ÜBER DIE KOEFFIZIENTEN DER SCHLICHTEN FUNKTIONEN (V)

KEN'ITI KOSEKI

In meiner früheren Arbeit[1] " ÜBER DIE KOEFFIZIENTEN DER SCHLICHTEN FUNKTIONEN (IV)" habe ich die Integralgleichung

$$\Im^{2)}\left[6\kappa^3(t)t^2+A\kappa(t)+2\kappa(t)\int_t^0\kappa^2(t_1)t_1dt_1+B\kappa^2(t)t-4\kappa^2(t)t\int_t^0 2\kappa(t_1)dt_1\right]$$

$$=0,\quad 0\leqq t\leqq 1. \tag{1}$$

eingeführt. Die Lösung dieser Gleichung ist ein Schlüssel zur Lösung von der Bieberbachschen Vermutung.

Die Berechnung in meiner Arbeit (IV) ist aber zu kompliziert. In der vorligenden Arbeit will ich die formale Lösung der Gleichung (I) gewinnen.

1. Schritt: Wir setzen

$$\kappa(t)=a_0+a_1t+\cdots\quad +a_nt^n+\cdots\quad =\sum_{n=0}^{\infty}a_nt^n$$

$$\kappa^2(t)=b_0+b_1t+\cdots\quad +b_nt^n+\cdots\quad =\sum_{n=0}^{\infty}b_nt^n,$$

so bekommen wir leicht

$$\int_t^0 2\kappa(t_1)dt_1=-\sum_{n=0}^{\infty}\frac{2a_n}{n+1}t^{n+1},\qquad \int_t^0\kappa^2(t_1)dt_1=-\sum_{n=0}^{\infty}\frac{b_n}{n+2}t^{n+2}.$$

Daraus und aus der Gleichung (1) folgt es ohne weiteres

$$\Im\Big[6\sum_{n=0}^{\infty}\Big(\sum_{j+k=n}a_jb_k\Big)t^{n+2}+A\sum_{n=0}^{\infty}a_nt^n-4\sum_{n=0}^{\infty}\Big(\sum_{j+k=n}\frac{a_jb_k}{k+2}\Big)t^{n+2}+B\sum_{n=0}^{\infty}b_nt^{n+1}$$

$$+8\sum_{n=0}^{\infty}\Big(\sum_{j+k=n}\frac{a_jb_k}{j+1}\Big)t^{n+2}\Big]=0. \tag{2}$$

Nach der Gleichung (2) besteht es

$$\Im Aa_0=0 \tag{3}$$

$$\Im(Aa_1+Bb_0)=0 \tag{4}$$

$$\Im\Big[6\sum_{j+k=n}a_jb_k+Aa_{n+2}-4\sum_{j+k=n}\frac{a_jb_k}{k+2}+Bb_{n+1}+\sum_{j+k=n}\frac{a_jb_k}{j+1}\Big]=0,\quad (n\geqq 0). \tag{5}$$

Wir setzen nun $a_n=\alpha_n+i\beta_n$, so aus der Eigenschaft $|\kappa(t)|=1$ folgt es leicht

[1] K. Koseki. Über Die Koeffizienten Der Schlichten Funktionen (IV). Math. Journ. Okay. Univ. Vol. II. (1963). S. 125.

[2] $\Im z$ bedeutet den imaginäre Teil von z.

$$\Big(\sum_{n=0}^{\infty}\alpha_n t^n\Big)^2+\Big(\sum_{n=0}^{\infty}\beta_n t^n\Big)^2=1.$$

Daher besteht es

$$\left.\begin{aligned}&\alpha_0^2+\beta_0^2=1\\ &\sum_{j+k=n}\alpha_j\alpha_k+\sum_{j+k=n}\beta_j\beta_k=0 \qquad (n\geqq 1).\end{aligned}\right\} \tag{6}$$

Da $b_n=\sum_{j+k=n}a_j a_k$ ist, ist es $\sum_{j+k=n}a_j b_k=\sum_{j+k+l=n}a_j a_k a_l$. Also bekommen wir aus (5)

$$A_1\beta_{n+2}+A_2\alpha_{n+2}=-\Im\Big[6\sum_{j+k+l=n}a_j a_k a_l-4\sum_{j+k+l=n}\frac{a_j a_k a_l}{n-j+2}+B\sum_{j+k=n+1}a_j a_k$$
$$+8\sum_{j+k+l=n}\frac{a_j a_k a_l}{j+1}\Big] \tag{7}$$

wo wir $A=A_1+iA_2$ und $B=B_1+iB_2$ setzen in (5).

Aus (6) folgt es ohne weiteres

$$\alpha_0\alpha_{n+2}+\beta_0\beta_{n+2}=-\frac{1}{2}\Big(\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\alpha_j\alpha_k+\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\beta_j\beta_k\Big). \tag{8}$$

Anderseits besteht es nach (7)

$$A_1\beta_{n+2}+A_2\alpha_{n+2}=-\Big[6\Big\{3\sum_{j+k+l=n}\alpha_j\alpha_k\beta_l-\sum_{j+k+l=n}\beta_j\beta_k\beta_l\Big\}-4\sum_{j+k+l=n}$$
$$\Big\{\Big(\frac{1}{n-j+2}+\frac{1}{n-l+2}+\frac{1}{n-k+2}\Big)\alpha_j\alpha_k\beta_l-\frac{1}{n-j+2}\beta_j\beta_k\beta_l\Big\}+2B_1\sum_{j+k=n+1}\alpha_k\beta_j+$$
$$B_2\sum_{j+k=n+1}(\alpha_j\alpha_k-\beta_j\beta_k)+8\sum_{j+k+l=n}\Big\{\Big(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{l+1}\Big)\alpha_j\alpha_k\beta_l-\frac{1}{j+1}\beta_j\beta_k\beta_l\Big\}\Big]. \tag{9}$$

Indem wir in (8) und (9) die β_{n+2} eliminieren, bekommen wir

$$\alpha_{n+2}(A_1\alpha_0-A_2\beta_0)=-\frac{A_1}{2}\Big(\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\alpha_j\alpha_k+\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\beta_j\beta_k\Big)+\beta_0\Big[6\Big\{3\sum_{j+k+l=n}\alpha_j\alpha_k\beta_l$$
$$-\sum_{j+k+l=n}\beta_j\beta_k\beta_l\Big\}-4\sum_{j+k+l=n}\Big\{\Big(\frac{1}{n-j+2}+\frac{1}{n-l+2}+\frac{1}{n-k+2}\Big)\alpha_j\alpha_k\beta_l$$
$$-\frac{1}{n-j+2}\beta_j\beta_k\beta_l\Big\}+2B_1\sum_{j+k=n+1}\alpha_k\beta_j+B_2\sum_{j+k=n+1}(\alpha_j\alpha_k-\beta_j\beta_k)+8\sum_{j+k+l=n}$$
$$\Big\{\Big(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{l+1}\Big)\alpha_j\alpha_k\beta_l-\frac{1}{j+1}\beta_j\beta_k\beta_l\Big\}\Big]. \tag{10}$$

Aus (3) entsteht es

$$A_1\beta_0+A_2\alpha_0=0.$$

Anderseits ist es offenbar

$$\alpha_0^2+\beta_0^2=1.$$

Wir nehmen zunächst an, dass $A_1^2+A_2^2\neq 0$ ist. Es entsteht dann aus den obigen zwei Beziehungen

$$\left.\begin{aligned}\alpha_0&=\pm\frac{A_1}{\sqrt{A_1^2+A_2^2}},\\ \beta_0&=\mp\frac{A_2}{\sqrt{A_1^2+A_2^2}}.\end{aligned}\right\}\tag{11}$$

Nach (10) und (11) besteht es

$$\begin{aligned}\alpha_{n+2}=&-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\Bigl(\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\alpha_j\alpha_k+\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\beta_j\beta_k\Bigr)-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\Bigl[6\Bigl\{3\sum_{j+k+l=n}\alpha_j\alpha_k\beta_l\\ &-\sum_{j+k+l=n}\beta_j\beta_k\beta_l\Bigr\}-4\sum_{j+k+l=n}\Bigl\{\Bigl(\frac{1}{n-j+2}+\frac{1}{n-l+2}+\frac{1}{n-k+2}\Bigr)\alpha_j\alpha_k\beta_l\\ &-\frac{1}{n-j+2}\beta_j\beta_k\beta_l\Bigr\}+2B_1\sum_{j+k=n+1}\alpha_k\beta_j+B_2\sum_{j+k=n+1}(\alpha_j\alpha_k-\beta_j\beta_k)+8\sum_{j+k+l=n}\\ &\Bigl\{\Bigl(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{l+1}\Bigr)\alpha_j\alpha_k\beta_l-\frac{1}{j+1}\beta_j\beta_k\beta_l\Bigr\}\Bigr],\quad(n\geqq 0),\end{aligned}\tag{12}$$

oder

$$\begin{aligned}\alpha_{n+2}=&\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\Bigl(\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\alpha_j\alpha_k+\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\beta_j\beta_k\Bigr)-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\Bigl[6\Bigl\{3\sum_{j+k+l=n}\alpha_j\alpha_k\beta_l\\ &-\sum_{j+k+l=n}\beta_j\beta_k\beta_l\Bigr\}-4\sum_{j+k+l=n}\Bigl\{\Bigl(\frac{1}{n-j+2}+\frac{1}{n-l+2}+\frac{1}{n-k+2}\Bigr)\alpha_j\alpha_k\beta_l\\ &-\frac{1}{n-j+2}\beta_j\beta_k\beta_l\Bigr\}+2B_1\sum_{j+k=n+1}\alpha_k\beta_j+B_2\sum_{j+k=n+1}(\alpha_j\alpha_k-\beta_j\beta_k)+8\sum_{j+k+l=n}\\ &\Bigl\{\Bigl(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{l+1}\Bigr)\alpha_j\alpha_k\beta_l-\frac{1}{j+1}\beta_j\beta_k\beta_l\Bigr\}\Bigr],\quad(n\geqq 0),\end{aligned}\tag{13}$$

jenachdem $\alpha_0=\frac{A_1}{\sqrt{A_1^2+A_2^2}}$ oder $\alpha_0=-\frac{A_1}{\sqrt{A_1^2+A_2^2}}$ ist.

Indem wir in (8) und (9) die α_{n+2} eliminieren, bekommen wir

$$\begin{aligned}\beta_{n+2}(A_2\beta_0-A_1\alpha_0)=&-\frac{A_2}{2}\Bigl(\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\alpha_j\alpha_k+\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\beta_j\beta_k\Bigr)+\alpha_0\Bigl[6\Bigl\{3\sum_{j+k+l=n}\alpha_j\alpha_k\beta_l\\ &-\sum_{j+k+l=n}\beta_j\beta_k\beta_l\Bigr\}-4\sum_{j+k+l=n}\Bigl\{\Bigl(\frac{1}{n-j+2}+\frac{1}{n-l+2}+\frac{1}{n-k+2}\Bigr)\alpha_j\alpha_k\beta_l\\ &-\frac{1}{n-j+2}\beta_j\beta_k\beta_l\Bigr\}+2B_1\sum_{j+k=n+1}\alpha_k\beta_j+B_2\sum_{j+k=n+1}(\alpha_j\alpha_k-\beta_j\beta_k)+8\sum_{j+k+l=n}\\ &\Bigl\{\Bigl(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{l+1}\Bigr)\alpha_j\alpha_k\beta_l-\frac{1}{j+1}\beta_j\beta_k\beta_l\Bigr\}\Bigr].\end{aligned}$$

Danach und nach (11) besteht es

$$\beta_{n+2}=\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\alpha_j\alpha_k+\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\beta_j\beta_k\Big)-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big[6\Big\{3\sum_{j+k+l=n}\alpha_j\alpha_k\beta_l$$

$$-\sum_{j+k+l=n}\beta_j\beta_k\beta_l\Big\}-4\sum_{j+k+l=n}\Big\{\Big(\frac{1}{n-j+2}+\frac{1}{n-l+2}+\frac{1}{n-k+2}\Big)\alpha_j\alpha_k\beta_l$$

$$-\frac{1}{n-j+2}\beta_j\beta_k\beta_l\Big\}+2B_1\sum_{j+k=n+1}\alpha_k\beta_j+B_2\sum_{j+k=n+1}(\alpha_j\alpha_k-\beta_j\beta_k)+8\sum_{j+k+l=n}$$

$$\Big\{\Big(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{l+1}\Big)\alpha_j\alpha_k\beta_l-\frac{1}{j+1}\beta_j\beta_k\beta_l\Big\}\Big],\qquad (n\geqq 0)\tag{14}$$

oder

$$\beta_{n+2}=-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\alpha_j\alpha_k+\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}\beta_j\beta_k\Big)$$

$$-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big[6\Big\{3\sum_{j+k+l=n}\alpha_j\alpha_k\beta_l-\sum_{j+k+l=n}\beta_j\beta_k\beta_l\Big\}-4\sum_{j+k+l=n}\Big\{\Big(\frac{1}{n-j+2}+\frac{1}{n-l+2}$$

$$+\frac{1}{n-k+2}\Big)\alpha_j\alpha_k\beta_l-\frac{1}{n-j+2}\beta_j\beta_k\beta_l\Big\}+2B_1\sum_{j+k=n+1}\alpha_k\beta_j+B_2\sum_{j+k=n+1}(\alpha_j\alpha_k-\beta_j\beta_k)$$

$$+8\sum_{j+k+l=n}\Big\{\Big(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{l+1}\Big)\alpha_j\alpha_k\beta_l-\frac{1}{j+1}\beta_j\beta_k\beta_l\Big\}\Big],\quad (n\geqq 0)\tag{15}$$

jenachdem $\alpha_0=\frac{A_1}{\sqrt{A_1^2+A_2^2}}$ oder $\alpha_0=-\frac{A_1}{\sqrt{A_1^2+A_2^2}}$ ist.

2. Schritt: Wir behandeln erstens den Fall, wo $\alpha_0=\frac{A_1}{\sqrt{A_1^2+A_2^2}}$ ist, und wir setzen

$$\sum_{\substack{j+k=n+2\\ j,k\geqq 1}}(\alpha_j\alpha_k+\beta_j\beta_k)=D_{n+2},$$

$$6\Big\{3\sum_{j+k+l=n}\alpha_j\alpha_k\beta_l-\sum_{j+k+l=n}\beta_j\beta_k\beta_l\Big\}-4\sum_{j+k+l=n}\Big\{\Big(\frac{1}{n-j+2}+\frac{1}{n-l+2}$$

$$+\frac{1}{n-k+2}\Big)\alpha_j\alpha_k\beta_l-\frac{1}{n-j+2}\beta_j\beta_k\beta_l\Big\}+2B_1\sum_{j+k=n+1}\alpha_k\beta_j+B_2\sum_{j+k=n+1}(\alpha_j\alpha_k-\beta_j\beta_k)$$

$$+8\sum_{j+k+l=n}\Big\{\Big(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{l+1}\Big)\alpha_j\alpha_k\beta_l-\frac{1}{j+1}\beta_j\beta_k\beta_l\Big\}=E_{n+2},\ (n\geqq 0).$$

Es besteht dann für $n\geqq 4$

$$D_{n+2}=2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)D_{n+1}$$

$$+2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)E_{n+1}+\sum_{\substack{j+k=n+2\\ j,k\geqq 2}}(\alpha_j\alpha_k+\beta_j\beta_k)$$

$$=2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)D_{n+1}$$

$$+2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)E_{n+1}+2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_2+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_2\Big)D_n$$

$$+2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_2-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_2\Big)E_n+\sum_{\substack{j+k=n+2\\ j,k\geq 3}}(\alpha_j\alpha_k+\beta_j\beta_k)$$

$$=2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)D_{n+1}+2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1$$

$$-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)E_{n+1}+2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_2+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_2\Big)D_n$$

$$+2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_2-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_2\Big)E_n+\sum_{\substack{j+k=n+2\\ j,k\geq 3}}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_j$$

$$-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)+\sum_{\substack{j+k=n+3\\ j,k\geq 3}}$$

$$\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big). \tag{16}$$

Für $n\geqq 7$ besteht es

$$E_{n+2}=\Big\{2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\Big)+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)\Big\}D_{n+1}+\Big\{2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\Big)$$

$$+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big)\Big\}E_{n+1}+\Big[2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_1$$

$$+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_1\Big)+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}$$

$$\Big\{6(3\alpha_0^2-\beta_0^2)-4\Big(\Big(\frac{1}{n+2}+\frac{1}{n+2}+\frac{1}{2}\Big)\alpha_0^2-\frac{1}{n+2}\beta_0^2\Big)+8\Big(\Big(\frac{1}{n+1}+\frac{1}{1}+\frac{1}{1}\Big)\alpha_0^2-\beta_0^2\Big)\Big\}$$

$$+6\beta_0\cdot 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}3\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)-4\beta_0\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n+2}+\frac{1}{n+2}$$

$$+\frac{1}{2}\Big)\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n+2}+\frac{1}{2}\Big)\beta_0\Big\}+8\beta_0\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{1}+\frac{1}{1}+\frac{1}{n+1}\Big)\alpha_0$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{1}+\frac{1}{n+1}\Big)\beta_0\Big\}\Big]D_n+\Big[2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1\Big)$$

$$+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big\{6(3\alpha_0^2-\beta_0^2)-4\Big(\Big(\frac{1}{n+2}$$

$$+\frac{1}{n+2}+\frac{1}{2}\Big)\alpha_0^2-\frac{1}{n+2}\beta_0^2\Big)+8\Big(\Big(\frac{1}{n+1}+\frac{1}{1}+\frac{1}{1}\Big)\alpha_0^2-\beta_0^2\Big)\Big\}$$

$$+6\beta_0\cdot 2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}3\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big)-4\beta_0\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n+2}+\frac{1}{n+2}$$

$$+\frac{1}{2}\Big)\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n+2}+\frac{1}{2}\Big)\beta_0\Big\}+8\beta_0\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{1}+\frac{1}{1}+\frac{1}{n+1}\Big)\alpha_0$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{1}+\frac{1}{n+1}\Big)\beta_0\Big\}\Big]E_n+\Big[2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_2+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_2\Big)$$

$$+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_2-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_2\Big)+\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)\Big\{6\cdot 2(3\alpha_0\alpha_1-\beta_0\beta_1)$$

$$-4\Big(2\Big(\frac{1}{3}+\frac{1}{n+2}+\frac{1}{n+1}\Big)\alpha_0\alpha_1-\Big(\frac{1}{n+2}+\frac{1}{n+1}\Big)\beta_0\beta_1\Big)+8\Big(2\Big(\frac{1}{n}+\frac{1}{1}+\frac{1}{2}\Big)\alpha_0\alpha_1$$

$$-\Big(\frac{1}{1}+\frac{1}{2}\Big)\beta_0\beta_1\Big)\Big\}+\Big\{6\beta_0\cdot 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}3\alpha_1-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)$$

$$+6\beta_1\cdot 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}3\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)\Big\}-4\beta_0\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n+2}+\frac{1}{n+1}$$

$$+\frac{1}{3}\Big)\alpha_1-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n+1}+\frac{1}{3}\Big)\beta_1\Big\}-4\beta_1\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{3}\Big)\alpha_0$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n+2}+\frac{1}{3}\Big)\beta_0\Big\}+8\beta_0\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n}+\frac{1}{2}+\frac{1}{1}\Big)\alpha_1$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n}+\frac{1}{2}\Big)\beta_1\Big\}+8\beta_1\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n}+\frac{1}{2}+\frac{1}{1}\Big)\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n}$$

$$+\frac{1}{1}\Big)\beta_0\Big\}\Big]D_{n+1}+\Big[2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_2-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_2\Big)+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_2$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_2\Big)+\Big(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big)\Big\{6\cdot 2(3\alpha_0\alpha_1-\beta_0\beta_1)-4\Big(2\Big(\frac{1}{3}+\frac{1}{n+2}$$

$$+\frac{1}{n+1}\Big)\alpha_0\alpha_1-\Big(\frac{1}{n+2}+\frac{1}{n+1}\Big)\beta_0\beta_1\Big)+8\Big(2\Big(\frac{1}{n}+\frac{1}{1}+\frac{1}{2}\Big)\alpha_0\alpha_1-\Big(\frac{1}{1}+\frac{1}{2}\Big)\beta_0\beta_1\Big)\Big\}$$

$$+\Big\{6\beta_0\cdot 2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}3\alpha_1+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)+6\beta_1\cdot 2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}3\alpha_0$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big)-4\beta_0\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n+2}+\frac{1}{n+1}+\frac{1}{3}\Big)\alpha_1+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n+1}$$

$$+\frac{1}{3}\Big)\beta_1\Big\}-4\beta_1\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{3}\Big)\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n+2}$$

$$+\frac{1}{3}\Big)\beta_0\Big\}+8\beta_0\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n}+\frac{1}{2}+\frac{1}{1}\Big)\alpha_1+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n}+\frac{1}{2}\Big)\beta_1\Big\}$$

$$+8\beta_1\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n}+\frac{1}{2}+\frac{1}{1}\Big)\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n}+\frac{1}{1}\Big)\beta_0\Big\}\Big]E_{n-1}$$

$$+\Big[\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)\Big\{6(3\alpha_1^2-\beta_1^2)+6\cdot 2(3\alpha_0\alpha_2-\beta_0\beta_2)-4\Big(\Big(\frac{1}{4}+\frac{1}{n+1}+\frac{1}{n+1}\Big)\alpha_1^2$$

$$-\frac{1}{n+1}\beta_1^2\Big)-4\Big(2\Big(\frac{1}{4}+\frac{1}{n}+\frac{1}{n+2}\Big)\alpha_0\alpha_2-\Big(\frac{1}{n+2}+\frac{1}{n}\Big)\beta_0\beta_2\Big)+8\Big(\Big(\frac{1}{n-1}+\frac{1}{2}$$

$$+\frac{1}{2}\Big)\alpha_1^2-\frac{1}{2}\beta_1^2\Big)+8\Big(2\Big(\frac{1}{n-1}+\frac{1}{1}+\frac{1}{3}\Big)\alpha_0\alpha_2-\Big(\frac{1}{1}+\frac{1}{3}\Big)\beta_0\beta_2\Big)\Big\}$$

$$+6\beta_0\cdot 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}3\alpha_2-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_2\Big)+6\beta_1\cdot 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}3\alpha_1$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)+6\beta_2\cdot 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}3\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)$$

$$-4\beta_0\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n+2}+\frac{1}{n}+\frac{1}{4}\Big)\alpha_2-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n}+\frac{1}{4}\Big)\beta_2\Big\}$$

$$-4\beta_1\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n+1}+\frac{1}{n+1}+\frac{1}{4}\Big)\alpha_1-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n+1}+\frac{1}{4}\Big)\beta_1\Big\}$$

$$-4\beta_2\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n}+\frac{1}{n+2}+\frac{1}{4}\Big)\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n+2}+\frac{1}{4}\Big)\beta_0\Big\}$$

$$+8\beta_0\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n-1}+\frac{1}{3}+\frac{1}{1}\Big)\alpha_2-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{3}+\frac{1}{n-1}\Big)\beta_2\Big\}$$

$$+8\beta_1\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n-1}+\frac{1}{2}+\frac{1}{2}\Big)\alpha_1-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{2}+\frac{1}{n-1}\Big)\beta_1\Big\}$$

$$+8\beta_2\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n-1}+\frac{1}{1}+\frac{1}{3}\Big)\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{1}+\frac{1}{n-1}\Big)\beta_0\Big\}\Big]D_{n-2}$$

