) = \sum_{k=1}^{m} ((x \otimes 1) a_{0k} b_{0k}) + f$ with some $f \in M$, and $(\sum_{k=1}^{m} ((\sum_{i=0}^{n-1} x^i a_{ik}) \otimes (\sum_{j=0}^{n-1} x^j b_{jk}))) x = \sum_{k=1}^{m} ((x \otimes 1) a_{1k}\, b\rho (b_{n-1k})) + g$ with some $g \in M$, Comparing the coefficients of $x \otimes 1$ of the both above, we have

$$(4) \quad \sum_{k=1}^{m} a_{0k} b_{0k} = \sum_{k=1}^{m} a_{1k} b\rho(b_{n-1k}) = \sum_{k=1}^{m} b\rho^n(a_{1k})\rho(b_{n-1k}).$$

Now, by (3) and (4), $1 = \sum_{k=0}^{m} a_{0k} b_{0k} + \sum_{k=1}^{m} b (\sum_{i+j=n} \rho^j (a_{ik}) b_{jk}) = \sum_{k=1}^{m} b\rho^n(a_{1k})\rho(b_{n-1k}) + \sum_{k=1}^{m} b (\sum_{i+j=n} \rho^j (a_{ik}) b_{jk}) = b (\sum_{k=1}^{m} (\rho^n(a_{1k})\rho(b_{n-1k}) + \sum_{i+j=n} \rho^j (a_{ik}) b_{jk}))$, namely, b has a right inverse. Since $bc = \rho^{-n}(c) b$ for any $c \in B$, we see that $b \in U(B)$.

Now, let A/B be a strongly G-cyclic extension with $A_B \oplus > B_B$. Then, A is BG-ring isomorphic to $\langle A_b \rangle \in \Omega_{\rho'}(B)$ for some automorphism ρ' and $b \in U(B)$ ([2]). Thus, if A/B is a strongly G-cyclic extension of ρ-automorphism type and $A_B \oplus > B_B$, then A is BG-ring isomorphic to some A_b with $\langle A_b \rangle \in U(P_\rho(B))$. Conversely, if $\langle A_b \rangle$ is in $U(P_\rho(B))$, then Prop. 1.1 and Th. 1.1 show that C/B is a strongly G-cyclic extension of ρ-automorphism type for any $C \in \langle A_b \rangle$. Summarizing those above, we obtain the following

Corollary 1.1. $U(P_\rho(B)) = \{ \langle A \rangle \in P_\rho (B) \mid A/B$ *is separable*$\}$ *represents the set of all BG-ring isomorphism classes of strongly G-cyclic extensions* A *of* ρ*-automorphism type with* $A_B \oplus > B_B$.

The next is also an easy consequence of Th. 1.1.

Corollary 1.2. *If the restriction* $\rho \mid Z$ *of* ρ *to* Z *coincides with the identity,* $P_\rho(B) \simeq Z/U(Z)^n \simeq P_1(B) \simeq P_1(Z)$. *In particular,* (1) *if* ρ *is inner then* $P_\rho(B) \simeq P_1(B)$, *and* (2) *if* B *is commutative then* $P_1(B) \simeq B/U(B)^n$.

If η is a ring isomorphism of B onto a ring B' with the center Z', then there exists a unique automorphism ρ' of B' with $\eta\rho = \rho'\eta$. Obviously, $\rho'^n = \widetilde{\eta(u)^{-1}}$, $\eta(u) \in U(B'^{\rho'}) \cap B'(\rho'^n)$, $\eta(Z_1) = Z'^{\rho'}$ and $\eta(LN_\rho(U(Z);n)) = LN_{\rho'}(U(Z');n)$. Accordingly, if $\bar{\rho}$ is the conjugate class of ρ in the group $\mathfrak{A}$ of all ring automorphisms of B and $\rho^* \in \bar{\rho}$ then $\rho^{*n} = \widetilde{u}^*$ with some $u^* \in U(B^{\rho^*}) \cap B(\rho^{*n})$. Now, the next is obvious.

Theorem 1.2. (1) *if* η *is a ring isomorphism of* B *onto a ring* B' *then there exists a unique automorphsim* ρ' *of* B' *such that* $\eta\rho' = \rho\eta$ *and* $P_\rho(B) \simeq P_{\rho'}(B')$. *In particular,* $P_1(B) \simeq P_1(B')$.

(2) $P_\rho(B) \simeq P_{\rho^*}(B)$ *for any* $\rho^* \in \bar{\rho}$.

