SOME RESULTS ON NORMAL BASES

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Throughout the present paper, A will represent a ring with 1, B a subring of A containing 1, and G a subgroup of the group $\hat{G}$ of B-ring automorphisms of A. Further, the centralizer $V_A(B)$ of B in A will be denoted by V, and for any subset S of A the right multiplication and the left multiplication by S (in A) respectively by S_R and S_L. We set $\mathfrak{A}=\mathrm{Hom}(A, A)$, which will be considered on the right side of A. If ${}_BA$ is completely faithful in the sense of [1], or equivalently, if B is a direct summand of ${}_BA$, then $V_{\mathfrak{A}}(V_{\mathfrak{A}}(B_L))=B_L$ (cf. [1]). In particular, if ${}_BA$ is free then B is a direct summand of ${}_BA$ and $V_{\mathfrak{A}}(V_{\mathfrak{A}}(B_L))=B_L$. Further, one may remark that if B is a direct summand of ${}_BA$ and $V_{\mathfrak{A}}(B_L)=GA_R$ then $J(G)=\{x\in A; x\sigma=x$ for every $\sigma\in G\}$ coincides with B. Those remarks will be used freely in the sequel.

If ${}_BA$ is finitely generated projective (resp. finitely generated free) and Hom $({}_BA_B, {}_BB_B)$ contains a left free A_R-basis h of Hom $({}_BA, {}_BB)$ then A/B is called a *Frobenius* (resp. *free Frobenius*) *extension*. Now, assume that A/B is a Frobenius extension and $h\in$ Hom $({}_BA_B, {}_BB_B)$ a left free A_R-basis of Hom $({}_BA, {}_BB)$. Then, h is also a left free A_L-basis of Hom (A_B, B_B) (cf. the proof of [13; Th. 1]). Moreover, in order that B be a direct summand of ${}_BA$ (or of A_B), it is necessary and sufficient that h be an epimorphism. In case B is a direct summand of ${}_BA$, A is semi-primary if and only if so is B ([9; Prop. 7.3]). Finally, let A be an Artinian simple ring, and G an N-group of A with $B=J(G)$. Then A/B is a free Frobenius extension with $V_{\mathfrak{A}}(B_L)=GA_R$ ([5; 3. Beispiele] and [13; Prop. 1]) and A is GB_R-homomorphic to GB_R ([7; Th. 1]).

In the present paper, we shall treat with a kind of free Frobenius extensions of semi-primary rings and the main theme of our discussion will concern the normal basis theorems. One of them is an extension of the last statement (Th. 3.7), and the original of the other will be found in [8] and [9] (cf. Ths. 2.1 and 2.2). Our theorems obtainted in §§2—4 will contain several results in [7] and [14].

1. A remark on Frobenius extensions

Assume that A/B is a Frobenius extension and $h\in$ Hom $({}_BA_B, {}_BB_B)$ a left free A_R-basis of Hom $({}_BA, {}_BB)$. Then, as was shown in the proof of [13; Th. 1], $(\sum x_{iR}hy_{iR})^*=\sum y_{iL}hx_{iL}$ defines an additive group isomorphism of $V_{\mathfrak{A}}(B_L)$ onto $V_{\mathfrak{A}}(B_R)$. We shall expose here the reciprocity between the conditions $V_{\mathfrak{A}}(B_L)=GA_R$ and $V_{\mathfrak{A}}(B_R)=GA_L$.

Proposition 1.1. *Let A/B be a Frobenius extension, and $h \in$* Hom $({}_BA_B, {}_BB_B)$ *a left free A_R-basis of* Hom $({}_BA, {}_BB)$. *If σ is a B-ring automorphism of A then $\sigma h = v_R h$ with a unit v in V* (cf. [6; p. 93]), *and $\sigma^* = \sigma^{-1} v_R = \sigma^{-1} \tilde{v}^{-1} v_L$ where $\tilde{v} = v_L v_R^{-1}$. In particular,* $1^* = 1$.