$$+\Big[\Big(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big)\Big\{6(3\alpha_1^2-\beta_1^2)+6\cdot 2(3\alpha_0\alpha_2-\beta_0\beta_2)-4\Big(\Big(\frac{1}{4}+\frac{1}{n+1}+\frac{1}{n+1}\Big)\alpha_1^2$$

$$-\frac{1}{n+1}\beta_1^2\Big)-4\Big(2\Big(\frac{1}{4}+\frac{1}{n}+\frac{1}{n+2}\Big)\alpha_0\alpha_2-\Big(\frac{1}{n+2}+\frac{1}{n}\Big)\beta_0\beta_2\Big)+8\Big(\Big(\frac{1}{n-1}+\frac{1}{2}$$

$$+\frac{1}{2}\Big)\alpha_1^2-\frac{1}{2}\beta_1^2\Big)+8\Big(2\Big(\frac{1}{n-1}+\frac{1}{1}+\frac{1}{3}\Big)\alpha_0\alpha_2-\Big(\frac{1}{1}+\frac{1}{3}\Big)\beta_0\beta_2\Big)\Big\}$$

$$+6\beta_0\cdot 2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}3\alpha_2+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_2\Big)+6\beta_1\cdot 2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}3\alpha_1$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)+6\beta_2\cdot 2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}3\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big)$$

$$-4\beta_0\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n+2}+\frac{1}{n}+\frac{1}{4}\Big)\alpha_2+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n}+\frac{1}{4}\Big)\beta_2\Big\}$$

$$-4\beta_1\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n+1}+\frac{1}{n+1}+\frac{1}{4}\Big)\alpha_1+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n+1}+\frac{1}{4}\Big)\beta_1\Big\}$$

$$-4\beta_2\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n}+\frac{1}{n+2}+\frac{1}{4}\Big)\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n+2}+\frac{1}{4}\Big)\beta_0\Big\}$$

$$+8\beta_0\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n-1}+\frac{1}{3}+\frac{1}{1}\Big)\alpha_2+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{3}+\frac{1}{n-1}\Big)\beta_2\Big\}$$

$$+8\beta_1\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n-1}+\frac{1}{2}+\frac{1}{2}\Big)\alpha_1+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{2}+\frac{1}{n-1}\Big)\beta_1\Big\}$$

$$+8\beta_2\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n-1}+\frac{1}{1}+\frac{1}{3}\Big)\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{1}+\frac{1}{n-1}\Big)\beta_0\Big\}\Big]E_{n-2}$$

$$+\Big[\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)\Big\{6\cdot 2(3\alpha_1\alpha_2-\beta_1\beta_2)-4\Big(2\Big(\frac{1}{5}+\frac{1}{n+1}+\frac{1}{n}\Big)\alpha_1\alpha_2$$

$$-\Big(\frac{1}{n+1}+\frac{1}{n}\Big)\beta_1\beta_2\Big\}+8\Big(2\Big(\frac{1}{n-2}+\frac{1}{2}+\frac{1}{3}\Big)\alpha_1\alpha_2-\Big(\frac{1}{2}+\frac{1}{3}\Big)\beta_1\beta_2\Big\}$$

$$+6\beta_1\cdot 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}3\alpha_2-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_2\Big)+6\beta_2\cdot 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}3\alpha_1$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)-4\beta_1\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n+1}+\frac{1}{n}+\frac{1}{5}\Big)\alpha_2-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n}+\frac{1}{5}\Big)\beta_2\Big\}$$

$$-4\beta_2\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n}+\frac{1}{n+1}+\frac{1}{5}\Big)\alpha_1-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n+1}+\frac{1}{5}\Big)\beta_1\Big\}$$

$$+8\beta_1\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{2}+\frac{1}{3}+\frac{1}{n-2}\Big)\alpha_2-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{3}+\frac{1}{n-2}\Big)\beta_2\Big\}$$

$$+8\beta_2\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{3}+\frac{1}{2}+\frac{1}{n-2}\Big)\alpha_1-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{2}+\frac{1}{n-2}\Big)\beta_1\Big\}\Big]D_{n-3}$$

$$+\Big[\Big(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big)\Big\{6\cdot 2(3\alpha_1\alpha_2-\beta_1\beta_2)-4\Big(2\Big(\frac{1}{5}+\frac{1}{n+1}+\frac{1}{n}\Big)\alpha_1\alpha_2$$

$$-\Big(\frac{1}{n+1}+\frac{1}{n}\Big)\beta_1\beta_2\Big)+8\Big(2\Big(\frac{1}{n-2}+\frac{1}{2}+\frac{1}{3}\Big)\alpha_1\alpha_2-\Big(\frac{1}{2}+\frac{1}{3}\Big)\beta_1\beta_2\Big\}$$

$$+6\beta_1\cdot 2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}3\alpha_2+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_2\Big)+6\beta_2\cdot 2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}3\alpha_1$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)-4\beta_1\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n+1}+\frac{1}{n}+\frac{1}{5}\Big)\alpha_2$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n}+\frac{1}{5}\Big)\beta_2\Big\}-4\beta_2\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n}+\frac{1}{n+1}+\frac{1}{5}\Big)\alpha_1$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n+1}+\frac{1}{5}\Big)\beta_1\Big\}+8\beta_1\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{2}+\frac{1}{3}+\frac{1}{n-2}\Big)\alpha_2$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{3}+\frac{1}{n-2}\Big)\beta_2\Big\}+8\beta_2\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{3}+\frac{1}{2}+\frac{1}{n-2}\Big)\alpha_1$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{2}+\frac{1}{n-2}\Big)\beta_1\Big\}\Big]E_{n-3}+\Big[\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)\Big\{6(3\alpha_2^2-\beta_2^2)$$

$$-4\Big(\Big(\frac{1}{6}+\frac{1}{n}+\frac{1}{n}\Big)\alpha_2^2-\frac{1}{n}\beta_2^2\Big)+8\Big(\Big(\frac{1}{3}+\frac{1}{3}+\frac{1}{n-3}\Big)\alpha_2^2-\frac{1}{3}\beta_2^2\Big)\Big\}$$

$$+6\beta_2\cdot 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}3\alpha_2-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_2\Big)-4\beta_2\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n}+\frac{1}{n}+\frac{1}{6}\Big)\alpha_2$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\left(\frac{1}{n}+\frac{1}{6}\right)\beta_2\Big\}+8\beta_2\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{n-3}\right)\alpha_2$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\left(\frac{1}{3}+\frac{1}{n-3}\right)\beta_2\Big\}\Big]D_{n-4}+\Big[\Big(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big)\Big\{6(3\alpha_2^2-\beta_2^2)-4\Big(\Big(\frac{1}{6}+\frac{1}{n}$$

$$+\frac{1}{n}\Big)\alpha_2^2-\frac{1}{n}\beta_2^2\Big)+8\Big(\Big(\frac{1}{3}+\frac{1}{3}+\frac{1}{n-3}\Big)\alpha_2^2-\frac{1}{3}\beta_2^2\Big)\Big\}+6\beta_2\cdot 2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}3\alpha_2$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_2\Big)-4\beta_2\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{n}+\frac{1}{n}+\frac{1}{6}\Big)\alpha_2+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n}+\frac{1}{6}\Big)\beta_2\Big\}$$

$$+8\beta_2\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\Big(\frac{1}{3}+\frac{1}{3}+\frac{1}{n-3}\Big)\alpha_2+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{3}+\frac{1}{n-3}\Big)\beta_2\Big\}\Big]E_{n-4}$$

$$+\sum_{s=0}^{2}6\beta_s\sum_{\substack{j+k=n-s\\ j,k\geqq 3}}\Big\{3\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_k$$

$$-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)-\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k$$

$$-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)\Big\}+\sum_{s=0}^{2}6\alpha_s\sum_{\substack{j+k=n-s\\ j,k\geqq 3}}3\cdot 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)$$

$$\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)-\sum_{s=0}^{2}6\beta_s\sum_{\substack{j+k=n-s\\ j,k\geqq 3}}2\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)$$

$$\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)-4\sum_{s=0}^{2}\beta_s\sum_{\substack{j+k=n-s\\ j,k\geqq 3}}\Big\{\Big(\frac{1}{n-s+2}+\frac{1}{n-j+2}+\frac{1}{n-k+2}\Big)$$

$$\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)$$

$$-\frac{1}{n-j+2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)\Big\}$$

$$-4\sum_{s=0}^{2}\alpha_s\sum_{\substack{j+k=n-s\\ j,k\geqq 3}}2\Big(\frac{1}{n-s+2}+\frac{1}{n-j+2}+\frac{1}{n-k+2}\Big)\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_j$$

$$-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)+4\sum_{s=0}^{2}\beta_s\sum_{\substack{j+k=n-s\\ j,k\geqq 3}}\Big(\frac{1}{n-j+2}$$

$$+\frac{1}{n-s+2}\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)$$

$$+8\sum_{s=0}^{2}\beta_s\sum_{\substack{j+k=n-s\\ j,k\geqq 3}}\Big\{\Big(\frac{1}{s+1}+\frac{1}{j+1}+\frac{1}{k+1}\Big)\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)$$

$$\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)-\frac{1}{j+1}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)$$

$$\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)\Big\}+8\sum_{s=0}^{2}\alpha_s\sum_{\substack{j+k=n-s\\ j,k\geqq 3}}2\Big(\frac{1}{s+1}+\frac{1}{j+1}+\frac{1}{k+1}\Big)$$

$$\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)$$

$$-8\sum_{s=0}^{3}\beta_s\sum_{\substack{j+k=n-s\\ j,k\geq 3}}\Big(\frac{1}{j+1}+\frac{1}{s+1}\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k$$

$$-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)\Big\}+2B_1\sum_{\substack{j+k=n+1\\ j,k\geq 3}}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)$$

$$\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)+B_2\sum_{\substack{j+k=n+1\\ j,k\geq 3}}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)$$

$$\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)-B_2\sum_{\substack{j+k=n+1\\ j,k\geq 3}}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)$$

$$\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)$$

$$+6\sum_{\substack{j+k+l=n\\ j,k,l\geq 3}}\Big\{3\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_k$$

$$-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)-\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k$$

$$-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)\Big\}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_l-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_l\Big)-4\sum_{\substack{j+k+l=n\\ j,k,l\geq 3}}\Big\{\Big(\frac{1}{n-j+2}$$

$$+\frac{1}{n-l+2}+\frac{1}{n-k+2}\Big)\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_k$$

$$-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)-\frac{1}{n-j+2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k$$

$$-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)\Big\}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_l-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_l\Big)+8\sum_{\substack{j+k+l=n\\ j,k,l\geq 3}}\Big\{\Big(\frac{1}{j+1}+\frac{1}{k+1}$$

$$+\frac{1}{l+1}\Big)\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)$$

$$-\frac{1}{j+1}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_j-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_j\Big)\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_k-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_k\Big)\Big\}$$

$$\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_l-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_l\Big). \tag{17}$$

Nach (16) und (17) lassen sich die D_{n+2} und E_{n+2} folgendermassen schreiben.

$$D_{n+2}=\sum_{i=1}^{6}a_i^{n+2}D_{n+2-i}+\sum_{i=1}^{6}b_i^{n+2}E_{n+2-i}+\sum_{q=2}^{3}{}^{3)}\sum_{i=1}^{h(n+2,q,1)}{}^{4)}\sum_{s_1+s_2=q}c^{n+2}_{q,n+2+q-1-i}$$

$$\Big\{(j_1,\cdots,j_{s_1}),(k_1,\cdots,k_{s_2})\Big\}\prod_{l=1}^{s_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)\prod_{m=1}^{s_2}$$

$$\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right),\quad (\text{für } n\geqq 4), \tag{18}$$

wo
$$a_1^{n+2}=2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\right),$$
$$a_2^{n+2}=2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_2+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_2\right),$$
$$a_3^{n+2}=a_4^{n+2}=a_5^{n+2}=a_6^{n+2}=0,$$
$$b_1^{n+2}=2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\right),$$
$$b_2^{n+2}=2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_2-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_2\right),$$
$$b_3^{n+2}=b_4^{n+2}=b_5^{n+2}=b_6^{n+2}=0,$$
$$c_{2,n+2}^{n+2}\{(j_1,j_2),(0,0)\}=1=c_{2,n+2}^{n+2}\{(0,0),(k_1,k_2)\},$$
$$c_{2,n+2}^{n+2}\{(j_1,0),(k_1,0)\}=0,$$
$$c_{q,n+2+q-1-i}^{n+2}=0 \qquad (q=3,\quad i=1,2,3,4,5)$$
$$(q=2,\quad i=2,3,4,5). \tag{19}$$

$$E_{n+2}=\sum_{i=1}^{6}d_i^{n+2}D_{n+2-i}+\sum_{i=1}^{6}e_i^{n+2}E_{n+2-i}+\sum_{q=2}^{3}\sum_{i=1}^{h(n+2,q,1)}\sum_{s_1+s_2=q}f_{q,n+2+q-1-i}^{n+2}$$

$$\Big\{(j_1,\cdots,j_{s_1}),(k_1,\cdots,k_{s_2})\Big\}\prod_{l=1}^{s_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)\prod_{m=1}^{s_2}$$

$$\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right),\quad (\text{für } n\geqq 7), \tag{20}$$

wo
$$d_1^{n+2}=2B_1\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\right)$$
$$+2B_2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0\right),$$

3) In $c_{q,n+2+q-1-i}^{n+2}\{(j_1,\cdots,j_{s_1}),(k_1,\cdots,k_{s_2})\}$ besteht es stets $s_1+s_2=q$ und die $j_1,\cdots,j_{s_1}$, $k_1,\cdots,k_{s_2}$ bedeuten alle positiven ganzen Zahlen unter der Bedingung, dass $j_1+\cdots+j_{s_1}+k_1+\cdots+k_{s_2}=n+2+q-1-i$ und $j_1,\cdots,j_{s_1}\geqq 3$, $k_1,\cdots,k_{s_2}\geqq 3$. Manchmal schreiben wir statt $c_{q,n+2+q-1-i}^{n+2}\{(j_1,\cdots,j_{s_1}),(k_1,\cdots,k_{s_2})\}$ ausführlich $c_{q,n+2+q-1-i}^{n+2}\{\underbrace{(j_1,\cdots,j_{s_1},0,0,\cdots,0)}_{q},\underbrace{(k_1,\cdots,k_{s_2},0,0,0,\cdots,0)}_{q}\}$

4) $h(n+2,q,p)=Min.(n+2-2q-p,5p)$

$$d_2^{n+2}=2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_1\Big)$$

$$+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)+\omega_0(n+2),$$

$$d_3^{n+2}=2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_2+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_2\Big)$$

$$+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_2-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_2\Big)+\omega_1(n+2),$$

$$d_4^{n+2}=\omega_2(n+2),$$

$$d_5^{n+2}=\omega_3(n+2),$$

$$d_6^{n+2}=\omega_4(n+2),$$

$$\omega_i(n+2)=\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)\Big[6\sum_{\substack{j+k=i\\ j,k\leq 2}}(3\alpha_j\alpha_k-\beta_j\beta_k)$$

$$-4\sum_{\substack{j+k=i\\ j,k\leq 2}}\Big\{\Big(\frac{1}{j+k+2}+\frac{1}{n+2-j}+\frac{1}{n+2-k}\Big)\alpha_j\alpha_k-\frac{1}{n+2-j}\beta_j\beta_k\Big\}$$

$$+8\sum_{\substack{j+k=i\\ j,k\leq 2}}\Big\{\Big(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{n-(j+k)+1}\Big)\alpha_j\alpha_k-\frac{1}{j+1}\beta_j\beta_k\Big\}\Big]$$

$$+6\sum_{\substack{j+k=i\\ j,k\leq 2}}\beta_j\cdot 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}3\alpha_k-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_k\Big)$$

$$-4\sum_{\substack{j+k=i\\ j,k\leq 2}}\beta_j\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n+2-j}+\frac{1}{j+k+2}+\frac{1}{n+2-k}\Big)\alpha_k$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n-k+2}+\frac{1}{i+2}\Big)\beta_k\Big\}$$

$$+8\sum_{\substack{j+k=i\\ j,k\leq 2}}\beta_j\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{n-(j+k)+1}\Big)\alpha_k$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{k+1}+\frac{1}{n-i+1}\Big)\beta_k\Big\},\quad (i=0,1,2,3,4),$$

$$e_1^{n+2}=2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\Big)$$

$$+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big),$$

$$e_2^{n+2}=2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1\Big)$$

$$+2B_2\Big(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)+\kappa_0(n+2),$$

$$e_3^{n+2}=2B_1\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_2-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_2\right)$$

$$+2B_2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_2+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_2\right)+\kappa_1(n+2),$$

$$e_4^{n+2}=\kappa_2(n+2),$$

$$e_5^{n+2}=\kappa_3(n+2),$$

$$e_6^{n+2}=\kappa_4(n+2),$$

$$\kappa_i(n+2)=\left(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\right)\left[6\sum_{\substack{j+k=i\\ j,k\leqq 2}}(3\alpha_j\alpha_k-\beta_j\beta_k)\right.$$

$$-4\sum_{\substack{j+k=i\\ j,k\leqq 2}}\left\{\left(\frac{1}{j+k+2}+\frac{1}{n+2-j}+\frac{1}{n+2-k}\right)\alpha_j\alpha_k-\frac{1}{n+2-j}\beta_j\beta_k\right\}$$

$$\left.+8\sum_{\substack{j+k=i\\ j,k\leqq 2}}\left\{\left(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{n-(j+k)+1}\right)\alpha_j\alpha_k-\frac{1}{j+1}\beta_j\beta_k\right\}\right]$$

$$+6\sum_{\substack{j+k=i\\ j,k\leqq 2}}\beta_j 2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}3\alpha_k+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_k\right)$$

$$-4\sum_{\substack{j+k=i\\ j,k\leqq 2}}\beta_j\left\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\left(\frac{1}{n+2-j}+\frac{1}{j+k+2}+\frac{1}{n+2-k}\right)\alpha_k\right.$$

$$\left.+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\left(\frac{1}{n-k+2}+\frac{1}{i+2}\right)\beta_k\right\}$$

$$+8\sum_{\substack{j+k=i\\ j,k\leqq 2}}\beta_j\left\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}2\left(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{n-(j+k)+1}\right)\alpha_k\right.$$

$$\left.+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\left(\frac{1}{k+1}+\frac{1}{n-i+1}\right)\beta_k\right\},\quad (i=0,1,2,3,4),$$

$$f^{n+2}_{2,n+2+2-1-1}=0,$$

$$f^{n+2}_{2,n+2+2-1-2}\{(j_1,j_2),(0,0)\}=B_2,$$

$$f^{n+2}_{2,n+2+2-1-2}\{(j_1,0),(k_1,0)\}=2B_1,$$

$$f^{n+2}_{2,n+2+2-1-2}\{(0,0),(k_1,k_2)\}=-B_2,$$

$$f^{n+2}_{2,n+2+2-1-(s+3)}\{(j_1,j_2),(0,0)\}=6\beta_s\cdot 3$$

$$-4\beta_s\left(\frac{1}{n-s+2}+\frac{1}{n-j_1+2}+\frac{1}{n-j_2+2}\right)+8\beta_s\left(\frac{1}{s+1}+\frac{1}{j_1+1}+\frac{1}{j_2+1}\right),$$

$(s=0,1,2)$,

$$f^{n+2}_{2,n+2+2-1-(s+3)}\{(j_1,0),(k_1,0)\}=6\alpha_s\cdot 3\cdot 2$$

$$-4\alpha_s 2\left(\frac{1}{n-s+2}+\frac{1}{n-j_1+2}+\frac{1}{n-k_1+2}\right)+8\alpha_s 2\left(\frac{1}{s+1}+\frac{1}{j_1+1}+\frac{1}{k_1+1}\right),$$

$(s=0,1,2)$,

$$f^{n+2}_{2,n+2+2-1-(s+3)}\{(0,0),(k_1,k_2)\}=-6\beta_s\cdot 3$$

$$+4\beta_s\frac{1}{n-k_1+2}+4\beta_s\left(\frac{1}{n-s+2}+\frac{1}{n-k_1+2}\right)-8\beta_s\frac{1}{k_1+1}-8\beta_s\left(\frac{1}{k_1+1}+\frac{1}{s+1}\right),$$

$(s=0,1,2)$,

$$f^{n+2}_{3,n+2+3-1-1}=f^{n+2}_{3,n+2+3-1-2}=f^{n+2}_{3,n+2+3-1-3}=0,$$

$$f^{n+2}_{3,n+2+3-1-4}\{(j_1,j_2,j_3),(0,0,0)\}=0,$$

$$f^{n+2}_{3,n+2+3-1-4}\{(j_1,j_2,0),(k_1,0,0)\}=6\cdot 3$$

$$-4\left(\frac{1}{n-j_1+2}+\frac{1}{n-j_2+2}+\frac{1}{n-k_1+2}\right)+8\left(\frac{1}{j_1+1}+\frac{1}{j_2+1}+\frac{1}{k_1+1}\right),$$

$$f^{n+2}_{3,n+2+3-1-4}\{(j_1,0,0),(k_1,k_2,0)\}=0,$$

$$f^{n+2}_{3,n+2+3-1-4}\{(0,0,0),(k_1,k_2,k_3)\}=6+4\frac{1}{n-k_1+2}-8\frac{1}{k_1+1},$$

$$f^{n+2}_{3,n+2+3-1-5}=0. \tag{21}$$

Für $4\leqq n\leqq 6$ besteht es

$$D_{n+2}=\sum_{i=1}^{n}a_i^{n+2}D_{n+2-i}+\sum_{i=1}^{n}b_i^{n+2}E_{n+2-i}+\sum_{q=2}^{2}\sum_{i=1}^{h(n+2,q,1)}\sum_{s_1+s_2=q}c^{n+2}_{q,n+2+q-1-i}$$

$$\left\{(j_1,\cdots,j_{s_1}),(k_1,\cdots,k_{s_2})\right\}\prod_{l=1}^{s_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)\prod_{m=1}^{s_2}$$

$$\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right),$$

und für $1\leqq n\leqq 3$ besteht es

$$D_{n+2}=\sum_{i=1}^{n}a_i^{n+2}D_{n+2-i}+\sum_{i=1}^{n}b_i^{n+2}E_{n+2-i}, \tag{18'}$$

wo $$a_1^{n+2}=2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\right),$$

$$a_2^{n+2}=\begin{cases}2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_2+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_2\right), & (n\geqq 3)\\ \left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_2+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_2\right), & (n=2),\end{cases}$$

$$a_i^{n+2}=0,\quad (i\geqq 3),$$

$$b_1^{n+2}=2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\right),$$

$$b_2^{n+2}=\begin{cases}2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_2-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_2\right), & (n\geqq 3)\\ \left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_2-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_2\right), & (n=2),\end{cases}$$

$$\begin{aligned}
&b_i^{n+2}=0,\quad (i\geqq 3),\\
&c_{2,n+2}^{n+2}\{(j_1,j_2),\ (0,0)\}=1=c_{2,n+2}^{n+2}\{(0,0),(k_1,k_2)\},\\
&c_{2,n+2}^{n+2}\{(j_1,0),(k_1,0)\}=0,\\
&c_{2,n+2+2-1-i}^{n+2}=0,\quad (i=2,3,4,5).
\end{aligned}\tag{19'}$$

