ON THE HYPERSPACE OF A QUASI-UNIFORM SPACE

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1. Introduction

1.1. In [4], E. Michael studied various topologies on the hyperspace including the uniform topology and the finite topology; J. R. Isbell also studied the uniform topology on hyperspaces in [1].

Given a quasi-uniform space $(X, \mathcal{U})$, we define a quasi-uniformity $2^{\mathcal{U}}$ on the hyperspace 2^X in a natural way and investigate some of its stable properties.

If we are given a topological space $(X, \mathcal{T})$ and let $\mathcal{P}(\mathcal{T})$ be Pervin's quasi-uniformity, then the quasi-uniform topology $\mathcal{T}(2^{\mathcal{P}(\mathcal{T})})$ is shown to coincide with $2^{\mathcal{T}}$, the finite topology on 2^X.

In § 3, we show that every quasi-uniform space has a compactification.

We shall use the definitions and properties of quasi-uniform spaces as developed by M. G. Murdeshwar and S. A. Naimpally in [5].

Finally, all spaces are presumed to be T_1.

1.2. Let $(X, \mathcal{T})$ be a topological space; 2^X denotes the set of all nonempty closed sets and is called the hyperspace of X. For each $A \subseteq X$, let $\langle A\rangle = \{E : E \in 2^X \text{ and } E \subseteq A\}$. $\overline{2}^{\mathcal{T}}$ is the topology for 2^X with $\{\langle O\rangle : O \in \mathcal{T}\}$ as base and is called the upper-semi-finite topology. For each $A \subseteq X$, let $\langle X, A\rangle = \{E : E \in 2^X \text{ and } A \cap E \neq \emptyset\}$; $\underline{2}^{\mathcal{T}}$ is the topology for 2^X with $\{\langle X, O\rangle : O \in \mathcal{T}\}$ as subbase and is called the lower-semi-finite topology. Finally, we let $2^{\mathcal{T}} = \underline{2}^{\mathcal{T}} \vee \overline{2}^{\mathcal{T}}$; $2^{\mathcal{T}}$ is called the finite topology for 2^X. We note that $\{x\} \in 2^X$ for each x in X since T_1 is presumed.

1.3. Let $(X; \mathcal{U})$ be a quasi-uniform space (all axioms for a uniform space hold except perhaps the symmetry axiom). For each $U \in \mathcal{U}$, we make the following definitions:

(i) $\overline{H}(U) = \{(A, B) : A, B \in 2^X \text{ and } B \subseteq U[A]\}$
(ii) $\underline{H}(U) = \{(A, B) : A, B \in 2^X \text{ and } A \subseteq U^{-1}[B]\}$
(iii) $H(U) = \overline{H}(U) \cap \underline{H}(U)$.

We note the asymmetry in the definitions of $\underline{H}(U)$ and $\overline{H}(U)$.

Theorem 1.3.1. *Let $(X, \mathcal{U})$ be a quasi-uniform space. Then* $\{\overline{H}(U) : U \in \mathcal{U}\}$, $\{\underline{H}(U) : U \in \mathcal{U}\}$ and $\{H(U) : U \in \mathcal{U}\}$ *are bases for quasi-uniformities for* 2^X *respectively denoted by* $\bar{2}^{\mathcal{U}}$, $\underline{2}^{\mathcal{U}}$ *and* $2^{\mathcal{U}}$. *These are called the upper-hyper-quasi-uniformity, the lower-hyper-quasi-uniformity and the hyper-quasi-uniformity respectively.*

Proof. The fact that $\{\overline{H}(U) : U \in \mathcal{U}\}$ is a base for a uniformity for 2^X follows from the identities :

(i) $\overline{H}(U) \circ \overline{H}(U) \subseteq \overline{H}(U \circ U)$ when $U \in \mathcal{U}$

(ii) $\triangle \subseteq \overline{H}(U) \subseteq \overline{H}(V)$ when $U \subseteq V$, $U, V \in \mathcal{U}$.

Similarly for $\{\underline{H}(U) : U \in \mathcal{U}\}$ and $\{H(U) : U \in \mathcal{U}\}$.

We note that when $(X, \mathcal{U})$ is a separated uniform space, then $2^{\mathcal{U}}$ is the uniformity studied by Michael and Isbell.