Proof. We set $\sigma^* = \sigma^{-1} g$. Since $x_R \sigma = \sigma(x\tau)_R$ for every x in A, we obtain $\sigma^{-1} g x_L = (x\sigma)_L \sigma^{-1} g = \sigma^{-1} x_L g$, namely, $g = u_R$ with some u in V. Hence, $\sigma^* = \sigma^{-1} u_R$. If $\sigma = \sum x_{iR} h y_{iR}$ then $\sigma^* = \sum y_{iL} h x_{iL}$, so that $\sigma^* h = \sum y_{iL} h (x_i h)_L$. Noting that $1 = \sum (x_i h) y_i$, we readily obtain $h = 1_L \cdot h = \sum y_{iL} h (x_i h)_L = \sigma^* h = \sigma^{-1} u_R h$, and so $v_R h = \sigma h = u_R h$. Hence, we obtain $u = v$.

The next is [13; Cor. 1].

Corollary. *In order that A/B be a Frobenius extension, it is necessary and sufficient that there exist some $h \in$* Hom $({}_BA_B, {}_BB_B)$ *and $x_1, \cdots, x_n$; $y_1, \cdots, y_n \in A$ such that $\sum x_{iR} h y_{iR} = \sum y_{iL} h x_{iL} = 1$.*

Proof. It remains only to prove the sufficiency. If f is in Hom$({}_BA, {}_BB)$ then $f = \sum x_{iR} h y_{iR} f = \sum (x_i \cdot y_i f) h$. Moreover, if $a_R h = 0$ $(a \in A)$ then $a = a(\sum y_{iL} h x_{iL}) = \sum y_i (a_R h x_{iL}) = 0$.

Corollary. *Let $\widetilde{V}$ be the set of all $\tilde{v}$ effected by units v contained in V. If A/B is a Frobenius extension* and $V_{\mathfrak{A}}(B_L) = GA_R$ *then $V_{\mathfrak{A}}(B_R) = G\widetilde{V}A_L = \widehat{G}A_L$.*

If A/B is G-Galois (cf. for instance [9]) then $h = \sum_{\sigma \in G} \sigma$ is a left free basis of Hom $({}_BA, {}_BB)$ (and so A/B is a Frobenius extension) and $Gh = h$. The next is also an easy consequence of Prop. 1.1.

Corollary. *Let A/B be a Frobenius extension, and $V_{\mathfrak{A}}(B_L) = GA_R$. If* Hom $({}_BA_B, {}_BB_B)$ *contains a left free A_R-basis h of* Hom $({}_BA, {}_BB)$ *such that $Gh \subset C_R h$ $(C = V_A(A))$ then $V_{\mathfrak{A}}(B_R) = GA_R$. In particular, if $V = C$ then $V_{\mathfrak{A}}(B_R) = GA_L$.*

2. Normal basis thorem

Let A be a *semi-primary ring*, namely, the residue class ring of A modulo the (Jacobson) radical $\mathfrak{R}(A)$ be Artinian. Assume that A/B is a free Frobenius extension and $V_{\mathfrak{A}}(B_L) = GA_R$. Then, the direct sum $A^{(s)}$ of s copies of A is GA_R-isomorphic to GA_R where $s = [A : B]$. Assume further that G contains a right free A_R-basis H of GA_R such that $GB_R = HB_R$. Then, to be easily seen, $(GB_R)^{(s)}$ is GB_R-isomorphic to GA_R, and so $A^{(s)}$ is GB_R-isomorphic to $(GB_R)^{(s)}$. Since GA_R is semi-primary, GB_R is also semi-primary by [9; Prop. 7.3]. Hence, as is well known, A is GB_R-isomorphic to GB_R, which proves the following that is fundamental in our subsequent study.

Theorem 2.1. *Let A/B be a free Frobenius extension, and $V_{\mathfrak{A}}(B_L) =$*

$\oplus_{\sigma\in H}\sigma A_R$ with a subset H of G. If A is semi-primary and $GB_R=HB_R$ then A is GB_R-isomorphic to GB_R.