Für $4\leqq n\leqq 6$ besteht es

$$E_{n+2}=\sum_{i=1}^{n}d_i^{n+2}D_{n+2-i}+\sum_{i=1}^{n}e_i^{n+2}E_{n+2-i}+\sum_{q=2}^{2}\sum_{i=1}^{h(n+2,q,1)}\sum_{s_1+s_2=q}f_{q,n+2+q-1-i}^{n+2}$$

$$\{(j_1,\cdots,j_{s_1}),(k_1,\cdots,k_{s_2})\}\prod_{=}^{s_1}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}$$

$$\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big),\tag{20'}$$

wo

$$\begin{aligned}
d_1^{n+2}&=2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\Big)\\
&\quad+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big),\\
d_2^{n+2}&=2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_1\Big)\\
&\quad+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)+\omega_0(n+2),\\
d_3^{n+2}&=2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_2+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_2\Big)\\
&\quad+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_2-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_2\Big)+\omega_1(n+2),\\
d_4^{n+2}&=\omega_2(n+2),\\
d_5^{n+2}&=\omega_3(n+2),\\
d_6^{n+2}&=\omega_4(n+2),\\
e_1^{n+2}&=2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\Big)\\
&\quad+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big),\\
e_2^{n+2}&=2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1\Big)\\
&\quad+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)+\kappa_0(n+2),\\
e_3^{n+2}&=2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_2-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_2\Big)
\end{aligned}$$

$$+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_2+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_2\Big)+\kappa_1(n+2),$$

$$e_4^{n+2}=\kappa_2(n+2),$$

$$e_5^{n+2}=\kappa_3(n+2),$$

$$e_6^{n+2}=\kappa_4(n+4),$$

$$\omega_i(n+2)=\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)\Big[6\sum_{\substack{j+k=i\\ j,k\leq 2}}(3\alpha_j\alpha_k-\beta_j\beta_k)$$

$$-4\sum_{\substack{j+k=i\\ j,k\leq 2}}\Big\{\Big(\frac{1}{j+k+2}+\frac{1}{n+2-j}+\frac{1}{n+2-k}\Big)\alpha_j\alpha_k-\frac{1}{n+2-j}\beta_j\beta_k\Big\}$$

$$+8\sum_{\substack{j+k=i\\ j,k\leq 2}}\Big\{\Big(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{n-(j+k)+1}\Big)\alpha_j\alpha_k-\frac{1}{j+1}\beta_j\beta_k\Big\}\Big]$$

$$+6\sum_{\substack{j+k=i\\ j,k\leq 2}}\beta_j 2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}3\alpha_k-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_k\Big)$$

$$-4\sum_{\substack{j+k=i\\ j,k\leq 2}}\beta_j\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{n+2-j}+\frac{1}{j+k+2}+\frac{1}{n+2-k}\Big)\alpha_k$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{n-k+2}+\frac{1}{i+2}\Big)\beta_k\Big\}$$

$$+8\sum_{\substack{j+k=i\\ j,k\leq 2}}\beta_j\Big\{-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}2\Big(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{n-(j+k)+1}\Big)\alpha_k$$

$$-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big(\frac{1}{k+1}+\frac{1}{n-i+1}\Big)\beta_k\Big\},\quad (i=0,1,2,3,4),\quad (n\geqq 7)$$

$$\kappa_i(n+2)=\Big(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big)\Big[6\sum_{\substack{j+k=i\\ j,k\leq 2}}(3\alpha_j\alpha_k-\beta_j\beta_k)$$

$$-4\sum_{\substack{j+k=i\\ j,k\leq 2}}\Big\{\Big(\frac{1}{j+k+2}+\frac{1}{n+2-j}+\frac{1}{n+2-k}\Big)\alpha_j\alpha_k-\frac{1}{n+2-j}\beta_j\beta_k\Big\}$$

$$+8\sum_{\substack{j+k=i\\ j,k\leq 2}}\Big\{\Big(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{n-(j+k)+1}\Big)\alpha_j\alpha_k-\frac{1}{j+1}\beta_j\beta_k\Big\}\Big]$$

$$+6\sum_{\substack{j+k=i\\ j,k\leq 2}}\beta_j 2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}3\alpha_k+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_k\Big)$$

$$-4\sum_{\substack{j+k=i\\ j,k\leq 2}}\beta_j\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^3}2\Big(\frac{1}{n+2-j}+\frac{1}{j+k+2}+\frac{1}{n+2-k}\Big)\alpha_k$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big(\frac{1}{n-k+2}+\frac{1}{i+2}\Big)\beta_k\Big\}$$

$$+8\sum_{\substack{j+k=i\\ j,k\leq 2}}\beta_j\Big\{-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^3}2\Big(\frac{1}{j+1}+\frac{1}{k+1}+\frac{1}{n-(j+k)+1}\Big)\alpha_k$$

$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\left(\frac{1}{k+1}+\frac{1}{n-i+1}\right)\beta_k\Big\},\quad (i=0,1,2,3,4),\quad (n\geqq 7)$$

$$f^{n+2}_{2,n+2+2-1-1}=0,$$

$$f^{n+2}_{2,n+2+2-1-2}\{(j_1,j_2),(0,0)\}=B_2,$$

$$f^{n+2}_{2,n+2+2-1-2}\{(j_1,0),(k_1,0)\}=2B_1,$$

$$f^{n+2}_{2,n+2+2-1-2}\{(0,0),(k_1,k_2)\}=-B_2,$$

$$f^{n+2}_{2,n+2+2-1-(s+3)}\{(j_1,j_2),(0,0)\}=6\beta_s\cdot 3-4\beta_s\Big(\frac{1}{n-s+2}+\frac{1}{n-j_1+2}$$

$$+\frac{1}{n-j_2+2}\Big)+8\beta_s\Big(\frac{1}{s+1}+\frac{1}{j_1+1}+\frac{1}{j_2+1}\Big),\quad (s=0,1,2).$$

$$f^{n+2}_{2,n+2+2-1-(s+3)}\{(j_1,0),(k_1,0)\}=6\alpha_s\cdot 3\cdot 2-4\alpha_s 2\Big(\frac{1}{n-s+2}+\frac{1}{n-j_1+2}$$

$$+\frac{1}{n-k_1+2}\Big)+8\alpha_s 2\Big(\frac{1}{s+1}+\frac{1}{j_1+1}+\frac{1}{k_1+1}\Big),\quad (s=0,1,2).$$

$$f^{n+2}_{2,n+2+2-1-(s+3)}\{(0,0),(k_1,k_2)\}=-6\beta_s\cdot 3+4\beta_s\frac{1}{n-k_1+2}+4\beta_s\Big(\frac{1}{n-s+2}$$

$$+\frac{1}{n-k_1+2}\Big)-8\beta_s\frac{1}{k_1+1}-8\beta_s\Big(\frac{1}{k_1+1}+\frac{1}{s+1}\Big),\quad (s=0,1,2).\qquad (21')$$

Wir können also die beiden α_{n+2} und β_{n+2} folgendermassen schreiben.

$$\alpha_{n+2}=\sum_{i=1}^{6}\mu_i^{n+2,1}D_{n+2-i}+\sum_{i=1}^{6}\nu_i^{n+2,1}E_{n+2-i}+\sum_{q=2}^{3}\sum_{i=1}^{h(n+2,q,1)}\sum_{s_1+s_2=q}$$

$$\omega^{n+2,1}_{q,n+2+q-1-i}\Big\{\Big(j_1,\cdots,j_{s_1}\Big),\Big(k_1,\cdots,k_{s_2}\Big)\Big\}\prod_{l=1}^{s_1}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)$$

$$\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big),\quad (n\geqq 7),\qquad (22)$$

$$\alpha_{n+2}=\sum_{i=1}^{n}\mu_i^{n+2,1}D_{n+2-i}+\sum_{i=1}^{n}\nu_i^{n+2,1}E_{n+2-i}+\sum_{q=2}^{2}\sum_{i=1}^{h(n+2,q,1)}\sum_{s_1+s_2=q}$$

$$\omega^{n+2,1}_{q,n+2+q-1-i}\Big\{\Big(j_1,\cdots,j_{s_1}\Big),\Big(k_1,\cdots,k_{s_2}\Big)\Big\}\prod_{l=1}^{s_1}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)$$

$$\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big),\quad (4\leqq n\leqq 6),\qquad (22')$$

wo $$\mu_i^{n+2,1}=-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}a_i^{n+2}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}d_i^{n+2},$$

$$\nu_i^{n+2,1}=-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}b_i^{n+2}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}e_i^{n+2},$$

$$\omega^{n+2,1}_{q,n+2+q-1-i}\Big\{\Big(j_1,\cdots,j_{s_1}\Big),\Big(k_1,\cdots,k_{s_2}\Big)\Big\}=-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}c^{n+2}_{q,n+2+q-1-i}$$

$$\left\{\left(j_1, \cdots, j_{s_1}\right), \left(k_1, \cdots, k_{s_2}\right)\right\} - \frac{A_2}{(\sqrt{A_1^2+A_2^2})^2} f^{n+2}_{q,n+2+q-1-i} \left\{\left(j_1, \cdots, j_{s_1}\right),\right.$$

$$\left.\left(k_1, \cdots, k_{s_2}\right)\right\}. \tag{23}$$

$$\beta_{n+2} = \sum_{i=1}^{6} \mu_i^{n+2,1,(1)} D_{n+2-i} + \sum_{i=1}^{6} \nu_i^{n+2,1,(1)} E_{n+2-i} + \sum_{q=2}^{3} \sum_{i=1}^{h(n+2,q,1)} \sum_{s_1+s_2=q}$$

$$\omega^{n+2,1,(1)}_{q,n+2+q-1-i} \left\{\left(j_1, \cdots, j_{s_1}\right), \left(k_1, \cdots, k_{s_2}\right)\right\} \prod_{l=1}^{s_1} \left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}} D_{j_l} - \frac{A_2}{(\sqrt{A_1^2+A_2^2})^2} E_{j_l}\right)$$

$$\prod_{m=1}^{s_2} \left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}} D_{k_m} - \frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} E_{k_m}\right), \quad (n \geqq 7), \tag{24}$$

$$\beta_{n+2} = \sum_{i=1}^{n} \mu_i^{n+2,1,(1)} D_{n+2-i} + \sum_{i=1}^{n} \nu_i^{n+2,1,(1)} E_{n+2-i} + \sum_{q=2}^{2} \sum_{i=1}^{h(n+2,q,1)}$$

$$\sum_{s_1+s_2=q} \omega^{n+2,1,(1)}_{q,n+2+q-1-i} \left\{\left(j_1, \cdots, j_{s_1}\right), \left(k_1, \cdots, k_{s_2}\right)\right\} \prod_{l=1}^{s_1} \left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}} D_{j_l} - \frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\right.$$

$$\left. E_{j_l}\right) \prod_{m=1}^{s_2} \left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}} D_{k_m} - \frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} E_{k_m}\right), \quad (4 \leqq n \leqq 6), \tag{24$'$}$$

$$\beta_{n+2} = \sum_{i=1}^{n} \mu_i^{n+2,1,(1)} D_{n+2-i} + \sum_{i=1}^{n} \nu_i^{n+2,1,(1)} E_{n+2-i}, \tag{24$''$}$$

wo $\mu_i^{n+2,1,(1)} = \frac{A_2}{2\sqrt{A_1^2+A_2^2}} a_i^{n+2} - \frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} d_i^{n+2}$,

$$\nu_i^{n+2,1,(1)} = \frac{A_2}{2\sqrt{A_1^2+A_2^2}} b_i^{n+2} - \frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} e_i^{n+2},$$

$$\omega^{n+2,1,(1)}_{q,n+2+q-1-i} \left\{\left(j_1, \cdots, j_{s_1}\right), \left(k_1, \cdots, k_{s_2}\right)\right\} = \frac{A_2}{2\sqrt{A_1^2+A_2^2}} c^{n+2}_{q,n+2+q-1-i}$$

$$\left\{\left(j_1, \cdots, j_{s_1}\right), \left(k_1, \cdots, k_{s_2}\right)\right\} - \frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} f^{n+2}_{q,n+2+q-1-i} \left\{\left(j_1, \cdots, j_{s_1}\right),\right.$$

$$\left.\left(k_1, \cdots, k_{s_2}\right)\right\}. \tag{25}$$

Wir nehmen nun an, dass die beiden α_{n+2} und β_{n+2} sich folgendermassen schreiben lassen, wenn p genug klein in bezug auf n und n genug gross ist.

$$\alpha_{n+2} = \sum_{i=1}^{5p+1} \mu_i^{n+2,p} D_{n+3-p-i} + \sum_{i=1}^{5p+1} \nu_i^{n+2,p} E_{n+3-p-i} + \sum_{q=2}^{2p+1} \sum_{i=1}^{5p} \sum_{s_1+s_2=q}$$

$$\omega^{n+2,p}_{q,n+2+q-p-i} \left\{\left(j_1, j_2, \cdots, j_{s_1}\right), \left(k_1, k_2, \cdots, k_{s_2}\right)\right\} \prod_{l=1}^{s_1} \left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}} D_{j_l}\right.$$

$$\left.-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2} E_{j_l}\right) \prod_{m=1}^{s_2} \left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}} D_{k_m} - \frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} E_{k_m}\right) + \sum_{q=2}^{2p+1} \sum_{i=1}^{5p} \sum_{s_1+s_2=q-1}^{5)}$$

$$\Big[\pi^{n+2,p}_{q,n+2+q-p-i}\Big\{\Big(j_1, j_2, \cdots, j_{s_1}\Big), \Big(k_1, k_2, \cdots, k_{s_2}\Big); r\Big\}D_r + \delta^{n+2,p}_{q,n+2+q-p-i}$$

$$\Big\{\Big(j_1, j_2, \cdots, j_{s_1}\Big), \Big(k_1, k_2, \cdots, k_{s_2}\Big); r\Big\}E_r\Big]\prod_{l=1}^{s_1}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}$$

$$-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big). \tag{26}$$

$$\beta_{n+2}=\sum_{i=1}^{5p+1}\mu_i^{n+2,p,(1)}D_{n+3-p-i}+\sum_{i=1}^{5p+1}\nu_i^{n+2,p,(1)}E_{n+3-p-i}+\sum_{q=2}^{2p+1}\sum_{i=1}^{5p}\sum_{s_1+s_2=q}$$

$$\omega^{n+2,p,(1)}_{q,n+2+q-p-i}\Big\{\Big(j_1, j_2, \cdots, j_{s_1}\Big), \Big(k_1, k_2, \cdots, k_{s_2}\Big)\Big\}\prod_{l=1}^{s_1}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}$$

$$-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big)$$

$$+\sum_{q=2}^{2p+1}\sum_{i=1}^{5p}\sum_{s_1+s_2=q-1}\Big[\pi^{n+2,p,(1)}_{q,n+2+q-p-i}\Big\{\Big(j_1, j_2, \cdots, j_{s_1}\Big), \Big(k_1, k_2, \cdots, k_{s_2}\Big); r\Big\}D_r$$

$$+\delta^{n+2,p,(1)}_{q,n+2+q-p-i}\Big\{\Big(j_1, j_2, \cdots, j_{s_1}\Big), \Big(k_1, k_2, \cdots, k_{s_2}\Big); r\Big\}E_r\Big]\prod_{l=1}^{s_1}$$

$$\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big). \tag{27}$$

Wir setzen nun (18) (18′) (20) (20′) in (26) ein, so bekommen wir

$$\alpha_{n+2}=\sum_{i_1=1}^{5p+1}\mu_{i_1}^{n+2,p}\Big\{\sum_{i_2=1}^{6}a_{i_2}^{n+3-p-i_1}D_{n+3-p-i_1-i_2}+\sum_{i_2=1}^{6}b_{i_2}^{n+3-p-i_1}E_{n+3-p-i_1-i_2}$$

$$+\sum_{q=2}^{3}\sum_{i_2=1}^{5}\sum_{s_1+s_2=q}c^{n+3-p-i_1}_{q,n+3-p-i_1+q-1-i_2}\Big\{\Big(j_1, j_2, \cdots, j_{s_1}\Big), \Big(k_1, k_2, \cdots, k_{s_2}\Big)\Big\}\prod_{l=1}^{s_1}$$

$$\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big)\Big\}$$

$$+\sum_{i_1=1}^{5p+1}\nu_{i_1}^{n+2,p}\Big\{\sum_{i_2=1}^{6}d_{i_2}^{n+3-p-i_1}D_{n+3-p-i_1-i_2}+\sum_{i_2=1}^{6}e_{i_2}^{n+3-p-i_1}E_{n+3-p-i_1-i_2}$$

$$+\sum_{q=2}^{3}\sum_{i_2=1}^{5}\sum_{s_1+s_2=q}f^{n+3-p-i_1}_{q,n+3-p-i_1+q-1-i_2}\Big\{\Big(j_1, j_2, \cdots, j_{s_1}\Big), \Big(k_1, k_2, \cdots, k_{s_2}\Big)\Big\}\prod_{l=1}^{s_1}$$

$$\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big)$$

5) Sowohl in $\pi^{n+2,p}_{q,n+2+q-p-i}\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r\}$ als auch in $\delta^{n+2,p}_{q,n+2+q-p-i}\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r\}$ besteht es stets $s_1+s_2=q-1$ und die $j_1, j_2, \cdots, j_{s_1}$, $k_1, k_2, \cdots, k_{s_2}$, r bedeuten alle positiven ganzen Zahlen unter der Bedingung, dass $j_1+j_2+\cdots+j_{s_1}+k_1+k_2+\cdots+k_{s_2}+r=n+2+q-p-i$ und $j_1, j_2, \cdots, j_{s_1}, k_1, k_2, \cdots, k_{s_2}, r \geqq 3$.

$$+\sum_{q=2}^{2p+1}\sum_{t_1=1}^{5p}\sum_{\substack{s_1+s_2=q\\ k_{s_2}\geqq 6}}\omega^{n+2,p}_{q,n+2+q-p-t_1}\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\right\}\prod_{l=1}^{s_1}$$

$$\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)\prod_{m=1}^{s_2-1}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right)$$

$$\left\{\sum_{t_2=1}^{(6,k_{s_2}-2)^{6)}}\mu_{t_2}^{k_{s_2},1,(1)}D_{k_{s_2}-t_2}+\sum_{t_2=1}^{(6,k_{s_2}-2)}\nu_{t_2}^{k_{s_2},1,(1)}E_{k_{s_2}-t_2}+\sum_{q'=2}^{3}\ {}^{7)}\sum_{t_2=1}^{h(k_{s_2},q',1)}\sum_{s'_1+s'_2=q'}\omega^{k_{s_2},1,(1)}_{q',k_{s_2}+q'-1-t_2}\right.$$

$$\left\{(j'_1, \cdots, j'_{s_1}),\ (k'_1\cdots k'_{s_2})\right\}\prod_{l=1}^{s'_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j'_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j'_l}\right)\prod_{m=1}^{s'_2}$$

$$\left.\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k'_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k'_m}\right)\right\}+\sum_{q=2}^{2p+1}\sum_{t_1=1}^{5p}\sum_{\substack{s_1+s_2=q\\ j_{s_1}\geqq 6}}\omega^{n+2,p}_{q,n+2+q-p-t_1}$$

$$\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\right\}\prod_{l=1}^{s_1-1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)\prod_{m=1}^{s_2}$$

$$\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right)\left\{\sum_{t_2=1}^{(6,j_{s_1}-2)}\mu_{t_2}^{j_{s_1},1}D_{j_{s_1}-t_2}+\sum_{t_2=1}^{(6,j_{s_1}-2)}\nu_{t_2}^{j_{s_1},1}E_{j_{s_1}-t_2}\right.$$

$$+\sum_{q'=2}^{3}\ {}^{8)}\sum_{t_2=1}^{h(j_{s_1},q',1)}\sum_{s'_1+s'_2=q'}\omega^{j_{s_1},1}_{q',j_{s_1}+q'-1-t_2}\left\{(j'_1, \cdots, j'_{s_1}),\ (k'_1, \cdots, k'_{s_2})\right\}\prod_{l=1}^{s'_1}$$

$$\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j'_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j'_l}\right)\prod_{m=1}^{s'_2}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k'_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k'_m}\right)$$

$$+\sum_{q=2}^{2p+1}\sum_{t_1=1}^{5p}\sum_{\substack{s_1+s_2=q\\ 3\leqq k_{s_2}\leqq 5}}\omega^{n+2,p}_{q,n+2+q-p-t_1}\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\right\}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\right.$$

$$\left.D_{k_{s_2}}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_{s_2}}\right)\prod_{l=1}^{s_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)\prod_{m=1}^{s_2-1}\left(\frac{A_2}{2\sqrt{A^2+A_2^2}}\right.$$

$$\left.D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right)+\sum_{q=2}^{2p+1}\sum_{t_1=1}^{5p}\sum_{\substack{s_1+s_2=q\\ 3\leqq j_{s_1}\leqq 5}}\omega^{n+2,p}_{q,n+2+q-p-t_1}\left\{(j_1, j_2, \cdots, j_{s_1}),\right.$$

$$\left.(k_1, k_2, \cdots, k_{s_2})\right\}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_{s_1}}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_{s_1}}\right)\prod_{l=1}^{s_1-1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}\right.$$

$$\left.-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)\prod_{m=1}^{s_2}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right)$$

---

6) $(6,\ k_{s_2}-2)$ bedeutet das Minimum von 6 und $k_{s_2}-2$.