Theorem 1. 3. 2. *Let $(X, \mathcal{U})$ be a quasi-uniform space and let* $\mathcal{U}^{-1} = \{U^{-1} : U \in \mathcal{U}\}$. *Then* (i) $(X, \mathcal{U}^{-1})$ *is a T_1-quasi-uniform space* (ii) $\bar{2}^{\mathcal{U}^{-1}} = (\underline{2}^{\mathcal{U}})^{-1}$ (iii) $\underline{2}^{\mathcal{U}^{-1}} = (\bar{2}^{\mathcal{U}})^{-1}$ (iv) $2^{\mathcal{U}} = \bar{2}^{\mathcal{U}} \vee \underline{2}^{\mathcal{U}}$, $\vee$ *denoting supremum* (v) $2^{\mathcal{U}^{-1}} = (2^{\mathcal{U}})^{-1}$.

Proof. (i) is cited in [5]. (ii) follows from the fact that $\overline{H}(U^{-1}) = (\underline{H}(U))^{-1}$ (iii) follows from the identity $\underline{H}(U^{-1}) = (\overline{H}(U))^{-1}$. (iv) $H(U) = \overline{H}(U) \cap \underline{H}(U)$ implies that $2^{\mathcal{U}} = \bar{2}^{\mathcal{U}} \vee \underline{2}^{\mathcal{U}}$. (v) follows from (ii) and (iii) and the fact that $(\mathcal{U} \vee \mathcal{V})^{-1} = \mathcal{U}^{-1} \vee \mathcal{V}^{-1}$ when $\mathcal{U}$ and $\mathcal{V}$ are quasi-uniformities (see [5]).

1. 4. In [4], Michael shows that when $(X, \mathcal{T})$ is a T_1-space, the function $i : X \to 2^X$ defined by $i(x) = \{x\}$ is a homeomorphism from $(X, \mathcal{T})$ into $(2^X, 2^{\mathcal{T}})$. For a separated uniform space the function $i : X \to 2^X$ is a unimorphism from $(X, \mathcal{U})$ into $(2^X, 2^{\mathcal{U}})$. In this sense, $2^{\mathcal{T}}$ is an admissible topology for 2^X and $2^{\mathcal{U}}$ is an admissible uniformity for 2^X.

We show next that in this sense, $\underline{2}^{\mathcal{U}}$, $\bar{2}^{\mathcal{U}}$ and $2^{\mathcal{U}}$ are admissible quasi-uniformities for 2^X when $(X, \mathcal{U})$ is a T_1 quasi-uniform space.

Theorem 1. 4. 1. *Let $(X, \mathcal{U})$ be a quasi-uniform space. Then each of the following are unimorphisms* :

(i) $i : (X, \mathcal{U}) \to (i[X], \bar{2}^{\mathcal{U}} \cap i[X] \times i[X])$

(ii) $i : (X, \mathcal{U}) \to (i[X], \underline{2}^{\mathcal{U}} \cap i[X] \times i[X])$

(iii) $i : (X, \mathcal{U}) \to (i[X], 2^{\mathcal{U}} \cap i[X] \times i[X])$.

Proof. (i) follows from the fact that $U = (i \times i)^{-1}[\overline{H}(U)]$ and

$(i\times i)[U]=\overline{H}(U)\cap i[X]\times i[X]$ when $U\in\mathcal{U}$. Similarly for (ii) and (iii).

Note that the assumption of T_1 is vital here.

Theorem 1. 4. 2. *Let* $(X, \mathcal{U})$ *be a quasi-uniform space and suppose that* $\mathcal{B}\subseteq\mathcal{U}$. *The following are equivalent* :

(i) $\mathcal{B}$ *is a base for* $\mathcal{U}$

(ii) $\{\overline{H}(B) : B\in\mathcal{B}\}$ *is a base for* $\overline{2}^{\mathcal{U}}$

(iii) $\{\underline{H}(B) : B\in\mathcal{B}\}$ *is a base for* $\underline{2}^{\mathcal{U}}$

(iv) $\{H(B) : B\in\mathcal{B}\}$ *is a base for* $2^{\mathcal{U}}$.

Proof. (i) is equivalent to (ii) since $B\subseteq U$ is equivalent to $\overline{H}(B)\subseteq\overline{H}(U)$. Similarly for the equivalence of (i) and (iii), and (i) and (iv).

The following example indicates that Theorem 1. 4. 2 cannot be generalized to subbase.

