A NOTE ON MORITA EQUIVALENCE OF POLYNOMIAL RINGS

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Throughout A and B will represent rings with 1. If A and B are Morita equivalent, namely, if there exist a positive integer n and an idempotent e of the complete matrix ring $(A)_n$ such that B is isomorphic to $e(A)_n e$ and $(A)_n e(A)_n = (A)_n$, then one will easily see that the polynomial rings $A[X]$ and $B[X]$ (in a commutative indeterminate X) are Morita equivalent. Needless to say, in case A and B are commutative, if A and B are Morita equivalent then they are isomorphic. Several authors studied the question: If $A[X] \cong B[X]$, does $A \cong B$ follow? A simple counter-example has been given by M. Hochster [4]. However, D. B. Coleman and E. E. Enochs proved

Proposition (see [3, p. 252]). *Assume that the Jacobson radical $J(A)$ of A is locally nilpotent and $A/J(A)$ is Artinian (A is semi-local). If $A[X] \cong B[X]$ then $A \cong B$.*

The purpose of this note is to prove the following theorem which is motivated by the last:

Theorem. *Let A be a semi-local ring with $J(A)$ locally nilpotent. If $A[X]$ and $B[X]$ are Morita equivalent, then so are A and B.*

In advance of proving our theorem, we state an easy lemma, which is contained in [2, Proposition 0. 2. 6].

Lemma. *Let D be a division ring, and $R = D[X]$. If e is an idempotent of $(R)_n$ then there exists a unit u of $(R)_n$ such that $ueu^{-1} =$* diag $\{1, \cdots, 1, 0, \cdots, 0\}$.

Proof. As is well known, R is a principal ideal domain, and every submodule of the left free R-module R^n of $1 \times n$ matrices with entries in R has a free R-basis. Regarding $(R)_n$ as the endomorphism ring of the left R-module R^n, we have $R^n = R^n e \oplus R^n(1-e)$. Now, let $\{u_1, \cdots, u_r\}$ and $\{u_{r+1}, \cdots, u_n\}$ be respective free R-bases of $R^n e$ and $R^n(1-e)$. Then $u = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix}$ is obviously a unit of $(R)_n$, and $ue = \text{diag}\ \{1, \cdots, 1, 0, \cdots, 0\}\, u$.

Proof of Theorem. There exist a positive integer n and an idempotent e of $(A)_n[X]$ $(\cong (A[X])_n)$ such that $B[X] \cong e(A)_n[X]e$ and $(A)_n[X] \cdot e \cdot (A)_n[X] = (A)_n[X]$. Obviously, $R = (A)_n$ is a semi-local ring with $J(R) = (J(A))_n$ locally nilpotent. Let $\bar{R} = R/J(R) = (D_1)_{n_1} \oplus \cdots \oplus (D_k)_{n_k}$, where D_i's are division rings. Since $J(R[X]) = J(R)[X]$ by [1, Theorem 1], we have $R[X]/J(R[X]) \cong \bar{R}[X] \cong (D_1)_{n_1}[X] \oplus \cdots \oplus (D_k)_{n_k}[X]$. Now, in virtue of Lemma, we can choose an idempotent $\bar{e}'$ of $\bar{R}$ which is isomorphic to the homomorphic image of e in $\bar{R}[X]$. The lifted idempotent $e' \in R$ is isomorphic to e. Hence, without loss of generality, we may assume from the beginning that e is in R. Then, $B[X] \cong eR[X]e = eRe[X]$ and $ReR = R$. Since $J(eRe) = eJ(R)e$ is locally nilpotent and $eRe/J(eRe)$ is Artinian, $B \cong eRe$ by Proposition. Thus, B is Morita equivalent to A.

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