Corollary. *Let A/B be a free Frobenius extension, and $V_{\mathfrak{A}}(B_L)=\oplus_1^s\sigma_iA_R$ with some $\sigma_i\in G$. If A is semi-primary and G is abelian then A is GB_R-isomorphic to GB_R.*

Proof. Let $\sigma=\sum\sigma_iy_{iR}$ ($y_i\in A$) be an arbitrary element of G. Then, for any $\tau\in G$ there holds $\sum\tau\sigma_iy_{iR}=\tau\sigma=\sigma\tau=\sum\tau\sigma_i(y_i\tau)_R$, whence it follows $y_i=y_i\tau$. We see therefore every y_i is in $J(G)=B$, namely, $GB_R=\oplus_1^s\sigma_iB_R$. Hence, A is GB_R-isomorphic to GB_R by Th. 2.1.

Next, we shall present the following whose proof is quite similar to that of [8; Th. 2.4] (cf. also [8; Example 2.2]).

Theorem 2.2. *Let A/B be a Frobenius extension such that B is a direct summand of A_B, and $V_{\mathfrak{A}}(B_L)=\oplus_1^s\sigma_iA_R$ and $GB_R=\oplus_1^s\sigma_iB_R$ with some $\sigma_i\in G$. If A is semi-primary and A_B can be generated by a subset of s elements then A is GB_R-isomorphic to GB_R.*

Proof. By the validity of Th. 2.1, it suffices to prove that A_B is free. There exists a right B-epimorphism f of $B^{(s)}$ onto A and a splitting exact sequence

$$0\longrightarrow \mathrm{Ker}\, f\longrightarrow B^{(s)}\xrightarrow{f} A\longrightarrow 0.$$

Obviously, the derived sequence

$$0\longrightarrow \mathrm{Ker}\, f\otimes_B A\longrightarrow B^{(s)}\otimes_B A\xrightarrow{f\otimes 1} A\otimes_B A\longrightarrow 0$$

is an exact sequence of right A-modules. As is well known, $V_{\mathfrak{A}}(B_L)=\oplus_1^s\sigma_iA_R$ is right A-isomorphic to $A\otimes_BA$ (cf. [13; Lemma 1]). Hence, $B^{(s)}\otimes_BA\otimes_A(A/\mathfrak{R}(A))$ and $A\otimes_BA\otimes_A(A/\mathfrak{R}(A))$ are isomorphic and have the same finite number of irreducible components. Therefore, $f\otimes1\otimes1$ has to be an isomorphism, and $f\otimes1$ is an isomorphism by [8; Lemma 1.7]. Hence, $\mathrm{Ker}\, f\otimes_BA=0$. Since B is a direct summand of ${}_BA$ too, we obtain $\mathrm{Ker}\, f=0$.

If A/B is G-Galois and A is GB_R-isomorphic to GB_R, then it is rather familiar that $H^i(G, A)=0$ for $i\geqq1$. By the way, one may remark here that if G is of finite order and there exists an element $a\in A$ such that $T_G(a)=\sum_{\sigma\in G}a\sigma=1$ then $H^1(G, A)=0$. In fact, if for every $\sigma\in G$ there corresponds an element $x_\sigma\in A$ and there holds $x_\tau+x_\sigma\tau=x_{\sigma\tau}$ ($\sigma, \tau\in G$), then for $x=\sum_{\tau\in G}x_\tau\cdot a\tau$ we have $x_\sigma=x-x\sigma$.

Theorem 2.3. *Let A be semi-primary, and $G=\{\sigma_1, \cdots, \sigma_s\}$ a finite group of ring automorphisms of A such that $B=J(G)$ is a direct summand of A_B. Assume that A/B is G-Galois and A_B can be generated by a subset of s elements. If a is a left G-normal basis element* (abbr. *G-n. b. e.*) *of*

A/B, namely, if $\{a\sigma_1, \cdots, a\sigma_s\}$ *is a left free B-basis of A, then the matrix* $(a\sigma_i\sigma_j)$ *is a unit of* $(A)_s$, *and conversely. In particular, if G is abelian then every right G-n. b. e. is a left G-n. b. e.*