Example 1. 4. 3. Let $(X, \mathcal{U})$ be the unit interval with the usual uniformity. Let $\mathcal{S}=\{U\cup\{a\}\times X : U\in\mathcal{U},\ a=0 \text{ or } a=1\}$. Then both $\mathcal{S}$ and $\mathcal{S}^{-1}$ are subbases for $\mathcal{U}$. But $\{\overline{H}(S) : S\in\mathcal{S}\}$ is not a subbase for $\overline{2}^{\mathcal{U}}$, $\{\underline{H}(S^{-1}) : S^{-1}\in\mathcal{S}^{-1}\}$ is not a subbase for $\underline{2}^{\mathcal{U}}$ and finally, $\{H(S) : S\in\mathcal{S}\}$ is not a subbase for $2^{\mathcal{U}}$. To see this, let $A=\{0, 1\}$ and $B=X$. Let $S\in\mathcal{S}$. Then $S[A]=X=S[B]$ and $S^{-1}[B]=X$ as the reader can easily see. Thus $(A, B)\in\overline{H}(S)$, $(B, A)\in\underline{H}(S^{-1})$ and $(A, B)\in H(S)$. Let $V=\{(x, y) : |x-y|<1/2\}$. Then $V\in\mathcal{U}$, but $(A, B)\notin H(V)$, $(A, B)\notin\overline{H}(V)$ and $(B, A)\notin\underline{H}(V)$.

Theorem 1. 4. 4. *Let* $(X, \mathcal{U})$ *be a quasi-uniform space. The following are equivalent* :

(i) $\mathcal{U}$ *is a uniformity*

(ii) $\overline{2}^{\mathcal{U}}=(\underline{2}^{\mathcal{U}})^{-1}$

(iii) $2^{\mathcal{U}}$ *is a uniformity.*

Proof. (i) implies (ii). If $\mathcal{U}$ is a uniformity, then $\mathcal{U}=\mathcal{U}^{-1}$. Then $\overline{2}^{\mathcal{U}}=\overline{2}^{\mathcal{U}^{-1}}=(\underline{2}^{\mathcal{U}})^{-1}$ by (ii) of Theorem 1. 3. 2.

(ii) implies (iii). $2^{\mathcal{U}}=\overline{2}^{\mathcal{U}}\vee\underline{2}^{\mathcal{U}}=(\underline{2}^{\mathcal{U}})^{-1}\vee(\overline{2}^{\mathcal{U}})^{-1}=(2^{\mathcal{U}})^{-1}$ by (iv) of Theorem 1. 3, 2

(iii) implies (i). If $2^{\mathcal{U}}$ is a uniformity, then $2^{\mathcal{U}}\cap i[x]\times i[x]$ is a uniformity and by (iii) of Theorem 1. 4. 1, $\mathcal{U}$ is a uniformity.

2. The Hyperspace of Pervin's Quasi-Uniformity

2. 1. For $A\subseteq X$, let $S(A)=A\times A\cup CA\times X$, C denoting the complement operator. In [6], Pervin showed that for a given topological space $(X, \mathcal{T})$, $\{S(O) : O \in \mathcal{T}\}$ is a subbase for a quasi-uniform space $(X, \mathcal{P}(\mathcal{T}))$ with the property that $\mathcal{T}(\mathcal{P}(\mathcal{T})) = \mathcal{T}$.

In this section, we will show that if $(X, \mathcal{T})$ is a topological space and $\mathcal{P}(\mathcal{T})$ is Pervin's quasi-uniformity, then $2^{\mathcal{T}} = \mathcal{T}(2^{\mathcal{P}(\mathcal{T})})$.

Several properties of Pervin's quasi-uniformity were developed by Levine in [3]. Applications were also made in [5].

It is worth noting that $S(A) = (S(\mathcal{C}A))^{-1}$ for all sets $A \subseteq X$.

Theorem 2.1.1. *Let* $(X, \mathcal{U})$ *be a quasi-uniform space and* $\mathcal{T} = \mathcal{T}(\mathcal{U})$. *Then*

$$\text{(i)} \quad \mathcal{T}(\bar{2}^{\mathcal{U}}) \subseteq \bar{2}^{\mathcal{T}} \quad \textit{and} \quad \text{(ii)} \quad \underline{2}^{\mathcal{T}} \subseteq \mathcal{T}(\underline{2}^{\mathcal{U}}).$$

Proof. (i). Let $E \in O \in \mathcal{T}(\bar{2}^{\mathcal{U}})$. There exists then a $U \in \mathcal{U}$ such that $\bar{H}(U)[E] \subseteq O$. But $E \in \langle \operatorname{Int} U[E] \rangle \subseteq \bar{H}(U)[E]$ as the reader can easily show.