7). 8). Wenn $6\leqq k_{s_2}$ (oder j_{s_1}) <9 ist, so soll $\sum\limits_{q'=2}^{3}$ durch $\sum\limits_{q'=2}^{2}$ ersetzt werden.

$$+\sum_{q=2}^{2p+1}\sum_{t_1=1}^{5p}\sum_{\substack{s_1+s_2=q-1\\ r\geq 6}}\prod_{l=1}^{s_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)\prod_{m=1}^{s_2}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\right.$$

$$\left.D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right)\pi^{n+2,p}_{q,n+2+q-p-t}\left\{(j_1,j_2,\cdots,j_{s_1}),\ (k_1,k_2,\cdots,k_{s_2})\,;r\right\}$$

$$\left\{\sum_{t_2=1}^{(6,r-2)}a^r_{t_2}D_{r-t_2}+\sum_{t_2=1}^{(6,r-2)}b^r_{t_2}E_{r-t_2}+\sum_{q'=2}^{3}{}^{9)}\sum_{t_2=1}^{h(r,q',1)}\sum_{s'_1+s'_2=q'}c^r_{q',r+q'-1-t_2}\left\{(j'_1,\cdots,j'_{s'_1}),\right.\right.$$

$$\left.\left.(k'_1,\cdots,k'_{s'_2})\right\}\prod_{l=1}^{s'_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j'_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j'_l}\right)\prod_{m=1}^{s'_2}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k'_m}\right.$$

$$\left.-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k'_m}\right)+\sum_{q=2}^{2p+1}\sum_{t_1=1}^{5p}\sum_{\substack{s_1+s_2=q-1\\ r\geq 6}}\prod_{l=1}^{s_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)$$

$$\prod_{m=1}^{s_2}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right)\delta^{n+2,p}_{q,n+2+q-p-t_1}\left\{(j_1,j_2,\cdots,j_{s_1}),\right.$$

$$\left.(k_1,k_2,\cdots,k_{s_2});\,r\right\}\left\{\sum_{t_2=1}^{(6,r-2)}d^r_{t_2}D_{r-t_2}+\sum_{t_2=1}^{(6,r-2)}e^r_{t_2}E_{r-t_2}+\sum_{q'=2}^{3}\sum_{t_2=1}^{h(r,q',1)}\sum_{s'_1+s'_2=q'}f^r_{q',r+q'-1-t_2}\right.$$

$$\left\{(j'_1,\cdots,j'_{s'_1}),(k'_1,\cdots,k'_{s'_2}\right\}\prod_{l=1}^{s'_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j'_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j'_l}\right)\prod_{m=1}^{s'_2}$$

$$\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k'_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k'_m}\right)$$

$$+\sum_{q=2}^{2p+1}\sum_{t=1}^{5p}\sum_{\substack{s_1+s_2=q-1\\ 3\leq r\leq 5}}\left[\pi^{n+2,p}_{q,n+2+q-p-t}\left\{(j_1,j_2,\cdots,j_{s_1}),\ (k_1,k_2,\cdots,k_{s_2})\,;\,r\right\}D_r\right.$$

$$\left.+\,\delta^{n+2,p}_{q,n+2+q-p-t}\left\{(j_1,j_1,\cdots,j_{s_1}),\ (k_1,k_2,\cdots,k_{s_2});\,r\right\}E_r\right]\prod_{l=1}^{s_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\right.$$

$$\left.D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)\prod_{m=1}^{s_2}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right).$$

Wir können also die α_{n+2} folgendermassen schreiben.

$$\alpha_{n+2}=\sum_{t=1}^{5(p+1)+1}\mu^{n+2,p+1}_t D_{n+3-(p+1)-t}+\sum_{t=1}^{5(p+1)+1}\nu^{n+2,p+1}_t E_{n+3-(p+1)-t}$$

$$+\sum_{q=2}^{2(p+1)+1}\sum_{t=1}^{5(p+1)}\sum_{s_1+s_2=q}\omega^{n+2,p+1}_{q,n+2+q-(p+1)-t}\left\{(j_1,j_2,\cdots,j_{s_1}),\ (k_1,k_2,\cdots,k_{s_2})\right\}\prod_{l=1}^{s_1}$$

$$\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)\prod_{m=1}^{s_2}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right)$$

$$+\sum_{q=2}^{2(p+1)+1}\sum_{t=1}^{h(n+2,q,p+1)}\sum_{s_1+s_2=q-1}\left[\pi^{n+2,p+1}_{q,n+2+q-(p+1)-t}\left\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2});\right.\right.$$

---

9) Wenn $6\leq r<9$ ist, so soll $\sum_{q'=2}^{3}$ durch $\sum_{q'=2}^{2}$ ersetzt werden.

$$r\Big\}D_r+\delta^{n+2,p+1}_{q,n+2+q-(p+1)-i}\Big\{(j_1,j_2,\cdots,j_{s_1}),\ (k_1,k_2,\cdots,k_{s_2});\ r\Big\}E_r\Big]\prod_{l=1}^{s_1}$$

$$\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^3}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^3}E_{k_m}\Big),$$

wo
$$\mu_i^{n+2,p+1}=\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\ge 1\\ i_2\le 6}}(\mu_{i_1}^{n+2,p}a_{i_2}^{n+3-p-i_1}+\nu_{i_1}^{n+2,p}d_{i_2}^{n+3-p-i_1})+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\ge 1\\ 4\le i_2\le 6}}$$

$$\Big[\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\Big)(\mu_{i_2}^{i_2+2,1,(1)}D_2+\nu_{i_2}^{i_2+2,1,(1)}E_2)\,\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1,0),\ (i_2+2,0)\Big\}$$

$$+\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)(\mu_{i_2}^{i_2+2,1,(1)}D_2+\nu_{i_2}^{i_2+2,1,(1)}E_2)\,\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(0,0),\ (k_1,i_2+2)\Big\}$$

$$+\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)(\mu_{i_2}^{i_2+2,1}D_2+\nu_{i_2}^{i_2+2,1}E_2)\,\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(i_2+2,0),\ (k_1,0)\Big\}$$

$$+\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\Big)(\mu_{i_2}^{i_2+2,1}D_2+\nu_{i_2}^{i_2+2,1}E_2)\,\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1,i_2+2),\ (0,0)\Big\}\Big]$$

$$+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\ge 1\\ 4\le i_2\le 6}}\Big[\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\Big)\Big\{(a_{i_2}^{i_2+2}D_2+b_{i_2}^{i_2+2}E_2)\,\pi^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1);\ i_2+2\Big\}$$

$$+(d_{i_2}^{i_2+2}D_2+e_{i_2}^{i_2+2}E_2)\,\delta^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1);\ i_2+2\Big\}\Big\}+\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)\Big\{(a_{i_2}^{i_2+2}D_2$$

$$+b_{i_2}^{i_2+2}E_2)\,\pi^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(k_1);\ i_2+2\Big\}+(d_{i_2}^{i_2+2}D_2+e_{i_2}^{i_2+2}E_2)\,\delta^{n+2,p}_{2,n+2+2-p-i_1}$$

$$\Big\{(k_1);\ i_2+2\Big\}\Big\}\Big]+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\ge 1\\ 1\le i_2\le 3}}\Big[\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\Big)\Big\{\pi^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1);\ i_2+2\Big\}D_{i_2+2}+$$

$$\delta^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1);\ i_2+2\Big\}E_{i_2+2}\Big\}+\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)\Big\{\pi^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(k_1);\ i_2+2\Big\}D_{i_2+2}$$

$$+\delta^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(k_1);\ i_2+2\Big\}E_{i_2+2}\Big\}\Big],$$

$$\nu_i^{n+2,p+1}=\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\ge 1\\ i_2\le 6}}(\mu_{i_1}^{n+2,p}b_{i_2}^{n+3-p-i_1}+\nu_{i_1}^{n+2,p}e_{i_2}^{n+3-p-i_1})+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\ge 1\\ 4\le i_2\le 6}}$$

$$\Big[\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^3}\Big)(\mu_{i_2}^{i_2+2,1,(1)}D_2+\nu_{i_2}^{i_2+2,1,(1)}E_2)\,\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1,0),\ (i_2+2,0)\Big\}\cdot$$

$$+\Big(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^3}\Big)(\mu_{i_2}^{i_2+2,1,(1)}D_2+\nu_{i_2}^{i_2+2,1,(1)}E_2)\,\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(0,0),\ (k_1,i_2+2)\Big\}$$

$$+\left(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\right)(\mu_{i_2}^{i_2+2,1} D_2+\nu_{i_2}^{i_2+2,1} E_2)\,\omega_{2,n+2+2-p-i_1}^{n+2,p}\Big\{(i_2+2,0),\ (k_1,0)\Big\}$$

$$+\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\right)(\mu_{i_2}^{i_2+2,1} D_2+\nu_{i_2}^{i_2+2,1} E_2)\,\omega_{2,n+2+2-p-i_1}^{n+2,p}\Big\{(j_1, i_2+2),\ (0,0)\Big\}\Big]$$

$$+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geq 1\\ 4\leq i_2\leq 6}}\Big[\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\right)\Big\{(a_{i_2}^{i_2+2} D_2+b_{i_2}^{i_2+2} E_2)\,\pi_{2,n+2+2-p-i_1}^{n+2,p}\Big\{(j_1);\ i_2+2\Big\}$$

$$+(d_{i_2}^{i_2+2} D_2+e_{i_2}^{i_2+2} E_2)\,\delta_{2,n+2+2-p-i_1}^{n+2,p}\Big\{(j_1);\ i_2+2\Big\}\Big\}+\left(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\right)\Big\{(a_{i_2}^{i_2+2} D_2$$

$$+b_{i_2}^{i_2+2} E_2)\,\pi_{2,n+2+2-p-i_1}^{n+2,p}\Big\{(k_1);\ i_2+2\Big\}+(d_{i_2}^{i_2+2} D_2+e_{i_2}^{i_2+2} E_2)\,\delta_{2,n+2+2-p-i_1}^{n+2,p}$$

$$\Big\{(k_1);\ i_2+2\Big\}\Big\}\Big]+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geq 1\\ 1\leq i_2\leq 3}}\Big[\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\right)\Big\{\pi_{2,n+2+2-p-i_1}^{n+2,p}\Big\{(j_1);\ i_2+2\Big\}D_{i_2+2}+$$

$$\delta_{2,n+2+2-p-i_1}^{n+2,p}\Big\{(j_1);\ i_2+2\Big\}E_{i_2+2}\Big\}+\left(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\right)\Big\{\pi_{2,n+2+2-p-i_1}^{n+2,p}\Big\{(k_1);\ i_2+2\Big\}$$

$$D_{i_2+2}+\delta_{2,n+2+2-p-i_1}^{n+2,p}\Big\{(k_1);\ i_2+2\Big\}E_{i_2+2}\Big\}\Big],$$

$$\omega_{2,n+2+2-(p+1)-i}^{n+2,p+1}\Big\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\Big\}=\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq 6}}$$

$$\Big[c_{2,n+3-p-i_1+2-1-i_2}^{n+3-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\Big\}\,\mu_{i_1}^{n+2,p}+f_{2,n+3-p-i_1+2-1-i_2}^{n+3-p-i_1}$$

$$\Big\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\Big\}\,\nu_{i_1}^{n+2,p}\Big]+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geq 1\\ 4\leq i_2\leq 6}}\Big[\omega_{3,n+2+3-p-i_1}^{n+2,p}$$

$$\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}, i_2+2)\Big\}(\mu_{i_2}^{i_2+2,1,(1)} D_2+\nu_{i_2}^{i_2+2,1,(1)} E_2)+\omega_{3,n+2+3-p-i_1}^{n+2,p}$$

$$\Big\{(j_1, j_2, \cdots, j_{s_1}, i_2+2),\ (k_1, k_2, \cdots, k_{s_2})\Big\}\,(\mu_{i_2}^{i_2+2,1} D_2+\nu_{i_2}^{i_2+2,1} E_2)\Big]$$

$$+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geq 1\\ 4\leq i_2\leq 6}}\Big[\pi_{3,n+2+3-p-i_1}^{n+2,p}\Big\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2});\ i_2+2\Big\}(a_{i_2}^{i_2+2} D_2$$

$$+b_{i_2}^{i_2+2} E_2)+\delta_{3,n+2+3-p-i_1}^{n+2,p}\Big\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2});\ i_2+2\Big\}(d_{i_2}^{i_2+2} D_2$$

$$+e_{i_2}^{i_2+2}E_2)\Big]+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geq 1\\ 1\leq i_2\leq 3}}\Big[\pi^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),\ (k_1,k_2,\cdots,k_{s_2});\ i_2+2\Big\}D_{i_2+2}$$

$$+\partial^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),\ (k_1,k_2,\cdots,k_{s_2});\ i_2+2\Big\}E_{i_2+2}\Big],$$

$$\omega^{n+2,p+1}_{3,n+2+3-(p+1)-i}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2})\Big\}=\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq 5}}\Big[c^{n+3-p-i_1}_{3,n+3-p-i_1+3-1-i_2}$$

$$\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2})\Big\}\mu^{n+2,p}_{i_1}+f^{n+3-p-i_1}_{3,n+3-p-i_1+3-1-i_2}\Big\{(j_1,j_2,\cdots,j_{s_1}),$$

$$(k_1,k_2,\cdots,k_{s_2})\Big\}\nu^{n+2,p}_{i_1}\Big]+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq h(k'_{s_2-1},2,1)}}\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),$$

$$(k_1,k_2,\cdots,k'_{s_2-1})\Big\}\omega^{k'_{s_2-1},1,(1)}_{2,k'_{s_2-1}+1-i_2}\Big\{(0,0),\ (k_{s_2-1},k_{s_2})\Big\}+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq h(k'_{s_2},2,1)}}\omega^{n+2,p}_{2,n+2+2-p-i_1}$$

$$\Big\{(j_1,j_2,\cdots,j_{s_1-1}),\ (k_1,k_2,\cdots,k'_{s_2})\Big\}\omega^{k'_{s_2},1,(1)}_{2,k'_{s_2}+1-i_2}\Big\{(j_{s_1},0),\ (k_{s_2},0)\Big\}+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq h(k'_{s_2+1},2,1)}}$$

$$\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1-2}),(k_1,k_2,\cdots,k'_{s_2+1})\,\omega^{k'_{s_2+1},1,(1)}_{2,k'_{s_2+1}+1-i_2}\Big\{(j_{s_1-1},j_{s_1}),(0,0)\Big\}$$

$$+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq h(j'_{s_1-1},2,1)}}\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1,j_2,\cdots,j'_{s_1-1}),\ (k_1,k_2,\cdots,k_{s_2})\Big\}\omega^{j'_{s_1-1},1}_{2,j'_{s_1-1}+1-i_2}$$

$$\Big\{(j_{s_1-1},j_{s_1}),\ (0,0)\Big\}+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq h(j'_{s_1},2,1)}}\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1,j_2,\cdots,j'_{s_1}),(k_1,k_2,\cdots k_{s_2-1})\Big\}$$

$$\omega^{j'_{s_1},1}_{2,j'_{s_1}+1-i_2}\Big\{(j_{s_1},0),\ (k_{s_2},0)\Big\}+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq h(j'_{s_1+1},2,1)}}\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1,j_2,\cdots,j'_{s_1+1}),$$

$$(k_1,k_2,\cdots,k_{s_2-2})\Big\}\omega^{j'_{s_1+1},1}_{2,j'_{s_1+1}+1-i_2}\Big\{(0,0),\ (k_{s_2-1},k_{s_2})\Big\}+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq h(r,2,1)}}\Big[\pi^{n+2,p}_{2,n+2+2-p-i_1}$$

$$\Big\{(j_1);\ r\Big\}c^{r}_{2,r+1-i_2}\Big\{(j_2,\cdots,j_{s_1}),\ (k_1,k_2,\cdots,k_{s_2})\Big\}+\pi^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(k_1);\ r\Big\}$$

$$c^{r}_{2,r+1-i_2}\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_2, \cdots, k_{s_2})\right\}+\delta^{n+2,p}_{2,n+2+2-p-i_1}\left\{(j_1)\,;\,r\right\}f^{r}_{2,r+1-i_2}$$

$$\left\{(j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\right\}+\delta^{n+2,p}_{2,n+2+2-p-i_1}\left\{(k_1)\,;\,r\right\}f^{r}_{2,r+1-i_2}\left\{(j_1, j_2, \cdots, j_{s_1}),\right.$$

$$\left.(k_2, \cdots, k_{s_2})\right\}\Big]$$

$$+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geqq 1\\ 4\leqq i_2\leqq 6}}\Big[\omega^{n+2,p}_{4,n+2+4-p-i_1}\left\{(j_1, j_2, \cdots, j_{s_1}, i_2+2),\ (k_1, k_2, \cdots, k_{s_2})\right\}(\mu^{i_2+2,1}_{i_2} D_2$$

$$+\nu^{i_2+2,1}_{i_2} E_2)+\omega^{n+2,p}_{4,n+2+4-p-i_1}\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2}, i_2+2)\right\}(\mu^{i_2+2,1,(1)}_{i_2} D_2$$

$$+\nu^{i_2+2,1,(1)}_{i_2} E_2)\Big]+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geqq 1\\ 4\leqq i_2\leqq 6}}\Big[\pi^{n+2,p}_{4,n+2+4-p-i_1}\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\,;\,i_2+2\right\}$$

$$(a^{i_2+2}_{i_2}D_2+b^{i_2+2}_{i_2}E_2)+\delta^{n+2,p}_{4,n+2+4-p-i_1}\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots k_{s_2})\,;\,i_2+2\right\}$$

$$(d^{i_2+2}_{i_2} D_2+e^{i_2+2}_{i_2} E_2)\Big]+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geqq 1\\ 1\leqq i_2\leqq 3}}\Big[\pi^{n+2,p}_{4,n+2+4-p-i_1}\left\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2});\, i_2+2\right\}$$

$$D_{i_2+2}+\delta^{n+2,p}_{4,n+2+4-p-i_1}\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2});\, i_2+2\right\}E_{i_2+2}\Big],$$

für $q\geqq 4$

$$\omega^{n+2,p+1}_{q,n+2+q-(p+1)-i}=\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geqq 1\\ i_2\leqq h(k'_{s_2-r_2+1},{}^{2,1})}}\ \sum_{r_1+r_2=2}\Big[\omega^{n+2,p}_{q-1,n+2+(q-1)-p-i_1}\left\{(j_1, j_2, \cdots, j_{s_1-r_1}),\right.$$

$$\left.(k_1,\ k_2,\ \cdots,\ k_{s_2-r_2},\ k'_{s_2-r_2+1})\right\}\omega^{k'_{s_2-r_2+1},1,(1)}_{2,k'_{s_2-r_2+1}+1-i_2}\left\{(j_{s_1-r_1+1},\ \cdots,\ j_{s_1}),\right.$$

$$\left.(k_{s_2-r_2+1},\ \cdots,\ k_{s_2})\right\}+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geqq 1\\ i_2\leqq h(j'_{s_1-r_1+1},{}^{2,1})}}\ \sum_{r_1+r_2=2}\omega^{n+2,p}_{q-1,n+2+(q-1)-p-i_1}\left\{(j_1,\ j_2,\ \cdots,\ j_{s_1-r_1},\right.$$

$$\left.j'_{s_1-r_1+1}),\ (k_1, k_2, \cdots,\ k_{s_2-r_2})\right\}\omega^{j'_{s_1-r_1+1},1}_{2,j'_{s_1-r_1+1}+1-i_2}\left\{(j_{s_1-r_1+1}, \cdots, j_{s_1}), (k_{s_2-r_2+1}, \cdots, k_{s_2})\right\}$$

$$+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geqq 1\\ i_2\leqq h(k'_{s_2-r_2+1},{}^{3,1})}}\ \sum_{r_1+r_2=3}\omega^{n+2,p}_{q-2,n+2+(q-2)-p-i_1}\left\{(j_1,\ j_2, \cdots,\ j_{s_1-r_1}),\ (k_1,\ k_2, \cdots,\ k_{s_2-r_2},\right.$$

$$k'_{s_2-r_2+1})\Big\}\omega^{k'_{s_2-r_2+1},1,(1)}_{3,k'_{s_2-r_2+1}+2-i_2}\Big\{(j_{s_1-r_1+1},\cdots,j_{s_1}),(k_{s_2-r_2+1},\cdots,k_{s_2})\Big\}+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq h(j'_{s_1-r_1+1},3,1)}}$$

$$\sum_{r_1+r_2=3}\omega^{n+2,p}_{q-2,n+2+(q-2)-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1-r_1},j'_{s_1-r_1+1}),(k_1,k_2,\cdots,k_{s_2-r_2})\Big\}$$

$$\omega^{j'_{s_1-r_1+1},1}_{3,j'_{s_1-r_1+1}+2-i_2}\Big\{(j_{s_1-r_1+1},\cdots,j_{s_1}),(k_{s_2-r_2+1},\cdots,k_{s_2})\Big\}+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geq 1\\ 4\leq i_2\leq 6}}\Big[$$

$$\omega^{n+2,p}_{q+1,n+2+(q+1)-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2},i_2+2)\Big\}(\mu^{i_2+2,1,(1)}_{i_2}$$

$$D_2+\nu^{i_2+2,1,(1)}_{i_2}E_2)+\omega^{n+2,p}_{q+1,n+2+(q+1)-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1},i_2+2),(k_1,k_2,\cdots$$

$$\cdots,k_{s_2})\Big\}(\mu^{i_2+2,1}_{i_2}D_2+\iota^{i_2+2,1}_{i_2}E_2)\Big]+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq h(r,2,1)}}\sum_{r_1+r_2=2}\Big[\pi^{n+2,p}_{q-1,n+2+(q-1)-p-i_1}$$

$$\Big\{(j_1,j_2,\cdots,j_{s_1-r_1}),(k_1,k_2,\cdots,k_{s_2-r_2});\ r\Big\}c^r_{2,r+1-i_2}\Big\{(j_{s_1-r_1+1},\cdots,j_{s_1}),$$

$$(k_{s_2-r_2+1},\cdots,k_{s_2})\Big\}+\delta^{n+2,p}_{q-1,n+2+(q-1)-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1-r_1}),(k_1,k_2,\cdots,$$

$$k_{s_2-r_2});\ r\Big\}f^r_{2,r+1-i_2}\Big\{(j_{s_1-r_1+1},\cdots,j_{s_1}),(k_{s_2-r_2+1},\cdots,k_{s_2})\Big\}\Big]+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ i_2\leq h(r,3,1)}}\sum_{r_1+r_2=3}$$

$$\Big[\pi^{n+2,p}_{q-2,n+2+(q-2)-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1-r_1}),(k_1,k_2,\cdots,k_{s_2-r_2});\ r\Big\}c^r_{3,r+2-i_2}$$

$$\Big\{(j_{s_1-r_1+1},\cdots,j_{s_1}),(k_{s_2-r_2+1},\cdots,k_{s_2})\Big\}+\delta^{n+2,p}_{q-2,n+2+(q-2)-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1-r_1}),$$

$$(k_1,k_2,\cdots,k_{s_2-r_2}),r\Big\}f^r_{3,r+2-i_2}\Big\{(j_{s_1-r_1+1},\cdots,j_{s_1}),(k_{s_2-r_2+1},\cdots,k_{s_2})\Big\}\Big]$$

$$+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geq 1\\ 4\leq i_2\leq 6}}\Big[\pi^{n+2,p}_{q+1,n+2+(q+1)-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2});\ i_2+2\Big\}$$

$$(a^{i_2+2}_{i_2}D_2+b^{i_2+2}_{i_2}E_2)+\delta^{n+2,p}_{q+1,n+2+(q+1)-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2});\ i_2+2\Big\}$$

$$(d^{i_2+2}_{i_2}D_2+e^{i_2+2}_{i_2}E_2)\Big]+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geq 1\\ 1\leq i_2\leq 3}}\Big[\pi^{n+2,p}_{q+1,n+2+(q+1)-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2});$$

$$i_2+2\Big\}D_{i_2+2}+\delta^{n+2,p}_{q+1,n+2+(q+1)-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2\cdots,k_{s_2});\ i_2+2\Big\}E_{i_2+2}\Big],$$

$$\pi^{n+2,p+1}_{2,n+2+2-(p+1)-i}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r\Big\} = \sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ 6-r\leq i_2\leq 6}} \Big[\omega^{n+2,p}_{2,n+2+2-p-i_1}$$

$$\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}, r+i_2)\Big\}\mu^{r+i_2,1,(1)}_{i_2} + \omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1},$$

$$r+i_2), (k_1, k_2, \cdots, k_{s_2})\Big\}\mu^{r+i_2,1}_{i_2}\Big] + \sum_{\substack{i_1+i_2=i+1,\\ i_1,i_2\geq 1\\ 6-r\leq i_2\leq 6}} \Big[\pi^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1}),$$

$$(k_1, k_2, \cdots, k_{s_2}); r+i_2\Big\}a^{r+i_2}_{i_2} + \delta^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r+i_2\Big\}$$

$$d^{r+i_2}_{i_2}\Big] + \sum_{\substack{i_1=i+1\\ r=3,4,5}} \Big[\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}, r)\Big\}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)$$

$$+\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1}, r), (k_1, k_2, \cdots, k_{s_2})\Big\}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\Big)\Big],$$

$$\delta^{n+2,p+1}_{2,n+2+2-(p+1)-i}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r\Big\} = \sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ 6-r\leq i_2\leq 6}} \Big[\omega^{n+2,p}_{2,n+2+2-p-i_1}$$

$$\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}, r+i_2)\Big\}\nu^{r+i_2,1,(1)}_{i_2} + \omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1},$$

$$r+i_2), (k_1, k_2, \cdots, k_{s_2})\Big\}\nu^{r+i_2,1}_{i_2}\Big] + \sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ 6-r\leq i_2\leq 6}} \Big[\pi^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1}),$$

$$(k_1, k_2, \cdots, k_{s_2}); r+i_2\Big\}b^{r+i_2}_{i_2} + \delta^{n+2,p}_{3,n+2+2-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2});$$

$$r+i_2\Big\}e^{r+i_2}_{i_2}\Big] + \sum_{\substack{i_1=i+1\\ r=3,4,5}} \Big[\omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}, r)\Big\}$$

$$\Big(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big) + \omega^{n+2,p}_{2,n+2+2-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1}, r), (k_1, k_2, \cdots, k_{s_2})\Big\}$$