2. Aclassification of G-cyclic extensions of D-derivation type. In this section, we assume that B is an alegebra over $GF(p)$, D a derivation of B such that $D^p = I_u$ with some $u \in B_0(D^p)$, and that G is a cyclic group of order p with a generator σ.

If $A = B[X; D]/(X^p - X - b)B[X; D] \in \Omega_D(B)$ then we write $A = A_b$ and $X + (X^p - X - b)B[X; D] = x_b$. As was mentioned in §0, A_b/B is a cyclic extension via $\sigma(x_b) = x_b + 1$. In all that follows, A_b will be understood as a left BG-module via $\sigma(\sum_{i=0}^{n-1} x_b^i b_i) = \sum_{i=0}^{n-1}(x_b + 1)^i b_i$. While, if C/B is a G-cyclic extension with $C_B \oplus > B_B$, then C is BG-ring isomorphic to $A_b \in \Omega_{D'}(B)$ with some derivation D' of B. Thus, the set $P_D(B)$ of all BG-ring isomorphism classes $\langle A \rangle$ of $A \in \Omega_D(B)$ may be regarded as the set of all BG-ring isomorphim classes of G-cyclic extensions A of D-derivation type with $A_B \oplus > B_B$.

Given $b \in B$, we put $\Delta_0^D(b) = 1$ and $\Delta_i^D(b) = D(\Delta_{i-1}^D(b)) + \Delta_{i-1}^D(b)b$ for $i \geqq 1$. Then, in $B[X; D]$ there holds

$$(X + b)^n = \sum_{i=0}^{n} X^i \binom{n}{i} \Delta_{n-i}^D(b)$$

(see [1]). From this we have

$$(X + b)^p = X^p + \Delta_p^D(b).$$

Lemma 2.1. (1) *If* $D = 0$ *then* $\Delta_p^D(b) = b^p$ *for all* $b \in B$.

(2) Δ_p^D *is an endomorphism of* $(Z_1, +)$.

Proof: It suffices to prove (2). Let $w, z \in Z$. Since $w(X + z) =$

$Xw + D(w) + wz = Xw + zw + D(w) = (X + z)w + D(w)$, one will easily see that $X^p + \Delta_p^D(z+w) = (X + (z+w))^p = ((X+z) + w)^p = (X+z)^p + \Delta_p^D(w) = X^p + \Delta_p^D(z) + \Delta_p^D(w)$. Hence Δ_k^D is a homomorphism of $(Z, +)$ into $(B, +)$. Next, we shall show that $\Delta_k^D(z) \in Z$. To our end, recalling that $\Delta_{k+1}^D(z) = \Delta_k^D(z)z + D(\Delta_k^D(z))$, it suffices to prove that if $\Delta_k^D(z)$ is in Z then $D(\Delta_k^D(z))$ also in Z. In fact, this is an easy combination of $D(c\Delta_k^D(z)) = D(\Delta_k^D(z)c)$ and $D(c)\Delta_k^D(z) = \Delta_k^D(z)D(c)$ $(c \in B)$.

Now, corresponding to Lemma 1. 1, we shall prove the following

Lemma 2.2. *If* $A_b, A_c \in \Omega_D(B)$, *then the following are equivalent*:

(1) $\langle A_b \rangle = \langle A_c \rangle$.

(2) *There exists a BG-ring isomorphism* ϕ *of* A_b *into* A_c *such that* $\phi(x_b) = x_c + \alpha$ *with some* $\alpha \in Z$.

(3) $b = c + \Delta_p^D(\beta) - \beta$ *for some* $\beta \in Z$.

Proof. (1) → (2). Let ϕ be a BG-ring isomorphism of A_b into A_c, and $\phi(x_b) = \sum_{i=0}^{p-1} x_c{}^i b_i$ $(b_i \in B)$. Then, $\sum_{i=0}^{p-1}(x_c + 1)^i b_i = \sigma\phi(x_b) = \phi\sigma(x_b) = \phi(x_b + 1) = \sum_{i=0}^{p-1} x_c{}^i b_i + 1$ implies $\phi(x_b) = x_c + b_0$. Noting here that $x_c + d + D(d) + db_0 = d(x_c + b_0) = d\phi(x_b) = \phi(dx_b d + D(d)) = x_c d + b_0 d + D(d)$ for all $d \in B$, we can see $b_0 \in Z$.