(ii) It suffices to show that $\langle X, O \rangle \in \mathcal{T}(\underline{2}^{\mathcal{U}})$ when $O \in \mathcal{T}$. Let $A \in \langle X, O \rangle$. Then $A \cap O \neq \emptyset$; let $a \in A \cap O$. There exists then a $U \in \mathcal{U}$ such that $U[a] \subseteq O$. We show now that $A \in \underline{H}(U)[A] \subseteq \langle X, O \rangle$. Let $B \in \underline{H}(U)[A]$. Then $A \subseteq U^{-1}[B]$ and hence $\emptyset \neq U[a] \cap B \subseteq B \cap O$. Thus $B \in \langle X, O \rangle$.

Theorem 2.1.2. *Let* $(X, \mathcal{T})$ *be a topological space and suppose that* $\mathcal{P}(\mathcal{T})$ *is Pervin's quasi-uniformity. If* $\mathcal{S} = \{S(O) : O \in \mathcal{T}\}$, *then* (i) $\{\bar{H}(S) : S \in \mathcal{S}\}$ *is a subbase for* $\bar{2}^{\mathcal{P}(\mathcal{T})}$, (ii) $\{\underline{H}(S) : S \in \mathcal{S}\}$ *is a subbase for* $\underline{2}^{\mathcal{P}(\mathcal{T})}$ *and* (iii) $\{H(S) : S \in \mathcal{S}\}$ *is a subbase for* $2^{\mathcal{P}(\mathcal{T})}$.

Proof. (i) Let $O_i \in \mathcal{T}$ for $1 \leqq i \leqq n$ and for $\emptyset \neq \delta \subseteq \{1, \cdots, n\}$, let $O_\delta = \cup \{O_i : i \in \delta\}$. It suffices to show that $\cap \{\bar{H}(S(O_\delta)) : \emptyset \neq \delta \subseteq \{1, \cdots, n\}\} \subseteq \bar{H}(S(O_1) \cap \cdots \cap S(O_n))$. Let (A, B) be a member of the left side and take $b \in B$. It suffices to show that there exists an a in A such that $(a, b) \in S(O_i)$ for $1 \leqq i \leqq n$.

Case 1. $b \in O_i$ for each i. Then any a in A will do.

Case 2. $b \notin \cap \{O_i : 1 \leqq i \leqq n\}$. Let $\delta = \{i : b \notin O_i\}$.

Then $(A, B) \in \bar{H}(S(O_\delta))$ and hence there exists an $a \in A$ such that $(a, b) \in S(O_\delta)$. If $(a, b) \notin S(O_j)$, then $a \in O_j$ and $b \notin O_j$ and hence $a \in O_\delta$. It follows then that $b \in O_\delta$, a contradiction.

(ii) Let $O_i \in \mathcal{T}$ for $1 \leqq i \leqq n$. For each $\emptyset \neq \delta \subseteq \{1, \cdots, n\}$, let $G_\delta = \cap \{O_i : i \in \delta\}$. It suffices to show that

$\cap \{\underline{H}(S(G_\delta)) : \emptyset \neq \delta \subseteq \{1, 2, \cdots, n\}\} \subseteq \underline{H}(S(O_1) \cap \cdots \cap S(O_n))$. Since $CG_\delta = \cup \{CO_i : i \in \delta\}$, it follows that $\cap \{\overline{H}(S(CG_\delta)) : \emptyset \neq \delta \subseteq \{1, \cdots, n\}\} \subseteq \overline{H}(S(CO_1) \cap \cdots \cap S(CO_n))$ using the argument in (i) above. Recalling that $S(A) = (S(CA))^{-1}$ (see § **2.1**) and $\overline{H}(U^{-1}) = (\overline{H}(U))^{-1}$, we have

$$\begin{aligned}\cap \{\underline{H}(S(G_\delta)) : \emptyset \neq \delta \subseteq \{1, \cdots, n\}\} &= \cap \{\underline{H}((S(CG_\delta))^{-1}) : \emptyset \neq \delta \subseteq \{1, \cdots, n\}\} \\ &= \cap \{(\overline{H}(S(CG_\delta)))^{-1} : \emptyset \neq \delta \subseteq \{1, \cdots, n\}\} \\ &= (\cap \{\overline{H}(S(CG_\delta)) : \emptyset \neq \delta \subseteq \{1, \cdots, n\}\})^{-1} \\ &\subseteq (\overline{H}(S(CO_1) \cap \cdots \cap S(CO_n)))^{-1} \\ &= \underline{H}(S(O_1) \cap \cdots \cap S(O_n))\end{aligned}$$