$$\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\Big)\Big],$$

$$\pi^{n+2,p+1}_{3,n+2+3-(p+1)-i}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r\Big\} = \sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geq 1\\ 6-r\leq i_2\leq 6}} \Big[\omega^{n+2,p}_{3,n+2+3-p-i_1}$$

$$\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}, r+i_2)\Big\}\mu^{r+i_2,1,(1)}_{i_2} + \omega^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1},$$

$$r+i_2),(k_1,k_2,\cdots,k_{s_2})\Big\}\mu_{i_2}^{r+i_2,1}\Big]+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geqq 1\\ 6-r\leqq i_2\leqq 6}}\Big[\pi^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),$$

$$(k_1,k_2,\cdots,k_{s_2});r+i_2\Big\}a^{r+i_2}_{i_2}+\delta^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2});r+i_2\Big\}$$

$$d^{r+i_2}_{i_2}\Big]+\sum_{\substack{i_1=i+1\\ r=3,4,5}}\Big[\omega^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2},r)\Big\}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)$$

$$+\omega^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1},r),(k_1,k_2,\cdots,k_{s_2})\Big\}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\Big)\Big],$$

$$\delta^{n+2,p+1}_{3,n+2+3-(p+1)-i}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2});r\Big\}=\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geqq 1\\ 6-r\leqq i_2\leqq 6}}\Big[\omega^{n+2,p}_{3,n+2+3-p-i_1}$$

$$\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2},r+i_2)\Big\}\nu^{r+i_2,1,(1)}_{i_2}+\omega^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1},$$

$$r+i_2),(k_1,k_2,\cdots,k_{s_2})\Big\}\nu^{r+i_2,1}_{i_2}\Big]+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geqq 1\\ 6-r\leqq i_2\leqq 6}}\Big[\pi^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),$$

$$(k_1,k_2\cdots,k_{s_2});r+i_2\Big\}b^{r+i_2}_{i_2}+\delta^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2},);r+i_2\Big\}$$

$$e^{r+i_2}_{i_2}\Big]+\sum_{\substack{i_1=i+1\\ r=3,4,5}}\Big[\omega^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2},r)\Big\}\Big(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\Big)$$

$$+\omega^{n+2,p}_{3,n+2+3-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1},r),(k_1,k_2\cdots,k_{s_2})\Big\}\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\Big),$$

im allgemeinen für $q\geqq 4$

$$\pi^{n+2,p+1}_{q,n+2+q-(p+1)-i}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2});r\Big\}=\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geqq 1\\ 6-r\leqq i_2\leqq 6}}\Big[\omega^{n+2,p}_{q,n+2+q-p-i_1}$$

$$\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2},r+i_2)\Big\}\mu^{r+i_2,1,(1)}_{i_2}+\omega^{n+2,p}_{q,n+2+q-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1},$$

$$r+i_2),(k_1,k_2,\cdots,k_{s_2})\Big\}\mu^{r+i_2,1}_{i_2}\Big]+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geqq 1\\ 6-r\leqq i_2\leqq 6}}\Big[\pi^{n+2,p}_{q,n+2+q-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),$$

$$(k_1,k_2,\cdots,k_{s_2});r+i_2\Big\}a^{r+i_2}_{i_2}+\delta^{n+2,p}_{q,n+2+q-(p+1)-i}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2});$$

$$r+i_2\Big\}d^{r+i_2}_{i_2}\Big]+\sum_{\substack{i_1=i+1\\ r=3,4,5}}\Big[\omega^{n+2,p}_{q,n+2+q-p-i_1}\Big\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2},r)\Big\}$$

$$\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\right)+\omega^{n+2,p}_{q,n+2+q-p-i_1}\left\{(j_1, j_2, \cdots, j_{s_1}, r), (k_1, k_2, \cdots, k_{s_2})\right\}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\right)\Bigg],$$

$$\delta^{n+2,p+1}_{q,n+2+q-(p+1)-i}\left\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r\right\}=\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geqq 1\\ 6-r\leqq i_2\leqq 6}}\Bigg[\omega^{n+2,p}_{q,n+2+q-p-i_1}$$

$$\left\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}, r+i_2)\right\}\nu^{r+i_2,1,(1)}_{i_2}+\omega^{n+2,p}_{q,n+2+q-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1},$$

$$r+i_2), (k_1, k_2, \cdots, k_{s_2})\Big\}\nu^{r+i_2,1}_{i_2}\Bigg]+\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geqq 1\\ 6-r\leqq i_2\leqq 6}}\Bigg[\pi^{n+2,p}_{q,n+2+q-p-i_1}\Big\{(j_1, j_2, \cdots, j_{s_1}),$$

$$(k_1, k_2, \cdots, k_{s_2}); r+i_2\Big\}b^{r+i_2}_{i_2}+\delta^{n+2,p}_{q,n+2+q-p-i_1}\left\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r+i_2\right\}$$

$$e^{r+i_2}_{i_2}\Bigg]+\sum_{\substack{i_1=i+1\\ r=3,4,5}}\Bigg[\omega^{n+2,p}_{q,n+2+q-p-i_1}\left\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}, r)\right\}\left(-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\right)$$

$$+\omega^{n+2,p}_{q,n+2+q-p-i_1}\left\{(j_1, j_2, \cdots, j_{s_1}, r), (k_1, k_2, \cdots, k_{s_2})\right\}\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\right)\Bigg]. \tag{28}$$

Wir setzen nun $n-9=6s+r_1$, $0\leqq r_1<6$ (s, r_1: ganze Zahl)

$n-7=5t+r_2$, $0\leqq r_2<5$ (t, r_2: ganze Zahl)

Die α_{n+2} lässt sich dann folgendermassen schreiben.

$$\alpha_{n+2}=\sum_{i=1}^{f(p)}\mu_i^{n+2,p}D_{n+3-p-i}+\sum_{i=1}^{f(p)}\nu_i^{n+2,p}E_{n+3-p-i}+\sum_{q=2}^{g(p)}\sum_{i=1}^{h(n+2,q,p)}\sum_{s_1+s_2=q}$$

$$\omega^{n+2,p}_{q,n+2+q-p-i}\left\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2})\right\}\prod_{l=1}^{s_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)$$

$$\prod_{m=1}^{s_2}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\right)+\sum_{q=2}^{g(p)}\sum_{i=1}^{h(n+2,q,p)}\sum_{s_1+s_2=q-1}\Bigg[\pi^{n+2,p}_{q,n+2+q-p-i}$$

$$\left\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r\right\}D_r+\delta^{n+2,p}_{q,n+2+q-p-i}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots,$$

$$k_{s_2}); r\Big\}E_r\Bigg]\prod_{l=1}^{s_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\right)\prod_{m=1}^{s_2}\Bigg(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}$$

$$-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Bigg).$$

(wenn $n-9=6s+r_1$, $r_1\neq 0$, $p\leqq s+1$ oder $r_1=0$, $p\leqq s$ ist)

$$\alpha_{n+2}=\sum_{i=5(s+1)+r_1}^{5(s+2)+1}\left\{\mu_i^{n+2,s+2}D_{n+3-(s+2)-i}+\nu_i^{n+2,s+2}E_{n+3-(s+2)-i}\right\}+\sum_{t_1=3}^{t}\sum_{i=5(s+1)+r_1-t_1+2}^{5(s+1)+r_1-t_1+5}$$

$$\left\{\mu_i^{n+2,s+t_1}D_{n+3-(s+t_1)-i}+\nu_i^{n+2,s+t_1}E_{n+3-(s+t_1)-i}\right\}+\sum_{i=1}^{f(p)}\mu_i^{n+2,p}D_{n+3-p-i}+\sum_{i=1}^{f(p)}\nu_i^{n+2,p}$$

$$E_{n+3-p-i}+\sum_{q=2}^{g(p)}\sum_{i=1}^{h(n+2,q,p)}\sum_{s_1+s_2=q}\omega_{q,n+2+q-p-i}^{n+2,p}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2})\Big\}\prod_{l=1}^{s_1}$$

$$\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big)$$

$$+\sum_{q=2}^{g(p)}\sum_{i=1}^{h(n+2,q,p)}\sum_{s_1+s_2=q-1}\Big[\pi_{q,n+2+q-p-i}^{n+2,p}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r\Big\}D_r$$

$$+\delta_{q,n+2+q-p-i}^{n+2,p}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r\Big\}E_r\Big]\prod_{l=1}^{s_1}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}$$

$$-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big).$$

(wenn $n-9=6s+r_1$, $r_1\geqq 3$, $p=s+t$, $t\geqq 2$ ist)

$$\alpha_{n+2}=\sum_{i=5(s+1)+r_1}^{5(s+1)+r_1+3}\Big\{\mu_i^{n+2,s+2}D_{n+3-(s+2)-i}+\nu_i^{n+2,s+2}E_{n+3-(s+2)-i}\Big\}+\sum_{t_1=3}^{t}$$

$$\sum_{i=5(s+1)+r_1-t_1+2}^{5(s+1)+r_1-t_1+5}\Big\{\mu_i^{n+2,s+t_1}D_{n+3-(s+t_1)-i}+\nu_i^{n+2,s+t_1}E_{n+3-(s+t_1)-i}\Big\}+\sum_{i=1}^{f(p)}\mu_i^{n+2,p}D_{n+3-p-i}$$

$$+\sum_{i=1}^{f(p)}\nu_i^{n+2,p}E_{n+3-p-i}+\sum_{q=2}^{g(p)}\sum_{i=1}^{h(n+2,q,p)}\sum_{s_1+s_2=q}\omega_{q,n+2+q-p-i}^{n+2,p}\Big\{(j_1, j_2, \cdots, j_{s_1}),$$

$$(k_1, k_2, \cdots, k_{s_2})\Big\}\prod_{l=1}^{s_1}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-$$

$$\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big)+\sum_{q=2}^{g(p)}\sum_{i=1}^{h(n+2,q,p)}\sum_{s_1+s_2=q-1}\Big[\pi_{q,n+2+q-p-i}^{n+2,p}\Big\{(j_1, j_2, \cdots, j_{s_1}),$$

$$(k_1, k_2, \cdots, k_{s_2}); r\Big\}D_r+\delta_{q,n+2+q-p-i}^{n+2,p}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r\Big\}E_r\Big]\prod_{l=1}^{s_1}$$

$$\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big).$$

(wenn $n-9=6s+r_1$, $0<r_1\leqq 2$, $p=s+t$, $t\geqq 2$ ist)

$$\alpha_{n+2}=\sum_{i=5s+6}\Big\{\mu_i^{n+2,s+1}D_{n+3-(s+1)-i}+\nu_i^{n+2,s+1}E_{n+3-(s+1)-i}\Big\}+\sum_{t_1=2}^{t}\sum_{i=5(s+1)+2-t_1}^{5(s+1)+5-t_1}$$

$$\Big\{\mu_i^{n+2,s+t_1}D_{n+3-(s+t_1)-i}+\nu_i^{n+2,s+t_1}E_{n+3-(s+t_1)-i}\Big\}+\sum_{i=1}^{f(p)}\mu_i^{n+2,p}D_{n+3-p-i}$$

$$+\sum_{i=1}^{f(p)}\nu_i^{n+2,p}E_{n+3-p-i}+\sum_{q=2}^{g(p)}\sum_{i=1}^{h(n+2,q,p)}\sum_{s_1+s_2=q}\omega_{q,n+2+q-p-i}^{n+3,p}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2})\Big\}$$

$$\prod_{l=1}^{s_1}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big)$$

$$+\sum_{q=2}^{g(p)}\sum_{i=1}^{h(n+2,q,p)}\sum_{s_1+s_2=q-1}\Big[\pi_{q,n+2+q-p-i}^{n+2,p}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}); r\Big\}D_r$$

$$+\delta^{n+2,p}_{q,n+2+q-p-t}\Big\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2}) ; r\Big\}E_r\Bigg]\prod_{l=1}^{s_1}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}$$

$$-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big). \tag{29}$$

(wenn $n-9=6s+r_1$, $r_1=0$, $p=s+t$, $t\geqq 1$ ist)

Es ist dabei

$$f(p)=\begin{cases} 5p+1 & (\text{wenn } n-9=6s+r_1,\ r_1\neq 0,\ p\leqq s+1 \text{ oder} \\ & r_1=0,\ p\leqq s \text{ ist}) \\ 5(s+1)+r_1-t+1 & (\text{wenn } n-9=6s+r_1,\ r_1\neq 0,\ p=s+t,\ t\geqq 2 \\ & \text{oder } r_1=0,\ p=s+t,\ t\geqq 1 \text{ ist}) \end{cases}$$

$$g(p)=\begin{cases} 2p+1, & (\text{wenn } n-7=5t+r_2,\ p\leqq t+1 \text{ ist}) \\ 2(t+1)+2, & (\text{wenn } n-7=5t+r_2,\ 2\leqq r_2<5,\ t+2\leqq p\leqq t \\ & +r_2 \text{ ist}) \\ 2(t+1)+1-s, & (\text{wenn } n-7=5t+r_2,\ 2\leqq r_2<5,\ t+r_2+2s+ \\ & 1\leqq p\leqq t+r_2+2s+2,\ s=0, 1, \cdots, 2t+1 \text{ ist}) \\ 2(t+1)+1 & (\text{wenn } n-7=5t+r_2,\ 0\leqq r_2<2,\ t+2\leqq p\leqq t \\ & +2+r_2 \text{ ist}) \\ 2(t+1)-s & (\text{wenn } n-7=5t+r_2,\ 0\leqq r_2<2,\ t+2+r_2+ \\ & 2s+1\leqq p\leqq t+2+r_2+2s+2,\ s=0, 1, \cdots, 2t \\ & \text{ist}). \end{cases} \tag{30}$$

Die $\mu_t^{n+2\ p}\nu_t^{n+2,p}\omega^{n+2,p}_{q,n+2+q-p-t}$, $\pi^{n+2,p}_{q,n+2+q-p-t}$ und $\delta^{n+2,p}_{q,n+2+q-p-t}$ werden durch (28) gegeben. Durch die mathematische Induktion können wir die obige Formel gewinnen.

Die ähnliche Formel gilt für β_{n+2}.

$$\beta_{n+2}=\sum_{t=1}^{f(p)}\mu_t^{n+2,p,(1)}D_{n+3-p-t}+\sum_{t=1}^{f(p)}\nu_t^{n+2,p,(1)}E_{n+3-p-t}+\sum_{q=2}^{g(p)}\sum_{t=1}^{h(n+2,q,p)}\sum_{s_1+s_2=q}$$

$$\omega^{n+2,p,(1)}_{q,n+2+q-p-t}\Big\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\Big\}\prod_{l=1}^{s_1}\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}D_{j_l}$$

$$-\frac{A_3}{(\sqrt{A_1^2+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}E_{k_m}\Big)+\sum_{q=2}^{g(p)}\sum_{t=1}^{h(n+2,q,p)}\sum_{s_1+s_2=q-1}$$

$$\Bigg[\pi^{n+2,p,(1)}_{q,n+2+q-p-t}\Big\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2}); r\Big\}D_r+\delta^{n+2,p,(1)}_{q,n+2+q-p-t}\Big\{(j_1, j_2, \cdots, j_{s_1}),$$

$$(k_1, k_2, \cdots, k_{s_2});\ r\Big\}E_r\Bigg]\prod_{l=1}^{s_1}\Big(-\frac{A_1}{2\sqrt{A_1^3+A_2^2}}D_{j_l}-\frac{A_2}{(\sqrt{A_1^3+A_2^2})^2}E_{j_l}\Big)\prod_{m=1}^{s_2}$$

$\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}} D_{k_m} - \frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} E_{k_m}\right).$ (wenn $n-9=6s+r_1$, $r_1 \neq 0$, $p \leqq s+1$ oder $r_1=0$, $p \leqq s$ ist)

$$\beta_{n+2} = \sum_{i=(5s+1)+r_1}^{5(s+2)+1} \left\{\mu_i^{n+2,s+2,(1)} D_{n+3-(s+2)-i} + \nu_i^{n+2,s+2,(1)} E_{n+3-(s+2)-i}\right\}$$

$$+\sum_{t_1=3}^{t} \sum_{i=5(s+1)+r_1-t_1+2}^{5(s+1)+r_1-t_1+5} \left\{\mu_i^{n+2,s+t_1,(1)} D_{n+3-(s+t_1)-i} + \nu_i^{n+2,s+t_1,(1)} E_{n+3-(s+t_1)-i}\right\}$$

$$+\sum_{i=1}^{f(p)} \mu_i^{n+2,p,(1)} D_{n+3-p-i} + \sum_{i=1}^{f(p)} \nu_i^{n+2,p,(1)} E_{n+3-p-i} + \sum_{q=2}^{g(p)} \sum_{i=1}^{h(n+2,q,p)} \sum_{s_1+s_2=q} \omega_{q,n+2+q-p-i}^{n+2,p}$$

$$\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\right\} \prod_{l=1}^{s_1} \left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}} D_{j_l} - \frac{A_2}{(\sqrt{A_1^2+A_2^2})^2} E_{j_l}\right) \prod_{m=1}^{s_2}$$

$$\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}} D_{k_m} - \frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} E_{k_m}\right) + \sum_{q=2}^{g(p)} \sum_{i=1}^{h(n+2,q,p)} \sum_{s_1+s_2=q-1} \Big[\pi_{q,n+2+q-p-i}^{n+2,p,(1)}$$

$$\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2});\ r\right\} D_r + \delta_{q,n+2+q-p-i}^{n+2,p,(1)} \left\{(j_1, j_2, \cdots, j_{s_1}),\right.$$

$$\left.(k_1, k_2, \cdots, k_{s_2});\ r\right\} E_r \Big] \prod_{l=1}^{s_1} \left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}} D_{j_l} - \frac{A_2}{(\sqrt{A_1^2+A_2^2})^2} E_{j_l}\right) \prod_{m=1}^{s_2} \left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\right.$$

$\left. D_{k_m} - \frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} E_{k_m}\right).$ (wenn $n-9=6\,s+r_1$, $r_1 \geqq 3$, $p=s+t$, $t \geqq 2$ ist)

$$\beta_{n+2} = \sum_{i=5(s+1)+r_1}^{5(s+1)+r_1+3} \left\{\mu_i^{n+2,s+2,(1)} D_{n+3-(s+2)-i} + \nu_i^{n+2,s+2,(1)} E_{n+3-(s+2)-i}\right\}$$

$$+\sum_{t_1=3}^{t} \sum_{i=5(s+1)+r_1-t_1+2}^{5(s+1)+r_1-t_1+5} \left\{\mu_i^{n+2,s+t_1,(1)} D_{n+3-(s+t_1)-i} + \nu_i^{n+2,s+t_1,(1)} E_{n+3-(s+t_1)-i}\right\}$$

$$+\sum_{i=1}^{f(p)} \mu_i^{n+2,p,(1)} D_{n+3-p-i} + \sum_{i=1}^{f(p)} \nu_i^{n+2,p,(1)} E_{n+3-p-i} + \sum_{q=2}^{g(p)} \sum_{i=1}^{h(n+2,q,p)} \sum_{s_1+s_2=q} \omega_{q,n+2+q-p-i}^{n+2,p,(1)}$$

$$\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\right\} \prod_{l=1}^{s_1} \left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}} D_{j_l} - \frac{A_2}{(\sqrt{A_1^2+A_2^2})^2} E_{j_l}\right) \prod_{m=1}^{s_2}$$

$$\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}} D_{k_m} - \frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} E_{k_m}\right) + \sum_{q=2}^{g(p)} \sum_{i=1}^{h(n+2,q,p)} \sum_{s_1+s_2=q-1} \Big[\pi_{q,n+2+q-p-i}^{n+2,p,(1)}$$

$$\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2});\ r\right\} D_r + \delta_{q,n+2+q-p-i}^{n+2,p,(1)} \left\{(j_1, j_2, \cdots, j_{s_1}), (k_1, k_2, \cdots, k_{s_2})\right.$$

$$\left.;\ r\right\} E_r \Big] \prod_{l=1}^{s_1} \left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}} D_{j_l} - \frac{A_2}{(\sqrt{A_1^2+A_2^2})^2} E_{j_l}\right) \prod_{m=1}^{s_2} \left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}} D_{k_m}\right.$$

$\left. - \frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} E_{k_m}\right).$ (wenn $n-9=6s+r_1$, $0 \leqq r_1 < 2$, $p=s+t$, $t \geqq 2$ ist)

$$\beta_{n+2} = \sum_{i=5s+6} \left\{\mu_i^{n+2,s+1,(1)} D_{n+3-(s+1)-i} + \nu_i^{n+2,s+1,(1)} E_{n+3-(s+1)-i}\right\}$$

$$+\sum_{t_1=2}^{t}\sum_{i=5(s+1)+2-t_1}^{5(s+1)+5-t_1}\left\{\mu_i^{n+2,s+t_1,(1)} D_{n+3-(s+t_1)-i}+\nu_i^{n+2,s+t_1,(1)} E_{n+3-(s+t_1)-i}\right\}$$

$$+\sum_{i=1}^{f(p)}\mu_i^{n+2,p,(1)} D_{n+3-p-i}+\sum_{i=1}^{f(p)}\nu_i^{n+2,p,(1)} E_{n+3-p-i}+\sum_{q=2}^{g(p)}\sum_{i=1}^{h(n+2,q,p)}\sum_{s_1+s_2=q}$$

$$\omega_{q,n+2+q-p-i}^{n+2,p,(1)}\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2})\right\}\prod_{l=1}^{s_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}} D_{j_l}\right.$$

$$\left.-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2} E_{j_l}\right)\prod_{m=1}^{s_2}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}} D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} E_{k_m}\right)+\sum_{q=2}^{g(p)}\sum_{i=1}^{h(n+2,q,p)}$$

$$\sum_{s_1+s_2=q-1}\left[\pi_{q,n+2+q-p-i}^{n+2,p,(1)}\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2});\ r\right\} D_r+\delta_{q,n+2+q-p-i}^{n+2,p,(1)}\right.$$

$$\left.\left\{(j_1, j_2, \cdots, j_{s_1}),\ (k_1, k_2, \cdots, k_{s_2});\ r\right\} E_r\right]\prod_{l=1}^{s_1}\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}} D_{j_l}-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2} E_{j_l}\right)$$

$$\prod_{m=1}^{s_2}\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}} D_{k_m}-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2} E_{k_m}\right). \quad (\text{wenn } n-9=6s+r_1,\ r_1=0,\ p=s+t,\ t\geqq 1 \text{ ist}) \tag{31}$$

Die analoge Formel wie in (28) gilt für $\mu_i^{n+2,p,(1)}$ usw. Z. b.