(2) → (3). To be easily seen, $b = x_b{}^p = \phi(x_b{}^p) = (x_c + \alpha)^p = x_c^p + \Delta_p^D(\alpha) - \alpha = c + \Delta_p^D(\alpha) - \alpha$.

(3) → (1). Let ϕ be the mapping of A_b into A_c defined by $\sum_{i=0}^{n-1} x_b{}^i b_i \to \sum_{i=0}^{n-1}(x_c + \beta)^i b_i$. Then $\phi(b) = \phi(x_b{}^p) = (x_c + \beta)^p = c + \Delta_p^D(\beta) - \beta = b$ and ϕ is a B-ring isomorphism. Moreover, $\sigma\phi(x_b) = \sigma(x_c + \beta) = x_c + 1 + \beta = \phi\sigma(x_b)$ shows that ϕ is a BG-ring isomorphism.

Lemma 2. 3. $B_0(D^p)$ *coincides with* $u + Z_0$.

Proof. Obviously, $u + Z_0 \subseteq B_0(D^p)$. Conversely, if b is an element of $B_0(D^p)$ then $I_b(c) = D^p(c) = I_u(c)$ for all $c \in B$. Hence, $u - b \in Z \cap B_0 = Z_0$.

Combining Lemma 2. 2 with Lemma 2. 3, we obtain the following

Theorem 2.1. (1) $P_D(B)$ *is an abelian group with the identity* $\langle A_u \rangle$ *under the composition* $*$ *defined by* $\langle A_b \rangle * \langle A_c \rangle = \langle A_{b+c-u} \rangle$.

(2) $P_D(B)$ *is isomorphic to the factor abelian group* $Z_0/\Delta^D(Z)$ *where* $\Delta^D(Z) = \{\Delta_p^D(\alpha) - \alpha \mid \alpha \in Z\} \cap Z_0$.

Proof. (1) Since $\langle A_b \rangle = \langle A_c \rangle$ if and only if $b = c + \Delta_p^D(\beta) - \beta$ with some $\beta \in Z$ (Lemma 2. 2), the assertion is evident by Lemma 2. 3.

(2) By Lemma 2.3, the mapping $f: z \to \langle A_{u+z} \rangle$ $(z \in Z_0)$ is a group epimorphism of Z_0 onto $P_D(B)$. Then $Z_0/\Delta^D(Z)$ is isomorphic to $P_D(B)$ by Lemma 2.2.

Recall here that if $D = 0$ then $\Delta^D(Z) = Z^{\mathfrak{p}}$ (Lemma 2.1), the next will be an immediate consequence of Th. 2.1.

Corollary 2.1. *If* $D|Z = 0$ *then* $P_D(B) \simeq Z/Z^{\mathfrak{p}} \simeq P_0(B) \simeq P_0(Z)$. *In particular,* (1) *if* D *is inner then* $P_D(B) \simeq P_0(B)$ *and* (2) *if* B *is commutative then* $P_0(B) \simeq B/B^{\mathfrak{p}}$.

If η is a ring isomorphism of B onto a ring B' with the center Z', then there exists a unique derivation D' of B' with $\eta D = D'\eta$. Obviously, $D'^{\mathfrak{p}} = I_{\eta(u)}$, $\eta(u) \in B'^{D'} \cap B'(D'^{\mathfrak{p}})$, $\eta(Z_0) = Z'^{D'}$ and $\eta(\Delta^D(Z)) = \Delta^{D'}(Z')$. Accordingly, if $\bar{E}$ is the similar class of E in the additive group $\mathfrak{D}$ of all derivations of B, where E and E^* in $\mathfrak{D}$ are similar if there exists a ring automorphism η of B with $\eta E = E^*\eta$, and if $E^{\mathfrak{p}} = I_v$ with $v \in B^E \cap B(E^{\mathfrak{p}})$ then for each $E^* \in \bar{E}$ we have $E^{*\mathfrak{p}} = I_{v^*}$ with some $v^* \in B^{E^*} \cap B(E^{*\mathfrak{p}})$. Now, the next is obvious.

Theorem 2.2. (1) *If* η *is a ring isomorphism of* B *onto a ring* B' *then there exists a unique derivation* D' *of* B' *such that* $\eta D = D'\eta$ *and* $P_D(B) \simeq P_{D'}(B')$. *In particular,* $P_0(B) \simeq P_0(B')$.

(2) $P_D(B) \simeq P_{D^*}(B)$ *for any* $D^* \in \bar{D}$.

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