(iii) Let $O_i \in \mathcal{T}$ for $1 \leqq i \leqq n$. Let O_δ and G_δ be defined as in (i) and (ii) above. Then

$$\begin{aligned}&\cap \{H(S(O_\delta)) : \emptyset \neq \delta \subseteq \{1, \cdots, n\}\} \cap \cap \{H(S(G_\delta)) : \emptyset \neq \delta \subseteq \{1, \cdots, n\}\} \\ &\subseteq \cap \{\overline{H}(S(O_\delta)) : \emptyset \neq \delta \subseteq \{1, \cdots, n\}\} \cap \cap \{\underline{H}(S(G_\delta)) : \emptyset \neq \delta \subseteq \{1, \cdots, n\}\} \\ &\subseteq \overline{H}(S(O_1) \cap \cdots \cap S(O_n)) \cap \underline{H}(S(O_1) \cap \cdots \cap S(O_n)) \\ &= H(S(O_1) \cap \cdots \cap S(O_n)).\end{aligned}$$

Theorem 2. 1. 3. *Let* $(X, \mathcal{T})$ *be a topological space and suppose that* $O \in \mathcal{T}$. *Then*

(i) $\overline{H}(S(O)) = S(\langle O \rangle)$

(ii) $\underline{H}(S(O)) = S(\langle X, O \rangle)$

(iii) $H(S(O)) = S(\langle O \rangle) \cap S(\langle X, O \rangle)$.

Proof. (i) It suffices to show that $\overline{H}(S(O)) = \langle O \rangle \times \langle O \rangle \cup \langle X, CO \rangle \times 2^X$. Let A, B be in 2^X.

Case 1. $A \subseteq O$. Then $(A, B) \in \overline{H}(S(O))$ iff $B \subseteq S(O)[A]$ iff $B \subseteq O$ iff $(A, B) \in \langle O \rangle \times \langle O \rangle$ iff $(A, B) \in \langle O \rangle \times \langle O \rangle \cup \langle X, CO \rangle \times 2^X$.

Case 2. $A \nsubseteq O$. Then $(A, B) \in \overline{H}(S(O))$ iff $B \subseteq S(O)[A]$ iff $B \subseteq X$ iff $(A, B) \in \langle X, CO \rangle \times 2^X$ iff $(A, B) \in \langle O \rangle \times \langle O \rangle \cup \langle X, CO \rangle \times 2^X$.

$$\begin{aligned}\text{(ii)}\quad \underline{H}(S(O)) &= (\overline{H}(S(CO)))^{-1} = (S(\langle CO \rangle))^{-1} \quad \text{(by (i))} \\ &= S(C\langle CO \rangle) = S(\langle X, O \rangle).\end{aligned}$$

$$\begin{aligned}\text{(iii)}\quad H(S(O)) &= \underline{H}(S(O)) \cap \overline{H}(S(O)) \\ &= S(\langle X, O \rangle) \cap S(\langle O \rangle)\end{aligned}$$

Corollary 2. 1. 4. *Let* $(X, \mathcal{T})$ *be a topological space. Then* (i) $\overline{2}^{\mathcal{P}(\mathcal{T})} \subseteq \mathcal{P}(\overline{2}^{\mathcal{T}})$ (ii) $\underline{2}^{\mathcal{P}(\mathcal{T})} \subseteq \mathcal{P}(\underline{2}^{\mathcal{T}})$ *and* (iii) $2^{\mathcal{P}(\mathcal{T})} \subseteq \mathcal{P}(2^{\mathcal{T}})$.

Proof. (i) Let $O \in \mathcal{T}$. By (i) of Theorem 2. 1. 2., $\overline{H}(S(O))$ is subbasic in $\overline{2}^{\mathcal{P}(\mathcal{T})}$. But $\overline{H}(S(O)) = S(\langle O \rangle)$ by (i) of Theorem

2. 1. 3., and $S(\langle O\rangle)\in\mathcal{P}(\bar{2}^{\mathcal{T}})$.