$$\mu_i^{n+2,p+1,(1)}=\sum_{\substack{i_1+i_2=i+1\\ i_1,i_2\geqq 1\\ i_2\leqq 6}}\left(\mu_{i_1}^{n+2,p,(1)} a_{i_2}^{n+3-p-i_1}+\nu_{i_1}^{n+2,p,(1)} d_{i_2}^{n+3-p-i_1}\right)$$

$$+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geqq 1\\ i_2\leqq 6}}\left[\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\right)\left(\mu_{i_2}^{i_2+2,1,(1)} D_2+\nu_{i_2}^{i_2+2,1,(1)} E_2\right)\omega_{2,n+2+2-p-i_1}^{n+2,p,(1)}\right.$$

$$\left\{(j_1, 0),\ (i_2+2, 0)\right\}+\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\right)\left(\mu_{i_2}^{i_2+2,1,(1)} D_2+\nu_{i_2}^{i_2+2,1,(1)} E_2\right)\omega_{2,n+2+2-p-i_1}^{n+2,p,(1)}$$

$$\left\{(0, 0),\ (k_1, i_2+2)\right\}+\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\right)\left(\mu_{i_2}^{i_2+2,1} D_2+\nu_{i_2}^{i_2+2,1} E_2\right)\omega_{2,n+2+2-p-i_1}^{n+2,p,(1)}$$

$$\left\{(i_2+2, 0),\ (k_1, 0)\right\}+\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\right)\left(\mu_{i_2}^{i_2+2,1} D_2+\nu_{i_2}^{i_2+2,1} E_2\right)\omega_{2,n+2+2-p-i_1}^{n+2,p,(1)}$$

$$\left.\left\{(j_1, i_2+2),\ (0, 0)\right\}\right]+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geqq 1\\ i_2\leqq 6}}\left[\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\right)\left\{\left(a_{i_2}^{i_2+2} D_2+b_{i_2}^{i_2+2} E_2\right)\pi_{2,n+2+2-p-i_1}^{n+2,p,(1)}\right.\right.$$

$$\left\{(j_1);\ i_2+2\right\}+\left(d_{i_2}^{i_2+2} D_2+e_{i_2}^{i_2+2} E_2\right)\delta_{2,n+2+2-p-i_1}^{n+2,p,(1)}\left\{(j_1);\ i_2+2\right\}+\left(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\right)$$

$$\left\{\left(a_{i_2}^{i_2+2} D_2+b_{i_2}^{i_2+2} E_2\right)\pi_{2,n+2+2-p-i_1}^{n+2,p,(1)}\left\{(k_1);\ i_2+2\right\}+\left(d_{i_2}^{i_2+2} D_2+e_{i_2}^{i_2+2} E_2\right)\delta_{2,n+2+2-p-i_1}^{n+2,p,(1)}\right.$$

$$\left.\left.\left\{(k_1);\ i_2+2\right\}\right\}\right]+\sum_{\substack{i_1+i_2=i\\ i_1,i_2\geqq 1\\ 1\leqq i_2\leqq 3}}\left[\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\right)\left\{\pi_{2,n+2+2-p-i_1}^{n+2,p,(1)}\left\{(j_1);\ i_2+2\right\}D_{i_2+2}\right.\right.$$

$$+\delta^{n+2,p,(1)}_{2,n+2+2-p-i_1}\Big\{(j_1);\, i_2+2\Big\}E_{i_2+2}\Big\}+\Big(\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\Big)\Big\{\pi^{n+2,p,(1)}_{2,n+2+2-p-i_1}\Big\{(k_1);\, i_2+2\Big\}D_{i_2+2}$$

$$+\delta^{n+2,p,(1)}_{2,n+2+2-p-i_1}\Big\{(k_1);\, i_3+2\Big\}E_{i_2+2}\Big\}\Big]$$

3. Schritt: Aus (28) bekommen wir für $p\geqq 1$

$$\mu_1^{n+2,p+1}=\mu_1^{n+2,p}\, a_1^{n+2-p}+\nu_1^{n+2,p}\, d_1^{n+2-p} \tag{32}$$

$$\nu_1^{n+2,p+1}=\mu_1^{n+2,p}\, b_1^{n+2-p}+\nu_1^{n+2,p}\, e_1^{n+2-p}. \tag{33}$$

Wir nehmen nun an, dass es

$$\mu_1^{n+2,p+1}=\mu_1^{n+2,1}\sum_{a+b+c+d=p}\Big\{2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)\Big\}^a$$

$$\Big\{2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\Big)+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)\Big\}^b$$

$$\Big\{2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)\Big\}^c\Big\{2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\Big)$$

$$+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big)\Big\}^d\alpha^p_{a,b,c,d}+\nu_1^{n+2,1}\sum_{a+b+c+d=p}$$

$$\Big\{2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)\Big\}^a\Big\{2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\Big)$$

$$+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)\Big\}^b\Big\{2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)\Big\}^c$$

$$\Big\{2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\Big)+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big)\Big\}^d$$

$$\alpha'^p_{a,b,c,d} \tag{34}$$

besteht, wobei $\sum_{a+b+c+d=p}$ über alle Null oder positiven ganzzahligen a, b, c, d unter der Nebenbedingung $a+b+c+d=p$ erstreckt wird und

$$\left\{\begin{array}{ll}\alpha^p_{p,0,0,0}=1, & \\ \alpha^p_{a,b,c,d}={}_{b+d-1}C_d\ {}_{p-(b+d)}C_b, & (b\geqq 1)\ (b=c), \\ \alpha^p_{a,0,0,d}=0, \quad (d\geqq 1) & \\ \text{andernfalls} & \\ \alpha^p_{a,b,c,d}=0. & \end{array}\right. \qquad \left\{\begin{array}{l}\alpha'^p_{a,b,c,d}={}_{b+a-1}C_a\ {}_{p-(b+a)}C_c \\ \qquad (c=b-1,\ p\geqq 1)\ ({}_0C_0=1) \\ \text{andernfalls} \\ \alpha'^p_{a,b,c,d}=0 \end{array}\right. \tag{35}$$

ist.

Ebenfalls nehmen wir an, dass es

$$\nu_1^{n+2,p+1}=\mu_1^{n+2,1}\sum_{a+b+c+d=p}\Big\{2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)\Big\}^a$$

$$\Big\{2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\Big)+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)\Big\}^b$$

$$\left\{2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\right)\right\}^c\left\{2B_1\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\right)\right.$$
$$\left.+2B_2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\right)\right\}^d \beta^p_{a,b,c,d}+\nu_1^{n+2,1}\sum_{a+b+c+d=p}$$
$$\left\{2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\right)\right\}^a\left\{2B_1\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\right)\right.$$
$$\left.+2B_2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\right)\right\}^b\left\{2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\right)\right\}^c$$
$$\left\{2B_1\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\right)+2B_2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\right)\right\}^d$$
$$\beta'^p_{a,b,c,d} \tag{36}$$

besteht, wo

$$\left\{\begin{array}{l} \beta^p_{a,b,c,d}={}_{c+d-1}C_{d\ p-(c+d)}C_b, \\ \qquad\qquad (b=c-1,\ p\geqq 1) \\ \text{andernfalls} \\ \beta^p_{a,b,c,d}=0 \end{array}\right. \qquad \left\{\begin{array}{l} \beta'^p_{a,b,c,d}={}_{b+a-1}C_{a\ p-(b+a)}C_b, \\ \qquad\qquad (b=c,\ b\geqq 1), \\ \beta'^p_{a,0,0,d}=0\ (a\geqq 1) \\ \beta'^p_{0,0,0,p}=1 \\ \text{andernfalls} \\ \beta'^p_{a,b,c,d}=0 \end{array}\right. \tag{37}$$

ist.

Indem wir die Relationen (34) (35) in (32) (33) einsetzen, bekommen wir

$$\alpha^{p+1}_{a,b,c,d}=\alpha^p_{a-1,b,c,d}+\beta^p_{a,b-1,c,d}, \qquad \text{(wenn } b=c\geqq 1,\ a\geqq 1 \text{ ist)},$$
$$\alpha'^{p+1}_{a,b,c,d}=\alpha'^p_{a-1,b,c,d}+\beta'^p_{a,b-1,c,d}, \qquad \text{(wenn } b-1=c,\ a\geqq 1 \text{ ist)},$$
$$\alpha^{p+1}_{a,b,c,d}=\beta^p_{a,b-1,c,d}, \qquad \text{(wenn } b=c\geqq 1,\ a=0 \text{ ist)},$$
$$\alpha'^{p+1}_{a,b,c,d}=\beta'^p_{a,b-1,c,d}, \qquad \text{(wenn } b-1=c,\ a=0 \text{ ist)},$$
$$\alpha^{p+1}_{p+1,0,0,0}=\alpha^p_{p,0,0,0}$$

andernfalls

$$\alpha^{p+1}_{a,b,c,d}=0,$$
$$\alpha'^{p+1}_{a,b,c,d}=0,$$
$$\beta^{p+1}_{a,b,c,d}=\alpha^p_{a,b,c-1,d}+\beta^p_{a,b,c,d-1}, \qquad \text{(wenn } b=c-1,\ d\geqq 1 \text{ ist)}$$
$$\beta'^{p+1}_{a,b,c,d}=\alpha'^p_{a,b,c-1,d}+\beta'^p_{a,b,c,d-1}, \qquad \text{(wenn } b=c\geqq 1,\ d\geqq 1 \text{ ist)}$$
$$\beta^{p+1}_{a,b,c,d}=\alpha^p_{a,b,c-1,d}, \qquad \text{(wenn } b=c-1,\ d=0 \text{ ist)}$$
$$\beta'^{p+1}_{a,b,c,d}=\alpha'^p_{a,b,c-1,d}, \qquad \text{(wenn } b=c\geqq 1,\ d=0 \text{ ist)}$$
$$\beta'^{p+1}_{0,0,0,p+1}=\beta'^p_{0,0,0,p}$$

andernfalls

$$\beta^{p+1}_{a,b,c,d}=0,$$

$\beta'^{p+1}_{a,b,c,d}=0.$

Aus den obigen Relationen folgt es sofort

$$\alpha^{p+1}_{a,b,c,d}={}_{b+d-1}C_{d}\ {}_{p-(b+d)}C_{b}+{}_{c+d-1}C_{d}\ {}_{p-(c+d)}C_{b-1}$$
$$={}_{b+d-1}C_{d}\ {}_{p+1-(b+d)}C_{b},\quad (\text{wenn } b=c\geqq 1,\ a\geqq 1 \text{ ist})$$
$$\alpha'^{p+1}_{a,b,c,d}={}_{b+a-2}C_{a-1}\ {}_{p-(b+a-1)}C_{c}+{}_{b-1+a-1}C_{a}\ {}_{p-(b-1+a)}C_{b-1}$$
$$={}_{b+a-1}C_{a}\ {}_{p+1-(b+a)}C_{c},\quad (\text{wenn } b-1=c,\ a\geqq 1 \text{ ist})$$
$$\alpha^{p+1}_{a,b,c,d}={}_{c+d-1}C_{d}\ {}_{p-(c+d)}C_{b-1}$$
$$={}_{b+d-1}C_{d}\ {}_{p+1-(b+d)}C_{b},\quad (\text{wenn } b=c\geqq 1,\ a=0 \text{ ist})$$
$$\alpha'^{p+1}_{a,b,c,d}={}_{b-1+a-1}C_{a}\ {}_{p-(b-1+a)}C_{b-1}$$
$$={}_{b+a-1}C_{a}\ {}_{p+1-(b+a)}C_{c},\quad (\text{wenn } b-1=c,\ a=0,\ b\geqq 2 \text{ ist})$$
$$\alpha'^{p+1}_{0,1,0,p}=\beta'^{p}_{0,0,0,p}=1={}_{b+a-1}C_{a}\ {}_{p+1-(b+a)}C_{c},\quad (\text{wenn } b-1=c,\ a=0,\ b=1 \text{ ist})$$
$$\beta^{p+1}_{a,b,c,d}={}_{b+d-1}C_{d}\ {}_{p-(b+d)}C_{b}+{}_{c+d-2}C_{d-1}\ {}_{p-(c+d-1)}C_{b}$$
$$={}_{c+d-1}C_{d}\ {}_{p+1-(c+d)}C_{b},\quad (\text{wenn } b=c-1,\ d\geqq 1 \text{ ist})$$
$$\beta'^{p+1}_{a,b,c,d}={}_{b+a-1}C_{a}\ {}_{p-(b+a)}C_{c-1}+{}_{b+a-1}C_{a}\ {}_{p-(b+a)}C_{b}$$
$$={}_{b+a-1}C_{a}\ {}_{p+1-(b+a)}C_{b},\quad (\text{wenn } b=c\geqq 1,\ d\geqq 1 \text{ ist})$$
$$\beta^{p+1}_{a,b,c,d}={}_{b+d-1}C_{d}\ {}_{p-(b+d)}C_{b}$$
$$={}_{c+d-1}C_{d}\ {}_{p+1-(c+d)}C_{b},\quad (\text{wenn } b=c-1,\ d=0,\ b\geqq 2 \text{ ist})$$
$$\beta^{p+1}_{a,b,c,d}=\alpha^{p}_{p,0,0,0}=1={}_{c+d-1}C_{d}\ {}_{p+1-(c+d)}C_{b},\quad (\text{wenn } b=c-1,\ d=0,\ b=1 \text{ ist})$$
$$\beta'^{p+1}_{a,b,c,d}={}_{b+a-1}C_{a}\ {}_{p-(b+a)}C_{c-1}$$
$$={}_{b+a-1}C_{a}\ {}_{p+1-(b+a)}C_{b},\quad (\text{wenn } b=c,\ d=0,\ b\geqq 1 \text{ ist})$$

Also gelten die Relationen (34) (35) (36) (37) auch für $p+1$ unter der Annahme, dass sie für p gelten. Offenbar gelten die Relationen (34) (36) für $p=1$.

Ebenfalls, da es

$$\mu_1^{n+2,p+1,(1)}=\mu_1^{n+2,p,(1)}\,a_1^{n+2-p}+\nu_1^{n+2,p,(1)}\,d_1^{n+2-p}$$
$$\nu_1^{n+2,p+1,(1)}=\mu_1^{n+2,p,(1)}\,b_1^{n+2-p}+\nu_1^{n+2,p,(1)}\,e_1^{n+2-p}$$

besteht, besteht es

$$\mu_1^{n+2,p+1,(1)}=\mu_1^{n+2,1,(1)}\sum_{a+b+c+d=p}\left\{2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\right)\right\}^a$$
$$\left\{2B_1\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\right)+2B_2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\right)\right\}^b$$
$$\left\{2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\right)\right\}^c\left\{2B_1\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\right)\right.$$
$$\left.+2B_2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\right)\right\}^d\alpha^p_{a,b,c,d}+\nu_1^{n+2,1,(1)}\sum_{a+b+c+d=p}$$

$$\left\{2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\right)\right\}^a\left\{2B_1\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\right)\right.$$

$$\left.+2B_2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\right)\right\}^b\left\{2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\right)\right\}^c$$

$$\left\{2B_1\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\right)+2B_2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\right)\right\}^d$$

$$\alpha'^p_{a,b,c,d} \tag{38}$$

$$\nu_1^{n+2,p+1,(1)}=\mu_1^{n+2,1,(1)}\sum_{a+b+c+d=p}\left\{2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\right)\right\}^a$$

$$\left\{2B_1\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\right)+2B_2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\right)\right\}^b$$

$$\left\{2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\right)\right\}^c\left\{2B_1\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\right)\right.$$

$$\left.+2B_2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\right)\right\}^d\beta^p_{a,b,c,d}+\nu_1^{n+2,1,(1)}\sum_{a+b+d+d=p}$$

$$\left\{2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\right)\right\}^a\left\{2B_1\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\right)\right.$$

$$\left.+2B_2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\right)\right\}^b\left\{2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\right)\right\}^c$$

$$\left\{2B_1\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\right)+2B_2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\right)\right\}^d$$

$$\beta'^p_{a,b,c,d}, \tag{39}$$

wo die $\alpha^p_{a,b,c,d}$, $\alpha'^p_{a,b,c,d}$, $\beta^p_{a,b,c,d}$ und $\beta'^p_{a,b,c,d}$ durch die Relationen (35) (37) gegeben werden.

4. Schritt: In (28) setzen wir $i=1$, so gewinnen wir

$$\omega^{n+2,p+1}_{2,n+2+2-(p+1)-1}\left\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots k_{s_2})\right\}=c^{n+3-p-1}_{2,n+2+2-(p+1)-1}$$

$$\left\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2})\right\}\mu_1^{n+2,p}+f^{n+3-p-1}_{2,n+2+2-(p+1)-1}\left\{(j_1,j_2,\cdots,j_{s_1}),\right.$$

$$\left.(k_1,k_2,\cdots,k_{s_2})\right\}\nu_1^{n+2,p}. \tag{40}$$

Ebenfalls besteht es

$$\omega^{n+2,p+1,(1)}_{2,n+2+2-(p+1)-1}\left\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2})\right\}=c^{n+3-p-1}_{2,n+2+2-(p+1)-1}$$

$$\left\{(j_1,j_2,\cdots,j_{s_1}),(k_1,k_2,\cdots,k_{s_2})\right\}\mu_1^{n+2,p,(1)}+f^{n+3-p-1}_{2,n+2+2-(p-1)-1}\left\{(j_1,j_2,\cdots,j_{s_1}),\right.$$

$$\left.(k_1,k_2,\cdots,k_{s_2})\right\}\nu_1^{n+2,p,(1)}. \tag{41}$$

Wir wollen nun die $\pi^{n+2,p+1}_{2,n+2+2-(p+1)-1}$ und $\delta^{n+2,p+1}_{2,n+2+2-(p+1)-1}$ berechnen. Aus (28) gewinnen wir

$$\pi^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(j_1);\ r\Big\}=\omega^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1,0),(r+1,0)\Big\}\mu_1^{r+1,1,(1)}$$
$$+\omega^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1,r+1),(0,0)\Big\}\mu_1^{r+1,1}+\pi^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1);\ r+1\Big\}a_1^{r+1}$$
$$+\delta^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1);\ r+1\Big\}d_1^{r+1}\qquad(r\geqq 6)\tag{42}$$

$$\delta^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(j_1);\ r\Big\}=\omega^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1,0),(r+1,0)\Big\}\nu_1^{r+1,1,(1)}$$
$$+\omega^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1,r+1),(0,0)\Big\}\nu_1^{r+1,1}+\pi^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1);\ r+1\Big\}b_1^{r+1}$$
$$+\delta^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1);\ r+1\Big\}e_1^{r+1}.\qquad(r\geqq 6)\tag{43}$$

Aus (42) und (43) entsteht es

$$\pi^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(j_1);\ r\Big\}=\omega^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1,0),(r+1,0)\Big\}\mu_1^{(1)}\ {}^{10)}$$
$$+\omega^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1,r+1),(0,0)\Big\}\mu_1+\Big\{\omega^{n+2,p-1}_{2,n+2+2-(p-1)-1}\Big\{(j_1,0),(r+2,0)\Big\}\mu_1^{(1)}$$
$$+\omega^{n+2,p-1}_{2,n+2+2-(p-1)-1}\Big\{(j_1,r+2),(0,0)\Big\}\mu_1+\pi^{n+2,p-1}_{2,n+2+2-(p-1)-1}\Big\{(j_1);\ r+2\Big\}a_1$$
$$+\delta^{n+2,p-1}_{2,n+2+2-(p-1)-1}\Big\{(j_1);\ r+2\Big\}d_1\Big\}\ a_1+\Big\{\omega^{n+2,p-1}_{2,n+2+2-(p-1)-1}\Big\{(j_1,0),(r+2,0)\Big\}$$
$$\nu_1^{(1)}+\omega^{n+2,p-1}_{2,n+2+2-(p-1)-1}\Big\{(j_1,r+2),(0,0)\Big\}\nu_1+\pi^{n+2,p-1}_{2,n+2+2-(p-1)-1}\Big\{(j_1);\ r+2\Big\}$$
$$b_1+\delta^{n+2,p-1}_{2,n+2+2-(p-1)-1}\Big\{(j_1);\ r+2\Big\}e_1\Big\}d_1$$
$$=\omega^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1,0),(r+1,0)\Big\}\mu_1^{(1)}+\omega^{n+2,p}_{2,n+2+2-p-1}\Big\{(j_1,r+1),(0,0)\Big\}\mu_1$$
$$+(\mu_1^{(1)}a_1+\nu_1^{(1)}d_1)\omega^{n+2,p-1}_{2,n+2+2-(p-1)-1}\Big\{(j_1,0),(r+2,0)\Big\}+(\mu_1a_1+\nu_1d_1)\omega^{n+2,p-1}_{2,n+2+2-(p-1)-1}$$
$$\Big\{(j_1,r+2),(0,0)\Big\}+(a_1^2+b_1d_1)\pi^{n+2,p-1}_{2,n+2+2-(p-1)-1}\Big\{(j_1);\ r+2\Big\}+(a_1d_1+e_1d_1)$$
$$\delta^{n+2,p-1}_{2,n+2+2-(p-1)-1}\Big\{(j_1);\ r+2\Big\}.$$

Wir setzen nun

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10). Manchmal schreiben wir statt $\mu_1^{r+1,1,(1)}$, $\mu_1^{r+1,1}$, $\nu_1^{r+1,1,(1)}$, $\nu_1^{r+1,1}$, a_1^{r+1}, d_1^{r+1}, b_1^{r+1}, e_1^{r+1} bloss $\mu_1^{(1)}$, μ_1, $\nu_1^{(1)}$, ν_1, a_1, d_1, b_1, e_1, da sie von $r+1$ unabhängig sind.

$$\left.\begin{aligned}&\tilde{\alpha}_{j+1}=a_1\tilde{\alpha}_j+b_1\tilde{\alpha}'_j,\\&\tilde{\alpha}'_{j+1}=d_1\tilde{\alpha}_j+e_1\tilde{\alpha}'_j,\\&\tilde{\alpha}_1=a_1,\\&\tilde{\alpha}'_1=d_1\end{aligned}\right\}\tag{44}$$

und wir nehmen nun an, dass es

$$\begin{aligned}\tilde{\alpha}_{j+1}=a_1\sum_{a+b+c+d=j}&\left\{2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\right)\right\}^a\\&\left\{2B_1\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\right)+2B_2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\right)\right\}^b\\&\left\{2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\right)\right\}^c\left\{2B_1\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\right)\right.\\&\left.+2B_2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\right)\right\}^d\gamma^j_{a,b,c,d}+b_1\sum_{a+b+c+d=j}\\&\left\{2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\right)\right\}^a\left\{2B_1\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\right)\right.\\&\left.+2B_2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\right)\right\}^b\left\{2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\right)\right\}^c\\&\left\{2B_1\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\right)+2B_2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\right)\right\}^d\\&\gamma'^j_{a,b,c,d}.\end{aligned}\tag{45}$$

$$\left\{\begin{aligned}&\gamma^j_{j,0,0,0}=1\\&\gamma^j_{a,b,c,d}={}_{b+d-1}C_{d}\ {}_{j-(b+d)}C_b,\quad (b=c,\ b\geqq1),\\&\gamma^j_{a,0,0,d}=0\ (d\geqq1),\\&\text{andernfalls}\\&\gamma^j_{a,b,c,d}=0\end{aligned}\right.\qquad\left\{\begin{aligned}&\gamma'^j_{a,b,c,d}={}_{c+d}C_d\ {}_{j-(b+d)}C_c\\&\qquad\qquad (c=b-1,\ j\geqq1)\\&\text{andernfalls}\\&\gamma'^j_{a,b,c,d}=0\end{aligned}\right.\tag{46}$$

$$\begin{aligned}\tilde{\alpha}'_{j+1}=d_1\sum_{a+b+c+d=j}&\left\{2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\right)\right\}^a\\&\left\{2B_1\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\right)+2B_2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\right)\right\}^b\\&\left\{2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\right)\right\}^c\left\{2B_1\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\right)\right.\\&\left.+2B_2\left(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\right)\right\}^d\gamma^j_{a,b,c,d}+e_1\sum_{a+b+c+d=j}\\&\left\{2\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\right)\right\}^a\left\{2B_1\left(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\right)\right.\end{aligned}$$

$$+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)\Big\}^b\Big\{2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)\Big\}^c$$

$$\Big\{2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\Big)+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big)\Big\}^d$$

$$\gamma'^{j}_{a,b,c,d}. \tag{47}$$

besteht.