Proof. By Th. 2.2, A is GB_R-isomorohic to GB_R, so that symmetrically there exists a left G-n. b. e. x of A/B. Since there exist some $f_j = y_{jR}\cdot\sum_1^s \sigma_k$ in Hom $({}_BA, {}_BB) = A_R\cdot\sum\sigma_k$ such that $\sum_k x\sigma_i\sigma_k\cdot y_j\sigma_k = x\sigma_i f_j = \delta_{ij}$ $(i, j = 1, \cdots, s)$, the matrix $(x\sigma_i\sigma_j)$ is a unit of $(A)_s$. Conversely, let a be an arbitrary element of A such that $(a\sigma_i\sigma_j)$ is a unit of $(A)_s$. If we set $a\sigma_i = \sum_j b_{ij}\cdot x\sigma_j$ $(b_{ij}\in B)$ then $(b_{ij}) = (a\sigma_i\sigma_j)\cdot(x\sigma_i\sigma_j)^{-1}$ is a unit of $(B)_s$, which means that a is a left G-n. b. e.

Finally, assume that A be an Artinian simple ring, and $G = \{\sigma_1, \cdots, \sigma_s\}$ an F-group of A with $B = J(G)$. Then, B is an Artinian simple ring and $[A : B] \leqslant s$. If $[A : B]$ coincides with s, A/B is defined to be *strictly Galois* with respect to G (cf. [10]). To be easily seen, A/B is strictly Golois with respect to G if and only if G-Galois. We obtain at once the following that contains [7 ; Th. 4].

Corollary. *Let* $G = \{\sigma_1, \cdots, \sigma_s\}$ *be an F-group of an Artinian simple ring A, and* $B = J(G)$. *In order that a be a left G-n. b. e. of* A/B, *it is necessary and sufficient that the matrix* $(a\sigma_i\sigma_j)$ *be a unit of* $(A)_s$.

3. Galois extensions of perfect rings

Following [2], a ring A is called *left* (resp. *right*) *perfect* if every left (resp. right) unital A-module possesses a projective cover. As was shown in [2], A is left perfect if and only if any of the following equivalent conditions is satisfied: (1) A is semi-primary and $\mathfrak{R}(A)$ is left T-nilpotent, and (2) the descending chain condition is satisfied for right principal ideals of A. Now, it will be easy to see that if A is left perfect then every right regular element (i. e. an element that is not a left zero-divisor) of A is a unit. Finally, A is called a *local ring* if the set of all non-units of A forms an ideal. The following will be found in [2] and [3].

Proposition 3.1. *Let A be a left perfect ring.*

(a) *If M is a left unital A-module then* $\mathfrak{R}(A)\cdot M \neq M$.

(b) *Every projective left unital A-module is a direct sum of directly indecomposable direct summands of* ${}_AA$, *and the numbers of isomorphic components are uniquely determined.*

(c) *If A is primary then A is a complete matrix ring over a local ring, and conversely.*

(d) $(A)_s$ *is left perfect. If e is a non-zero idempotent of A then eAe is left perfect.*

Now, by the validity of Prop. 3.1 (a) and (b), we can prove the following, whose proof proceeds in the same way as in [12].

Proposition 3.2. *Let A be a right perfect ring, and let N and P be right unital A-modules.*

(a) *If P is projective and $N^{(n)}$ is A-homorphic to $P^{(p)}$ with positive integers $n \geqslant p$ then N is A-homomorphic to P.*

(b) *If $N^{(n)}$ is A-isomorphic to $A^{(\omega)}$ with a positive integer n and an infinite cardinal number ω then N is A-isomorphic to $A^{(\omega)}$.*

(c) *If $N^{(n)}$ is A-isomorphic to $A^{(a)}$ with positive integers n, a, and $a = nq + r$ $(0 \leqslant r < n)$, then N is A-isomorphic to $A^{(q)} \oplus N_0$ for an A-homomorphic image N_0 of A such that $N_0^{(n)}$ is A-isomorphic to $A^{(r)}$.*

Proposition 3.3. *Let B be a direct summand of ${}_BA$.*

(a) *If A is left perfect then so is B. In particular, A is left perfect if and only if so is $(A)_s$.*

(b) *If ${}_BA$ is finitely generated projective and B is a right perfect ring then A is right perfect. Particularly, in case A/B is a Frobenius extension, A is a (left and right) perfect ring when and only when so is B.*

Proof. (a) If $\mathfrak{r}$ is a right ideal of B then $\mathfrak{r}A \cap B = \mathfrak{r}$. Hence. the descending chain condition is valid for right principal ideals of B. The latter will be obvious by Prop. 3.1 (d).