(ii) Let $O\in\mathcal{T}$. By (ii) of Theorem 2. 1. 2, $\underline{H}(S(O))$ is subbasic in $\underline{2}^{\mathcal{P}(\mathcal{T})}$. By (ii) of Theorem 2. 1. 3, $\underline{H}(S(O))=S(\langle X, O\rangle)\in\mathcal{P}(\underline{2}^{\mathcal{T}})$

(iii) Let $O\in\mathcal{T}$. By (iii) of Theorem 2. 1. 2, $H(S(O))$ is subbasic in $2^{\mathcal{P}(\mathcal{T})}$. By (iii) of Theorem 2. 1. 3, $H(S(O))=S(\langle O\rangle)\cap S(\langle X, O\rangle)\in\mathcal{P}(2^{\mathcal{T}})$.

Theorem 2. 1. 5. *Let* $(X, \mathcal{T})$ *be a topological space. Then* (i) $\mathcal{T}(\bar{2}^{\mathcal{P}(\mathcal{T})})=\bar{2}^{\mathcal{T}}$ (ii) $\mathcal{T}(2^{\mathcal{P}(\mathcal{T})})=\underline{2}^{\mathcal{T}}$ *and* (iii) $\mathcal{T}(2^{\mathcal{P}(\mathcal{T})})=2^{\mathcal{T}}$.

Proof. (i) By (i) of Corollary 2. 1. 4, $\mathcal{T}(\bar{2}^{\mathcal{P}(\mathcal{T})})\subseteq\bar{2}^{\mathcal{T}}$. Hence it suffices to show that $\bar{2}^{\mathcal{T}}\subseteq\mathcal{T}(\bar{2}^{\mathcal{P}(\mathcal{T})})$ or that $\langle O\rangle\in\mathcal{T}(\bar{2}^{\mathcal{P}(\mathcal{T})}$ when $O\in\mathcal{T}$. Let $A\in\langle O\rangle$; by (i) of Theorem 2. 1. 3, $\bar{H}(S(O))[A]=S(\langle O\rangle)[A]\subseteq\langle O\rangle$.

(ii) By (ii) of Corollary 2. 1. 4, $\mathcal{T}(\underline{2}^{\mathcal{P}(\mathcal{T})})\subseteq\underline{2}^{\mathcal{T}}$ and from (ii) of Theorem 2. 1. 1, $\underline{2}^{\mathcal{T}}\subseteq\mathcal{T}(\underline{2}^{\mathcal{P}(\mathcal{T})})$.

(iii) Since $2^{\mathcal{P}(\mathcal{T})}=\bar{2}^{\mathcal{P}(\mathcal{T})}\vee\underline{2}^{\mathcal{P}(\mathcal{T})}$, it follows that $\mathcal{T}(2^{\mathcal{P}(\mathcal{T})})=\mathcal{T}(\bar{2}^{\mathcal{P}(\mathcal{T})})\vee\mathcal{T}(\underline{2}^{\mathcal{P}(\mathcal{T})})=\bar{2}^{\mathcal{T}}\vee\underline{2}^{\mathcal{T}}=2^{\mathcal{T}}$.

3. A Compactification of a Quasi-Uniform Space

Lemma 3. 1. *Let* $(X, \mathcal{T})$ *be a topological space. Then* $(2^X, \bar{2}^{\mathcal{T}})$ *is compact.*

This fact is well known and the easy proof is omitted.

Theorem 3. 2. *Let* $(X, \mathcal{U})$ *be a* T_1*-quasi-uniform space. Then* $(X, \mathcal{U})$ *has a compactification.*

Proof. By (i) of Theorem 1. 4. 1, $i:(X, \mathcal{U})\to(i[X], \bar{2}^{\mathcal{U}}\cap i[X]\times i[X])$ is a quasi-unimorphism and by (i) of Theorem 2. 1. 1, $\mathcal{T}(\bar{2}^{\mathcal{U}})\subseteq\bar{2}^{\mathcal{T}(\mathcal{U})}$. Thus $\mathcal{T}(\bar{2}^{\mathcal{U}})$ is a compact topology by Lemma 3.1 and hence $(i, c(i[X]))$ is a compactification of $(X, \mathcal{U})$.

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(Received March 9, 1971)