Indem wir die Relationen (45) und (47) in (44) einsetzen, bekommen wir

$$\gamma^{j+1}_{a,b,c,d}=\gamma^{j}_{a-1,b,c,d}+\gamma'^{j}_{a,b,c-1,d} \quad (\text{wenn } b=c\geqq 1,\ a\geqq 1 \text{ ist})$$

$$\gamma'^{j+1}_{a,b,c,d}=\gamma^{j}_{a,b-1,c,d}+\gamma'^{j}_{a,b,c,d-1}, \quad (\text{wenn } c=b-1,\ d\geqq 1 \text{ ist})$$

$$\gamma^{j+1}_{a,b,c,d}=\gamma'^{j}_{a,b,c-1,d}, \quad (\text{wenn } b=c\geqq 1,\ a=0 \text{ ist})$$

$$\gamma'^{j+1}_{a,b,c,d}=\gamma^{j}_{a,b-1,c,d}, \quad (\text{wenn } c=b-1,\ d=0 \text{ ist})$$

$$\gamma^{j+1}_{j+1,0,0,0}=\gamma^{j}_{j,0,0,0},$$

andernfalls

$$\gamma^{j+1}_{a,b,c,d}=0,$$

$$\gamma'^{j+1}_{a,b,c,d}=0.$$

Daraus folgt es sofort

$$\gamma^{j+1}_{a,b,c,d}={}_{b+d-1}C_{d}\ {}_{j-(b+d)}C_{b}+{}_{c-1+d}C_{d}\ {}_{j-(b+d)}C_{c-1}$$

$$={}_{b+d-1}C_{d}\ {}_{j+1-(b+d)}C_{b}, \quad (\text{wenn } b=c\geqq 1,\ a\geqq 1 \text{ ist})$$

$$\gamma'^{j+1}_{a,b,c,d}={}_{b-1+d-1}C_{d}\ {}_{j-(b-1+d)}C_{b-1}+{}_{c+d-1}C_{d-1}\ {}_{j-(b+d-1)}C_{c}$$

$$={}_{c+d}C_{d}\ {}_{j+1-(b+d)}C_{c}, \quad (\text{wenn } c=b-1,\ d\geqq 1 \text{ ist})$$

$$\gamma^{j+1}_{a,b,c,d}={}_{b-1+d}C_{d}\ {}_{j-(b+d)}C_{b-1}={}_{b+d-1}C_{d}\ {}_{j+1-(b+d)}C_{b}, \quad (\text{wenn } b=c\geqq 1,\ a=0 \text{ ist})$$

$$\gamma'^{j+1}_{a,b,c,d}={}_{b-1+d-1}C_{d}\ {}_{j-(b-1+d)}C_{b-1}$$

$$={}_{c+d}C_{d}\ {}_{j+1-(b+d)}C_{c}, \quad (\text{wenn } c=b-1,\ d=0,\ b\geqq 2 \text{ ist})$$

$$\gamma'^{j+1}_{j,1,0,0}=\gamma^{j}_{j,0,0,0}=1={}_{c+d}C_{d}\ {}_{j+1-(b+d)}C_{c}. \quad (\text{wenn } c=b-1,\ d=0,\ b=1 \text{ ist})$$

Folglich gelten die Relationen (45) (46) und (47) auch für $j+2$ unter der Annahme, dass sie für $j+1$ gelten. Offenbar gelten sie für $j=1$.

wir setzen nun

$$\left.\begin{aligned}&\tilde\beta_{j+1}=a_1\tilde\beta_j+b_1\tilde\beta'_j,\\&\tilde\beta'_{j+1}=d_1\tilde\beta_j+e_1\tilde\beta'_j,\\&\tilde\beta_1=b_1,\\&\tilde\beta'_1=e_1\end{aligned}\right\}\tag{48}$$

und wir nehmen an, dass es

$$\tilde\beta_{j+1}=a_1\sum_{a+b+c+d=j}\Big\{2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)\Big\}^a\Big\{2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0$$

$$+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\Big)+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)\Big\}^b\Big\{2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1$$
$$-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)\Big\}^c\Big\{2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\Big)+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0$$
$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big)\Big\}^d\delta^j_{a,b,c,d}+b_1\sum_{a+b+c+d=j}\Big\{2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)\Big\}^a$$
$$\Big\{2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\Big)+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)\Big\}^b$$
$$\Big\{2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)\Big\}^c\Big\{2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\Big)$$
$$+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big)\Big\}^d\delta'^j_{a,b,c,d} \tag{49}$$

$$\left\{\begin{array}{l}\delta^j_{a,b,c,d}={}_{c+a-1}C_a\;{}_{j-(c+a)}C_b\\ \qquad(b=c-1,\ j\geqq 1)\\ \text{andernfalls}\\ \delta^j_{a,b,c,d}=0.\end{array}\right.\qquad\left\{\begin{array}{l}\delta'^j_{0,0,0,j}=1,\\ \delta'^j_{a,b,c,d}={}_{b+a-1}C_a\;{}_{j-(b+a)}C_b,\\ \qquad(b=c,\ b\geqq 1),\\ \delta'^j_{a,0,0,d}=0,\ \ (a\geqq 1),\\ \text{andernfalls}\\ \delta'^j_{a,b,c,d}=0.\end{array}\right. \tag{50}$$

$$\tilde\beta'_{j+1}=d_1\sum_{a+b+c+d=j}\Big\{2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)\Big\}^a\Big\{2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0$$
$$+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\Big)+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)\Big\}^b\Big\{2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1$$
$$-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)\Big\}^c\Big\{2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\Big)+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0$$
$$+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big)\Big\}^d\delta^j_{a,b,c,d}+e_1\sum_{a+b+c+d=j}\Big\{2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_1+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_1\Big)\Big\}^a$$
$$\Big\{2B_1\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\beta_0+\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\alpha_0\Big)+2B_2\Big(-\frac{A_1}{2\sqrt{A_1^2+A_2^2}}\alpha_0-\frac{A_2}{2\sqrt{A_1^2+A_2^2}}\beta_0\Big)\Big\}^b$$
$$\Big\{2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_1-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_1\Big)\Big\}^c\Big\{2B_1\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\beta_0-\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0\Big)$$
$$+2B_2\Big(-\frac{A_2}{(\sqrt{A_1^2+A_2^2})^2}\alpha_0+\frac{A_1}{(\sqrt{A_1^2+A_2^2})^2}\beta_0\Big)\Big\}^d\delta'^j_{a,b,c,d}\,. \tag{51}$$

Indem wir die Relationen (49) und (51) in (48) einsetzen, bekommen wir

$$\delta^{j+1}_{a,b,c,d}=\delta^j_{a-1,b,c,d}+\delta'^j_{a,b,c-1,d},\quad(\text{wenn } b=c-1,\ a\geqq 1 \text{ ist})$$
$$\delta'^{j+1}_{a,b,c,d}=\delta^j_{a,b-1,c,d}+\delta'^j_{a,b,c,d-1},\quad(\text{wenn } b=c\geqq 1,\ d\geqq 1 \text{ ist})$$
$$\delta^{j+1}_{a,b,c,d}=\delta'^j_{a,b,c-1,d},\quad(\text{wenn } b=c-1,\ a=0 \text{ ist})$$

$$\delta'^{j+1}_{a,b,c,d}=\delta^{j}_{a,b-1,c,d}, \quad (\text{wenn } \ b=c\geqq 1, \ d=0 \ \text{ ist})$$

$$\delta'^{j+1}_{0,0,0,j+1}=\delta'^{j}_{0,0,0,j}$$

andernfalls

$$\delta^{j+1}_{a,b,c,d}=0,$$

$$\delta'^{j+1}_{a,b,c,d}=0.$$

Daraus folgt es sofort

$$\begin{aligned}
\delta^{j+1}_{a,b,c,d} &= {}_{c+a-2}C_{a-1}\ {}_{j-(c+a-1)}C_b + {}_{b+a-1}C_a\ {}_{j-(b+a)}C_b \\
&= {}_{c+a-1}C_a\ {}_{j+1-(c+a)}C_b, \quad (\text{wenn } \ b=c-1\geqq 1, \ \ a\geqq 1 \ \text{ ist}) \\
\delta^{j+1}_{a,b,c,d} &= {}_{c+a-2}C_{a-1}\ {}_{j-(c+a-1)}C_b = 1 \\
&= {}_{c+a-1}C_a\ {}_{j+1-(c+a)}C_b, \quad (\text{wenn } \ b=c-1=0, \ \ a\geqq 1 \ \text{ ist}) \\
\delta'^{j+1}_{a,b,c,d} &= {}_{c+a-1}C_a\ {}_{j-(c+a)}C_{b-1} + {}_{b+a-1}C_a\ {}_{j-(b+a)}C_b \\
&= {}_{b+a-1}C_a\ {}_{j+1-(b+a)}C_b, \quad (\text{wenn } \ b=c\geqq 1, \ \ d\geqq 1 \ \text{ ist}) \\
\delta^{j+1}_{a,b,c,d} &= {}_{b+a-1}C_a\ {}_{j-(b+a)}C_b = {}_{j-(c-1+a)}C_b = {}_{j+1-(c+a)}C_b \\
&= {}_{c+a-1}C_a\ {}_{j+1-(c+a)}C_b, \quad (\text{wenn } \ b=c-1\geqq 1, \ \ a=0 \ \text{ ist}) \\
\delta^{j+1}_{a,b,c,d} &= \delta'^{j}_{0,0,0,j} = 1 = {}_{c+a-1}C_a\ {}_{j-(c+a)}C_b, \quad (\text{wenn } \ b=c-1=0, \ \ a=0 \ \text{ ist}) \\
\delta'^{j+1}_{a,b,c,d} &= {}_{c+a-1}C_a\ {}_{j-(c+a)}C_{b-1} = {}_{b+a-1}C_a\ {}_{j+1-(c+a)}C_b. \quad (\text{wenn } b=c\geqq 1, \ d=0 \text{ ist})
\end{aligned}$$

Folglich gelten die Relationen (49) (50) und (51) anch für $j+2$ unter der Annahme, dass sie für $j+1$ gelten. Offenbar gelten sie für $j=1$.

Der Bequemlichkeitshalber setzen wir $\tilde{\alpha}_0=1$, $\tilde{\alpha}'_0=0$, $\tilde{\beta}_0=0$, $\tilde{\beta}'_0=1$. Wir nehmen nun an, dass die $\pi^{n+2,p+1}_{2,n+2+2-(p+1)-1}\{(j_1);r\}$ und $\delta^{n+2,p+1}_{2,n+2+2-(p+1)-1}\{(j_1);r\}$ sich folgendermassen schreiben lassen.

$$\pi^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(j_1)\,;\,r\Big\}=\sum_{i=0}^{k-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(j_1,\ 0),\ (r+1+i,\ 0)\Big\}(\mu^{(1)}_1\tilde{\alpha}_i+\nu^{(1)}_1\tilde{\alpha}'_i)+\sum_{i=0}^{k-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(j_1,\ r+1+i),\ (0,0)\Big\}(\mu_1\tilde{\alpha}_i+\nu_1\tilde{\alpha}'_i)+\tilde{\alpha}_k\pi^{n+2,p+1-k}_{2,n+2+2-(p+1-k)-1}\Big\{(j_1)\,;\,r+k\Big\}+\tilde{\alpha}'_k\delta^{n+2,p+1-k}_{2,n+2+2-(p+1-k)-1}\Big\{(j_1)\,;\,r+k\Big\}.$$

$$\delta^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(j_1)\,;\,r\Big\}=\sum_{i=0}^{k-1}\omega^{n+2,p-i}_{3,n+2+2-(p-i)-1}\Big\{(j_1,\ 0),\ (r+1+i,\ 0)\Big\}(\mu^{(1)}_1\tilde{\beta}_i+\nu^{(1)}_1\tilde{\beta}'_i)+\sum_{i=0}^{k-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(j_1,\ r+1+i),\ (0,0)\Big\}(\mu_1\tilde{\beta}_i+\nu_1\tilde{\beta}'_i)+\tilde{\beta}_k\pi^{n+2,p+1-k}_{2,n+2+2-(p+1-k)-1}\Big\{(j_1)\,;\,r+k\Big\}+\tilde{\beta}'_k\delta^{n+2,p+1-k}_{2,n+2+2-(p+1-k)-1}\Big\{(j_1)\,;\,r+k\Big\}.$$

In den obigen setzen wir die Relationen (42) und (43) ein, so bekommen wir

$$\pi^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(j_1);r\Big\} = \sum_{i=0}^{k-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0),(r+1+i,0)\Big\}(\mu_1^{(1)}\tilde{\alpha}_i+\nu_1^{(1)}\tilde{\alpha}'_i)+\sum_{i=0}^{k-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(j_1,r+1+i),(0,0)\Big\}(\mu_1\tilde{\alpha}_i+\nu_1\tilde{\alpha}'_i)+\tilde{\alpha}_k\Big[\omega^{n+2,p-k}_{2,n+2+2-(p-k)-1}\Big\{(j_1,0),(r+k+1,0)\Big\}\mu_1^{(1)}+\omega^{n+2,p-k}_{2,n+2+2-(p-k)-1}\Big\{(j_1,r+k+1),(0,0)\Big\}\mu_1+\pi^{n+2,p-k}_{2,n+2+2-(p-k)-1}\Big\{(j_1);r+k+1\Big\}a_1+\delta^{n+2,p-k}_{2,n+2+2-(p-k)-1}\Big\{(j_1);r+k+1\Big\}d_1\Big]+\tilde{\alpha}'_k\Big[\omega^{n+2,p-k}_{2,n+2+2-(p-k)-1}\Big\{(j_1,0),(r+k+1,0)\Big\}\nu_1^{(1)}+\omega^{n+2,p-k}_{2,n+2+2-(p-k)-1}\Big\{(j_1,r+k+1),(0,0)\Big\}\nu_1+\pi^{n+2,p-k}_{2,n+2+2-(p-k)-1}\Big\{(j_1);r+k+1\Big\}b_1+\delta^{n+2,p-k}_{2,n+2+2-(p-k)-1}\Big\{(j_1);r+k+1\Big\}e_1\Big].$$

Die rechte Seite dieser Relation lässt sich durch (44) folgendermassen schreiben.

$$\pi^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(j_1);r\Big\}=\sum_{i=0}^{k}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0),(r+1+i,0)\Big\}(\mu_1^{(1)}\tilde{\alpha}_i+\nu_1^{(1)}\tilde{\alpha}'_i)+\sum_{i=0}^{k}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(j_1,r+1+i),(0,0)\Big\}(\mu_1\tilde{\alpha}_i+\nu_1\tilde{\alpha}'_i)+\tilde{\alpha}_{k+1}\pi^{n+2,p-k}_{2,n+2+2-(p-k)-1}\Big\{(j_1);r+k+1\Big\}+\tilde{\alpha}'_{k+1}\delta^{n+2,p-k}_{2,n+2+2-(p-k)-1}\Big\{(j_1);r+k+1\Big\}.$$

Dieses Verfahren wiederholen wir p-malig, so gewinnen[11] wir

$$\pi^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(j_1);r\Big\}=\sum_{i=0}^{p-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0),(r+1+i,0)\Big\}(\mu_1^{(1)}\tilde{\alpha}_i+\nu_1^{(1)}\tilde{\alpha}'_i)+\sum_{i=0}^{p-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(j_1,r+1+i),(0,0)\Big\}(\mu_1\tilde{\alpha}_i+\nu_1\tilde{\alpha}'_i). \tag{52}$$

Ebenfalls gilt es

$$\delta^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(j_1);r\Big\}=\sum_{i=0}^{p-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0),(r+1+i,0)\Big\}(\mu_1^{(1)}\tilde{\beta}_i+\nu_1^{(1)}\tilde{\beta}'_i)+\sum_{i=0}^{p-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(j_1,r+1+i),(0,0)\Big\}(\mu_1\tilde{\beta}_i+\nu_1\tilde{\beta}'_i). \tag{53}$$

Durch (40) und (41) können wir die $\pi^{n+2,p+1}_{2,n+2+2-(p+1)-1}\{(j_1);r\}$ und $\delta^{n+2,p+1}_{2,n+2+2-(p+1)-1}\{(j_1);r\}$ in der folgenden Form schreiben

$$\pi^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(j_1);r\Big\}=\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0),(r+1+i,0)\Big\}\mu_1^{n+2,p-i-1}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0),(r+1+i,0)\Big\}\nu_1^{n+2,p-i-1}\Big](\mu_1^{(1)}\tilde{\alpha}_i+\nu_1^{(1)}\tilde{\alpha}'_i)+$$

11) $\pi^{n+2,1}_{q,n+2+q-1-i}=0$, $\delta^{n+2,1}_{q,n+2+q-1-i}=0$

$$\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(j_1, r+1+i), (0,0)\Big\}\mu_1^{n+2,p-i-1}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(j_1, r+1+i), (0,0)\Big\}\nu_1^{n+2,p-i-1}\Big](\mu_1\tilde{\alpha}_i+\nu_1\tilde{\alpha}'_i), \quad (r\geqq 6), \tag{54}$$

$$\delta^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(j_1)\,;\,r\Big\}=\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0), (r+1+i,0)\Big\}$$

$$\mu_1^{n+2,p-i-1}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0), (r+1+i,0)\Big\}\nu_1^{n+2,p-i-1}\Big](\mu_1^{(1)}\tilde{\beta}_i+\nu_1^{(1)}\tilde{\beta}'_i)$$

$$+\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(j_1, r+1+i), (0,0)\Big\}\mu_1^{n+2,p-i-1}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(j_1, r+1+i), (0,0)\Big\}\nu_1^{n+2,p-i-1}\Big](\mu_1\tilde{\beta}_i+\nu_1\tilde{\beta}'_i), \qquad (r\geqq 6), \tag{55}$$

wo wir $\mu_1^{n+2,0}=-\dfrac{A_1}{2\sqrt{A_1^2+A_2^2}}$, $\nu_1^{n+2,0}=-\dfrac{A_2}{(\sqrt{A_1^2+A_2^2})^2}$, $\mu_1^{n+2,0,(1)}=\dfrac{A_2}{2\sqrt{A_1^2+A_2^2}}$, $\nu_1^{n+2,0,(1)}=-\dfrac{A_1}{(\sqrt{A_1^2+A_2^2})^2}$ setzen.

Ebenfalls besteht es

$$\pi^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(k_1)\,;\,r\Big\}=\sum_{i=0}^{p-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(0,0), (k_1, r+1+i)\Big\}(\mu_1^{(1)}\tilde{\alpha}_i$$

$$+\nu_1^{(1)}\tilde{\alpha}'_i)+\sum_{i=0}^{p-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(0, r+1+i), (k_1,0)\Big\}(\mu_1\tilde{\alpha}_i+\nu_1\tilde{\alpha}'_i)$$

$$=\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(0,0), (k_1, r+1+i)\Big\}\mu_1^{n+2,p-i-1}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(0,0), (k_1, r+1+i)\Big\}\nu_1^{n+2,p-i-1}\Big](\mu_1^{(1)}\tilde{\alpha}_i+\nu_1^{(1)}\tilde{\alpha}'_i)+\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(0, r+1+i), (k_1,0)\Big\}\mu_1^{n+2,p-i-1}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(0, r+1+i), (k_1,0)\Big\}\nu_1^{n+2,p-i-1}\Big]$$

$$(\mu_1\tilde{\alpha}_i+\nu_1\tilde{\alpha}'_i), \quad (r\geqq 6).$$

$$\delta^{n+2,p+1}_{2,n+2+2-(p+1)-1}\Big\{(k_1)\,;\,r\Big\}=\sum_{i=0}^{p-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(0,0), (k_1, r+1+i)\Big\}$$

$$(\mu_1^{(1)}\tilde{\beta}_i+\nu_1^{(1)}\tilde{\beta}'_i)+\sum_{i=0}^{p-1}\omega^{n+2,p-i}_{2,n+2+2-(p-i)-1}\Big\{(0, r+1+i), (k_1,0)\Big\}(\mu_1\tilde{\beta}_i+\nu_1\tilde{\beta}'_i)$$

$$=\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(0,0), (k_1, r+1+i)\Big\}\mu_1^{n+2,p-i-1}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(0,0), (k_1, r+1+i)\Big\}\nu_1^{n+2,p-i-1}\Big](\mu_1^{(1)}\tilde{\beta}_i+\nu_1^{(1)}\tilde{\beta}'_i)+\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(0, r+1+i), (k_1,0)\Big\}\mu_1^{n+2,p-i-1}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(0, r+1+i), (k_1,0)\Big\}\nu_1^{n+2,p-i-1}\Big]$$

$$(\mu_1\tilde{\beta}_i+\nu_1\tilde{\beta}'_i), \quad (r\geqq 6).$$

$$\tilde{\pi}^{n+2,p+1,(1)}_{2,n+2+2-(p+1)-1}\Big\{(j_1)\,;\,r\Big\}=\sum_{i=0}^{p-1}\omega^{n+2,p-i,(1)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0),\,(r+1+i,0)\Big\}(\mu_1^{(1)}\tilde{\alpha}_i$$

$$+\nu_1^{(1)}\tilde{\alpha}'_i)+\sum_{i=0}^{p-1}\omega^{n+2,p-i,(1)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,r+1+i),\,(0,0)\Big\}(\mu_1\tilde{\alpha}_i+\nu_1\tilde{\alpha}'_i)$$

$$=\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0),\,(r+1+i,0)\Big\}\mu_1^{n+2,p-i-1,(1)}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(j_1,0),\,(r+1+i,0)\Big\}\nu_1^{n+2,p-i-1,(1)}\Big](\mu_1^{(1)}\tilde{\alpha}_i+\nu_1^{(1)}\tilde{\alpha}'_i)+\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(j_1,r+1+i),\,(0,0)\Big\}\mu_1^{n+2,p-i-1,(1)}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,r+1+i),\,(0,0)\Big\}$$

$$\nu_1^{n+2,p-i-1,(1)}\Big](\mu_1\tilde{\alpha}_i+\nu_1\tilde{\alpha}'_i), \quad (r\geqq 6).$$

$$\tilde{\delta}^{n+2,p+1,(1)}_{2,n+2+2-(p+1)-1}\Big\{(j_1)\,;\,r\Big\}=\sum_{i=0}^{p-1}\omega^{n+2,p-i,(1)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0),\,(r+1+i,0)\Big\}(\mu_1^{(1)}\tilde{\beta}_i$$

$$+\nu_1^{(1)}\tilde{\beta}'_i)+\sum_{i=0}^{p-1}\omega^{n+2,p-i,(1)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,r+1+i),\,(0,0)\Big\}(\mu_1\tilde{\beta}_i+\nu_1\tilde{\beta}'_i)$$

$$=\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,0),\,(r+1+i,0)\Big\}\mu_1^{n+2,p-i-1,(1)}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(j_1,0),\,(r+1+i,0)\Big\}\nu_1^{n+2,p-i-1,(1)}\Big](\mu_1^{(1)}\tilde{\beta}_i+\nu_1^{(1)}\tilde{\beta}'_i)+\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(j_1,r+1+i),\,(0,0)\Big\}\mu_1^{n+2,p-i-1,(1)}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(j_1,r+1+i),\,(0,0)\Big\}$$

$$\nu_1^{n+2,p-i-1,(1)}\Big](\mu_1\tilde{\beta}_i+\nu_1\tilde{\beta}'_i), \quad (r\geqq 6).$$

$$\tilde{\pi}^{n+2,p+1,(1)}_{2,n+2+2-(p+1)-1}\Big\{(k_1)\,;\,r\Big\}=\sum_{i=0}^{p-1}\omega^{n+2,p-i,(1)}_{2,n+2+2-(p-i)-1}\Big\{(0,0),\,(k_1,r+1+i)\Big\}(\mu_1^{(1)}\tilde{\alpha}_i$$

$$+\nu_1^{(1)}\tilde{\alpha}'_i)+\sum_{i=0}^{p-1}\omega^{n+2,p-i,(1)}_{2,n+2+2-(p-i)-1}\Big\{(0,r+1+i),\,(k_1,0)\Big\}(\mu_1\tilde{\alpha}_i+\nu_1\tilde{\alpha}'_i)$$

$$=\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(0,0),\,(k_1,r+1+i)\Big\}\mu_1^{n+2,p-i-1,(1)}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(0,0),\,(k_1,r+1+i)\Big\}\nu_1^{n+2,p-i-1,(1)}\Big](\mu_1^{(1)}\tilde{\alpha}_i+\nu_1^{(1)}\tilde{\alpha}'_i)+\sum_{i=0}^{p-1}\Big[c^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}$$

$$\Big\{(0,r+1+i),\,(k_1,0)\Big\}\mu_1^{n+2,p-i-1,(1)}+f^{n+3-(p-i)}_{2,n+2+2-(p-i)-1}\Big\{(0,r+1+i),\,(k_1,0)\Big\}$$

$$\nu_1^{n+2,p-i-1,(1)}\Big](\mu_1\tilde{\alpha}_i+\nu_1\tilde{\alpha}'_i), \quad (r\geqq 6).$$

$$\tilde{\delta}^{n+2,p+1,(1)}_{2,n+2+2-(p+1)-1}\Big\{(k_1)\,;\,r\Big\}=\sum_{i=0}^{p-1}\omega^{n+2,p-i,(1)}_{2,n+2+2-(p-i)-1}\Big\{(0,0),\,(k_1,r+1+i)\Big\}(\mu_1^{(1)}\tilde{\beta}_i$$

$$+\nu_1^{(1)}\tilde{\beta}'_i)+\sum_{i=0}^{p-1}\omega_{2,n+2+2-(p-i)-1}^{n+2,p-i,(1)}\Big\{(0,r+1+i),\ (k_1,0)\Big\}(\mu_1\tilde{\beta}_i+\nu_1\tilde{\beta}'_i)$$

$$=\sum_{i=0}^{p-1}\Big[c_{2,n+2+2-(p-i)-1}^{n+3-(p-i)}\Big\{(0,0),\ (k_1,r+1+i)\Big\}\mu_1^{n+2,p-i-1,(1)}+f_{2,n+2+2-(p-i)-1}^{n+3-(p-i)}$$

$$\Big\{(0,0),\ (k_1,r+1+i)\Big\}\nu_1^{n+2,p-i-1,(1)}\Big](\mu_1^{(1)}\tilde{\beta}_i+\nu_1^{(1)}\tilde{\beta}'_i)+\sum_{i=0}^{p-1}\Big[c_{2,n+2+2-(p-i)-1}^{n+3-(p-i)}$$

$$\Big\{(0,r+1+i),\ (k_1,0)\Big\}\mu_1^{n+2,p-i-1,(1)}+f_{2,n+2+2-(p-i)-1}^{n+3-(p-i)}$$

$$\Big\{(0,r+1+i),\ (k_1,0)\Big\}\nu_1^{n+2,p-i-1,(1)}\Big](\mu_1\tilde{\beta}_i+\nu_1\tilde{\beta}'_i),\qquad (r\geqq 6). \tag{56}$$