(b) In virtue of Prop. 3.1 (d), $V_{\mathfrak{A}}(B_L)$ is right perfect. Since A_R is a direct summand of the right A_R-module $V_{\mathfrak{A}}(B_L)$ (cf, [9; Lemma 6.8]), A_R is itself right perfect by (a).

Proposition 3.4. *Let A be left perfect, and M_A finitely generated free. If N is a finitely generated free submodule of M_A with $[M:A]_R = [N:A]_R$ then $M = N$.*

Proof. Let $\{m_1, \cdots, m_s\}$ and $\{n_1, \cdots, n_s\}$ be right free A-bases of M and N, respectively. If $n_j = \sum m_i a_{ij} (a_{ij} \in A)$ then the matrix (a_{ij}) is obviously a right regular element of the left perfect ring $(A)_s$ (Prop. 3.1 (d)). Hence, (a_{ij}) is a unit of $(A)_s$, and $\{n_1, \cdots, n_s\}$ is a right free A-basis of M.

As an easy consequence of Props. 3.3 (b) and 3.4, we obtain the next:

Corollary. *Let $G = \{\sigma_1, \cdots \sigma_s\}$, B a right perfect ring, and $[A:B]_L = s$. If $(a\sigma_i\sigma_j)$ is a unit of $(A)_s$ then a is a left G-n. b. e. of A/B.*

Corresponding to Th. 2.1. we obtain the following (cf. [7; Th. 2]):

Theorem 3.5. *Let A/B be a free Frobenius extension, and $V_{\mathfrak{A}}(B_L) = \oplus_{\sigma \in H} \sigma A_R$ with a subset H of G. If A is left perfect then the following* *conditions are equivalent*: (1) A *is* GB_R*-isomorphic to* GB_R, *and* (2) $GB_R = HB_R$.

Proof. By Th. 2.1, it suffices to prove (1) $\Rightarrow$ (2). By Prop. 3.3 (a), B is left perfect. Since $[GB_R : B_R]_R = [A : B]_R = [A : B]_L = [HA_R : A_R]_R = [HB_R : B_R]_R$, (2) is a consequence of Prop. 3.4.

The next is a generalization of [7 ; Th. 3].

Theorem 3.6. *Let* A/B *be* G*-Galois, and* U *a* G*-invariant right perfect subring of* A *such that* A_U *possesses a free basis* $\{y_\lambda ; \lambda \in \Lambda\}$.

(a) *If* Λ *is infinite then there exists a subset* $\{x_\lambda ; \lambda \in \Lambda\}$ *such that* $\{x_\lambda \sigma ; \lambda \in \Lambda, \sigma \in G\}$ *is a free basis of* A_U.

(b) *Let* G *be of order* s, *and* $\sharp\Lambda = t < \infty$. *If* $t = sq + r$ $(0 \leqslant r < s)$ *then* A *contains a subset* $X = \{x_1, \cdots, x_q\}$ *and a* GU_R*-homomorphic image* M *of* GU_R *such that* XG *is right* U*-free,* $M^{(s)}$ *is* GU_R*-isomorphic to* $(GU_R)^{(r)}$ *and that* $A = (XG)U \oplus M$.

Proof. Since $GU_R = \oplus_{\sigma \in G} U_R \sigma$ is right perfect (Prop. 3.3 (b)) and $A^{(s)}$ is GU_R-isomorphic to $(GU_R)^{(\sharp\Lambda)}$, (a) and (b) are direct consequences of Prop. 3.2 (b) and (c), respectively.

We shall conclude this section with the following that contains [7 ; Th. 1]:

Theorem 3.7. *Let* A/B *be a free Frobenius extension, and* $V_{\mathfrak{A}}(B_L) = GA_R$. *If* U *is a* G*-invariant right Artinian subring of* A *such that* A_U *possesses a generating system* $\{x_1, \cdots, x_t\}$ *with* $t \leqslant [A : B]$, *then* A *is* GU_R*-homomorphic to* GU_R. *In particular, if* A *is right Artinian then* A *is* GB_R*-homomorphic to* GB_R.

Proof. If $s = [A : B]$ then $A^{(s)}$ is GA_R-isomorphic to GA_R. Since GA_R is right finite over U_R, GU_R is right Artinian. Noting that $GA_R = A_R G = \sum_1^t x_{iR} U_R G$, we see that GA_R is GU_R-homomorphic to $(GU_R)^{(t)}$, and so $A^{(s)}$ is GU_R-homomorphic to $(GU_R)^{(t)}$. Hence, by Prop. 3.2 (a), A is GU_R-homomorphic to GU_R.

4. A special type of Galois extensions of left perfect primary rings

The present section is devoted exclusively to the treaty of a special type of Galois extensions of left perfect primary rings.

Proposition 4.1. *Let* A *be a* G*-Galois extension of a left perfect primary ring* B. *If* G' *is a normal subgroup of* G, $\bar{G} = G/G'$ *and* $B' = J(G')$, *then* B'/B *is free* $\bar{G}$*-Galois and* B' *is left perfect.*

Proof. Let $B = \sum B_0 e_{ij}$, where $\{e_{ij}\}$ is a system of matrix units and $B_0 = V_B(\{e_{ij}\})$ a local ring (Prop. 3. (c)). If $A_0 = V_A(\{e_{ij}\})$ then A_0/B_0 is

G-Galois by [9; Th. 5.8]. Hence, By [4; Th. 2], A_0/B_0 is free G-Galois, which means that A/B is free G-Galois. Accordingly, B'/B is $\bar{G}$-Galois (cf. for instance [9]), and then free by the above argument. Finally, B' is left perfect by Prop. 3.3 (b).

If B is a left perfect primary ring then, as is well known, the center Z of B is a perfect local ring and the characteristic of B is either 0 or a power of a prime. Now, we shall present a slight generalization of [14; Th. 2].

Theorem 4.2. *Let A/B be G-Galois, and $G' \neq \{1\}$ a normal subgroup of G with $B'=J(G')$. If B is a left perfect primary ring and $\bar{G}=G/G'$ then the following conditions are equivalent:* (1) *a is a right G-n. b. e. of A/B whenever $T_{G'}(a)=\sum_{\sigma' \in G'} a\sigma'$ is a right $\bar{G}$-n. b. e. of B'/B, and* (2) *the characteristic of B and the order of G' are powers of a prime p.*

Proof. By Prop. 4.1, A/B is free G-Galois and A is left perfect. Hence, there exists a right G-n. b. e. u (Th. 2.1), and $T_{G'}(u)$ is a right $\bar{G}$-n. b. e. (Props. 3.4 and 4.1). As in the proof of [11; Th.], the mapping $\varphi: \sum \sigma x_{\sigma R} \longrightarrow \sum \bar{\sigma} x_{\sigma R}$ is a ring homomorphism of GB_R (isomorphic to the group ring GB) onto $\bar{G}B_R$, and one will easily see that $T_{G'}(u\alpha)=(T_{G'}(u))(\alpha\varphi)$ for every $\alpha \in GB_R$. As GB_R is left perfect (Prop. 3.3 (b)). $u\alpha$ is again a right G-n. b. e. when and only when α is a unit of GB_R. Similarly, $T_{G'}(u\alpha)$ is again a right $\bar{G}$-n. b. e. when and only when $\alpha\varphi$ is a unit of $\bar{G}B_R$. Our equivalence is therefore evident by Th. of [11].

Corollary. *Let A/B be G-Galois. If B is a left perfect primary proper subring of A then the following conditions are equivalent:* (1) *a is a right G-n. b. e. of A/B whenever $T_G(a)$ is a unit of B, and* (2) *the characteristic of B and the order of G are powers of a prime p. In particular, if B is a perfect primary ring of characteristic p^c and G is of order p^e then every left G-n. b. e. is a right G-n. b. e.*

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