5. Schritt: Es ist bemerkenswert, dass die in dem 4. Schritte eingeführten $\tilde{\alpha}_i$, $\tilde{\alpha}'_i$, $\tilde{\beta}_i$, $\tilde{\beta}'_i$ nicht voneinander unabhängig sind. Wir wollen zunächst in diesem Schritte zeigen, dass es

$$\left.\begin{aligned} &d_1\tilde{\beta}_j=b_1\tilde{\alpha}'_j,\\ &b_1\tilde{\alpha}_j+e_1\tilde{\beta}_j=\tilde{\beta}_{j+1},\\ &a_1\tilde{\alpha}'_j+d_1\tilde{\beta}'_j=\tilde{\alpha}'_{j+1},\\ &b_1\tilde{\alpha}'_j+e_1\tilde{\beta}'_j=\tilde{\beta}'_{j+1}\end{aligned}\right\} \tag{57}$$

für $j=0, 1, 2, \cdots$ besteht.

Wenn $j=0$ ist, so ist $d_1\tilde{\beta}_0=d_1\cdot 0=0=b_1\cdot 0=b_1\tilde{\alpha}'_0$, $b_1\tilde{\alpha}_0+e_1\tilde{\beta}_0=b_1=\tilde{\beta}_1$. $a_1\tilde{\alpha}'_0+d_1\tilde{\beta}'_0=d_1=\tilde{\alpha}'_1$, $b_1\tilde{\alpha}'_0+e_1\tilde{\beta}'_0=e_1=\tilde{\beta}'_1$. Wenn $j=1$ ist, so ist $d_1\tilde{\beta}_1=d_1b_1=b_1\tilde{\alpha}'_1$, $b_1\tilde{\alpha}_1+e_1\tilde{\beta}_1=b_1a_1+e_1b_1=a_1\tilde{\beta}_1+b_1\tilde{\beta}'_1=\tilde{\beta}_2$, $a_1\tilde{\alpha}'_1+d_1\tilde{\beta}'_1=a_1d_1+d_1e_1=d_1\tilde{\alpha}_1+e_1\tilde{\alpha}'_1=\tilde{\alpha}'_2$, $b_1\tilde{\alpha}'_1+e_1\tilde{\beta}'_1=b_1d_1+e_1e_1=d_1\tilde{\beta}_1+e_1\tilde{\beta}'_1=\tilde{\beta}'_2$.

Wir nehmen nun an, dass die Relation (57) für $j=k$ richtig ist.

$d_1\tilde{\beta}_{k+1}=d_1(a_1\tilde{\beta}_k+b_1\tilde{\beta}'_k)=a_1d_1\tilde{\beta}_k+d_1b_1\tilde{\beta}'_k=a_1b_1\tilde{\alpha}'_k+d_1b_1\tilde{\beta}'_k=b_1(a_1\tilde{\alpha}'_k+d_1\tilde{\beta}'_k)=b_1\tilde{\alpha}'_{k+1}$.

$b_1\tilde{\alpha}_{k+1}+e_1\tilde{\beta}_{k+1}=b_1(a_1\tilde{\alpha}_k+b_1\tilde{\alpha}'_k)+e_1(a_1\tilde{\beta}_k+b_1\tilde{\beta}'_k)=a_1(b_1\tilde{\alpha}_k+e_1\tilde{\beta}_k)+b_1(b_1\tilde{\alpha}'_k+e_1\tilde{\beta}'_k)=a_1\tilde{\beta}_{k+1}+b_1\tilde{\beta}'_{k+1}=\tilde{\beta}_{k+2}$.

$a_1\tilde{\alpha}'_{k+1}+d_1\tilde{\beta}'_{k+1}=a_1(d_1\tilde{\alpha}_k+e_1\tilde{\alpha}'_k)+d_1(d_1\tilde{\beta}_k+e_1\tilde{\beta}'_k)=d_1(a_1\tilde{\alpha}_k+b_1\tilde{\alpha}'_k)+e_1(a_1\tilde{\alpha}'_k+d_1\tilde{\beta}'_k)=d_1\tilde{\alpha}_{k+1}+e_1\tilde{\alpha}'_{k+1}=\tilde{\alpha}'_{k+2}$.

$b_1\tilde{\alpha}'_{k+1}+e_1\tilde{\beta}'_{k+1}=b_1(d_1\tilde{\alpha}_k+e_1\tilde{\alpha}'_k)+e_1(d_1\tilde{\beta}_k+e_1\tilde{\beta}'_k)=d_1(b_1\tilde{\alpha}_k+e_1\tilde{\beta}_k)+e_1(b_1\tilde{\alpha}'_k+e_1\tilde{\beta}'_k)=d_1\tilde{\beta}_{k+1}+e_1\tilde{\beta}'_{k+1}=\tilde{\beta}'_{k+2}$.

Also ist die Relation (57) richtig auch für $j=k+1$ unter der Annahme, dass es für $j=k$ richtig ist. Daher besteht die Relation (57) für $j=0, 1, 2, \cdots$.

Die $\tilde{\alpha}_i, \tilde{\alpha}'_i, \tilde{\beta}_i, \tilde{\beta}'_i, \mu_1^{n+2,i+1}$ und $\nu_1^{n+2,i+1}$ sind nicht voneinander unabhängig. Wir wollen zeigen, dass es

$$\left.\begin{aligned}\tilde{\alpha}_j\mu_1+\tilde{\alpha}'_j\nu_1&=\mu_1^{n+2,j+1}\\ \tilde{\beta}_j\mu_1+\tilde{\beta}'_j\nu_1&=\nu_1^{n+2,j+1}\end{aligned}\right\} \tag{58}$$

für $j=0, 1, 2, \cdots\cdots$ besteht.

Wenn $j=0$ ist, so ist $\tilde{\alpha}_0\mu_1+\tilde{\alpha}'_0\nu_1=\mu_1=\mu_1^{n+2,1}$, $\tilde{\beta}_0\mu_1+\tilde{\beta}'_0\nu_1=\nu_1=\nu_1^{n+2,1}$. Wenn $j=1$ ist, so ist $\tilde{\alpha}_1\mu_1+\tilde{\alpha}'_1\nu_1=a_1\mu_1+d_1\nu_1=\mu_1^{n+2,2}$, $\tilde{\beta}_1\mu_1+\tilde{\beta}'_1\nu_1=b_1\mu_1+e_1\nu_1=\nu_1^{n+2,2}$.

Wir nehmen nun an, dass die Relation (58) für $j=k$ richtig ist.

$\tilde{\alpha}_{k+1}\mu_1+\tilde{\alpha}'_{k+1}\nu_1=(a_1\tilde{\alpha}_k+b_1\tilde{\alpha}'_k)\mu_1+(d_1\tilde{\alpha}_k+e_1\tilde{\alpha}'_k)\nu_1$. Das zweite Glied $(d_1\tilde{\alpha}_k+e_1\tilde{\alpha}'_k)\nu_1=(a_1\tilde{\alpha}'_k+d_1\tilde{\beta}'_k)\nu_1$ nach der (57). Daher ist $\tilde{\alpha}_{k+1}\mu_1+\tilde{\alpha}'_{k+1}\nu_1=(a_1\tilde{\alpha}_k+b_1\tilde{\alpha}'_k)\mu_1+(a_1\tilde{\alpha}'_k+d_1\tilde{\beta}'_k)\nu_1=a_1(\tilde{\alpha}_k\mu_1+\tilde{\alpha}'_k\nu_1)+(b_1\tilde{\alpha}'_k\mu_1+d_1\tilde{\beta}'_k\nu_1)=a_1(\tilde{\alpha}_k\mu_1+\tilde{\alpha}'_k\nu_1)+d_1(\tilde{\beta}_k\mu_1+\tilde{\beta}'_k\nu_1)$ (nach der (57)) $=a_1\mu_1^{n+2,k+1}+d_1\nu_1^{n+2,k+1}=\mu_1^{n+2,k+2}$ (nach der (32)).

$\tilde{\beta}_{k+1}\mu_1+\tilde{\beta}'_{k+1}\nu_1=(a_1\tilde{\beta}_k+b_1\tilde{\beta}'_k)\mu_1+(d_1\tilde{\beta}_k+e_1\tilde{\beta}'_k)\nu_1=(a_1\tilde{\beta}_k+b_1\tilde{\beta}'_k)\mu_1+(b_1\tilde{\alpha}'_k+e_1\tilde{\beta}'_k)\nu_1$ (nach der (57)) $=(b_1\tilde{\alpha}_k+e_1\tilde{\beta}_k)\mu_1+(b_1\tilde{\alpha}'_k+e_1\tilde{\beta}'_k)\nu_1$ (nach der (57)) $=b_1(\tilde{\alpha}_k\mu_1+\tilde{\alpha}'_k\nu_1)+e_1(\tilde{\beta}_k\mu_1+\tilde{\beta}'_k\nu_1)=b_1\mu_1^{n+2,k+1}+e_1\nu_1^{n+2,k+1}=\nu_1^{n+2,k+2}$ (nach der (33)). Also besteht die Relation (58) für $j=0, 1, 2, \cdots$.

Ebenfalls können wir zeigen, dass es

$$\left.\begin{aligned}\tilde{\alpha}_j\mu_1^{(1)}+\tilde{\alpha}'_j\nu_1^{(1)}&=\mu_1^{n+2,j+1,(1)}\\ \tilde{\beta}_j\mu_1^{(1)}+\tilde{\beta}'_j\nu_1^{(1)}&=\nu_1^{n+2,j+1,(1)}\end{aligned}\right\} \tag{59}$$

für $j=0, 1, 2, \cdots$ besteht.

Wenn $j=0$ ist, so ist $\tilde{\alpha}_0\mu_1^{(1)}+\tilde{\alpha}'_0\nu_1^{(1)}=\mu_1^{(1)}=\mu_1^{n+2,1,(1)}$, $\tilde{\beta}_0\mu_1^{(1)}+\tilde{\beta}'_0\nu_1^{(1)}=\nu_1^{(1)}=\nu_1^{n+2,1,(1)}$. Wenn $j=1$ ist, so ist $\tilde{\alpha}_1\mu_1^{(1)}+\tilde{\alpha}'_1\nu_1^{(1)}=a_1\mu_1^{(1)}+d_1\nu_1^{(1)}=\mu_1^{n+2,2,(1)}$, $\tilde{\beta}_1\mu_1^{(1)}+\tilde{\beta}'_1\nu_1^{(1)}=b_1\mu_1^{(1)}+e_1\nu_1^{(1)}=\nu_1^{n+2,2,(1)}$.

Wir nehmen nun an, dass die (59) für $j=k$ richtig ist.

$\tilde{\alpha}_{k+1}\mu_1^{(1)}+\tilde{\alpha}'_{k+1}\nu_1^{(1)}=(a_1\tilde{\alpha}_k+b_1\tilde{\alpha}'_k)\mu_1^{(1)}+(d_1\tilde{\alpha}_k+e_1\tilde{\alpha}'_k)\nu_1^{(1)}=(a_1\tilde{\alpha}_k+b_1\tilde{\alpha}'_k)\mu_1^{(1)}+(a_1\tilde{\alpha}'_k+d_1\tilde{\beta}'_k)\nu_1^{(1)}$ (nach der (57)) $=a_1(\tilde{\alpha}_k\mu_1^{(1)}+\tilde{\alpha}'_k\nu_1^{(1)})+d_1(\tilde{\beta}_k\mu_1^{(1)}+\tilde{\beta}'_k\nu_1^{(1)})$ (nach der (57)) $=a_1\mu_1^{n+2,k+1,(1)}+d_1\nu_1^{n+2,k+1,(1)}=\mu_1^{n+2,k+2,(1)}$.

$\tilde{\beta}_{k+1}\mu_1^{(1)}+\tilde{\beta}'_{k+1}\nu_1^{(1)}=(a_1\tilde{\beta}_k+b_1\tilde{\beta}'_k)\mu_1^{(1)}+(d_1\tilde{\beta}_k+e_1\tilde{\beta}'_k)\nu_1^{(1)}=(a_1\tilde{\beta}_k+b_1\tilde{\beta}'_k)\mu_1^{(1)}+(b_1\tilde{\alpha}'_k+e_1\tilde{\beta}'_k)\nu_1^{(1)}$ (nach der (57)) $=b_1(\tilde{\alpha}_k\mu_1^{(1)}+\tilde{\alpha}'_k\nu_1^{(1)})+e_1(\tilde{\beta}_k\mu_1^{(1)}+\tilde{\beta}'_k\nu_1^{(1)})=b_1\mu_1^{n+2,k+1,(1)}+e_1\nu_1^{n+2,k+1,(1)}=\nu_1^{n+2,k+2,(1)}$.

Also besteht die Relation (57) für $j=0, 1, 2\cdots$.

Wir können nun die Relationen (44) und (48) verallgemeinern folgendermassen.

$$\left.\begin{aligned}&\tilde{\alpha}_i\tilde{\alpha}_j+\tilde{\beta}_i\tilde{\alpha}'_j=\tilde{\alpha}_{i+j}\\&\tilde{\alpha}'_i\tilde{\alpha}_j+\tilde{\beta}'_i\tilde{\alpha}'_j=\tilde{\alpha}'_{i+j}\\&\tilde{\alpha}_i\tilde{\beta}_j+\tilde{\beta}_i\tilde{\beta}'_j=\tilde{\beta}_{i+j}\\&\tilde{\alpha}'_i\tilde{\beta}_j+\tilde{\beta}'_i\tilde{\beta}'_j=\tilde{\beta}'_{i+j}\end{aligned}\right\}\tag{60}$$

$$\text{für } i=0, 1, 2, \cdots, \quad j=0, 1, 2, \cdots.$$

Die Relation (60) ist offenbar richtig für $i=0$ und $i=1$. Wir wollen zeigen, dass es für $i=k+1$ besteht unter der Annahme, dass es für $i=k$ richtig ist.

$\tilde{\alpha}_{k+1}\tilde{\alpha}_j+\tilde{\beta}_{k+1}\tilde{\alpha}'_j=(a_1\alpha_k+b_1\tilde{\alpha}'_k)\tilde{\alpha}_j+(a_1\tilde{\beta}_k+b_1\tilde{\beta}'_k)\tilde{\alpha}'_j=a_1(\tilde{\alpha}_k\tilde{\alpha}_j+\tilde{\beta}_k\tilde{\alpha}'_j)+b_1(\tilde{\alpha}'_k\alpha_j+\tilde{\beta}'_k\tilde{\alpha}'_j)=a_1\tilde{\alpha}_{k+j}+b_1\tilde{\alpha}'_{k+j}=\tilde{\alpha}_{k+j+1}=\tilde{\alpha}_{(k+1)+j}$.

$\tilde{\alpha}'_{k+1}\tilde{\alpha}_j+\tilde{\beta}'_{k+1}\tilde{\alpha}'_j=(d_1\tilde{\alpha}_k+e_1\tilde{\alpha}'_k)\tilde{\alpha}_j+(d_1\tilde{\beta}_k+e_1\tilde{\beta}'_k)\alpha'_j=d_1(\tilde{\alpha}_k\tilde{\alpha}_j+\tilde{\beta}_k\tilde{\alpha}'_j)+e_1(\tilde{\alpha}'_k\tilde{\alpha}_j+\tilde{\beta}'_k\tilde{\alpha}'_j)=d_1\tilde{\alpha}_{k+j}+e_1\tilde{\alpha}'_{k+j}=\tilde{\alpha}'_{k+j+1}=\tilde{\alpha}'_{(k+1)+j}$.

$\alpha_{k+1}\tilde{\beta}_j+\tilde{\beta}_{k+1}\tilde{\beta}'_j=(a_1\tilde{\alpha}_k+b_1\tilde{\alpha}'_k)\tilde{\beta}_j+(a_1\tilde{\beta}_k+b_1\tilde{\beta}'_k)\tilde{\beta}'_j=a_1(\tilde{\alpha}_k\tilde{\beta}_j+\tilde{\beta}_k\tilde{\beta}'_j)+b_1(\tilde{\alpha}'_k\tilde{\beta}_j+\tilde{\beta}'_k\tilde{\beta}'_j)=a_1\tilde{\beta}_{k+j}+b_1\tilde{\beta}'_{k+j}=\tilde{\beta}_{k+j+1}=\tilde{\beta}_{(k+1)+j}$.

$\tilde{\alpha}'_{k+1}\tilde{\beta}_j+\tilde{\beta}'_{k+1}\tilde{\beta}'_j=(d_1\tilde{\alpha}_k+e_1\tilde{\alpha}'_k)\tilde{\beta}_j+(d_1\tilde{\beta}_k+e_1\tilde{\beta}'_k)\tilde{\beta}'_j=d_1(\tilde{\alpha}_k\tilde{\beta}_j+\tilde{\beta}_k\tilde{\beta}'_j)+e_1(\tilde{\alpha}'_k\tilde{\beta}_j+\tilde{\beta}'_k\tilde{\beta}'_j)=d_1\tilde{\beta}_{k+j}+e_1\tilde{\beta}'_{k+j}=\tilde{\beta}'_{k+j+1}=\tilde{\beta}'_{(k+1)+j}$.

Also ist die Relation (60) richtig für $i=0, 1, 2, \cdots$, $j=0, 1, 2, \cdots$.

Wir können nun die Relation (57) verallgemeinern folgendermassen.

$$\left.\begin{aligned}&\tilde{\alpha}'_i\tilde{\beta}_j=\tilde{\beta}_i\tilde{\alpha}'_j\\&\tilde{\beta}_i\tilde{\alpha}_j+\tilde{\beta}'_i\tilde{\beta}_j=\tilde{\beta}_{i+j}\\&\tilde{\alpha}_i\tilde{\alpha}'_j+\tilde{\alpha}'_i\tilde{\beta}'_j=\tilde{\alpha}'_{i+j}\\&\tilde{\beta}_i\tilde{\alpha}'_j+\tilde{\beta}'_i\tilde{\beta}'_j=\tilde{\beta}'_{i+j}\end{aligned}\right\}\tag{61}$$

$$\text{für } i=0, 1, 2, \cdots, \quad j=0, 1, 2, \cdots.$$

Die Relation (61) ist offenbar richtig für $i=0$ und $i=1$. Wir wollen zeigen, dass es für $i=k+1$ besteht unter der Annahme, dass es für $i=k$ richtig ist.

$\tilde{\alpha}'_{k+1}\tilde{\beta}_j=(a_1\tilde{\alpha}'_k+d_1\tilde{\beta}'_k)\tilde{\beta}_j$ (nach der (57)) $=a_1\tilde{\alpha}'_k\tilde{\beta}_j+d_1\tilde{\beta}'_k\tilde{\beta}_j=a_1\tilde{\beta}_k\tilde{\alpha}'_j+d_1\tilde{\beta}'_k\tilde{\beta}_j$ (nach der (61)) $=a_1\tilde{\beta}_k\tilde{\alpha}'_j+b_1\tilde{\alpha}_j'\tilde{\beta}_k'$ (nach der (57)) $=(a_1\tilde{\beta}_k+b_1\tilde{\beta}'_k)\tilde{\alpha}'_j=\tilde{\beta}_{k+1}\tilde{\alpha}'_j$.

$\tilde{\beta}_{k+1}\tilde{\alpha}_j+\tilde{\beta}'_{k+1}\tilde{\beta}_j=\tilde{\alpha}_j\tilde{\beta}_{k+1}+\tilde{\beta}_j\tilde{\beta}'_{k+1}=\tilde{\beta}_{j+(k+1)}$ (nach der (60)) $=\tilde{\beta}_{(k+1)+j}$.

$\tilde{\alpha}_{k+1}\tilde{\alpha}'_j+\tilde{\alpha}'_{k+1}\tilde{\beta}'_j=\tilde{\alpha}'_j\alpha_{k+1}+\tilde{\beta}'_j\tilde{\alpha}'_{k+1}=\tilde{\alpha}'_{j+(k+1)}$ (nach der (60)) $=\tilde{\alpha}'_{(k+1)+j}$.
$\tilde{\beta}_{k+1}\tilde{\alpha}'_j+\tilde{\beta}'_{k+1}\tilde{\beta}'_j=\tilde{\alpha}'_j\tilde{\beta}_{k+1}+\tilde{\beta}'_j\tilde{\beta}'_{k+1}=\tilde{\beta}'_{j+(k+1)}$ (nach der (60)) $=\tilde{\beta}'_{(k+1)+j}$.

(Fortgesetzt)

MATHEMATISCHES INSTITUT
UNIVERSITÄT ZU OKAYAMA.

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