ÜBER DIE KOEFFIZIENTEN DER SCHLICHTEN FUNKTIONEN (IV).

KEN'ITI KOSEKI

Diese Arbeit ist die Fortsetzung von meiner früheren Arbeit[1] "ÜBER DIE KOEFFIZIENTEN DER SCHLICHTEN FUNKTIONEN (III)".

Ich will nun die Integralgleichung[2]

$$\mathfrak{J}\Big[6\kappa^3(s)e^{-2s}+12\kappa(s)\int_\infty^0 2\kappa(s_1)e^{-s_1}\int_\infty^{s_1}2\kappa(s_2)e^{-s_2}ds_2ds_1+6\kappa(s)\int_\infty^s 2\kappa^2(s_1)e^{-2s_1}ds_1+4\kappa(s)\int_s^0 2\kappa^2(s_1)e^{-2s_1}ds_1+12\kappa^2(s)e^{-s}\int_s^0 2\kappa(s_1)e^{-s_1}ds_1+8\kappa^2(s)e^{-s}\int_\infty^s 2\kappa(s_1)e^{-s_1}ds_1\Big]=0,\quad 0\leqq s<\infty, \tag{1}$$

unter keine Bedingung lösen.

Aus der Gleichung (1) folgt es ohne weiteres

$$\mathfrak{J}\Big[6\kappa^3(s)e^{-2s}+\Big\{12\int_\infty^0 2\kappa(s_1)e^{-s_1}\int_\infty^{s_1}2\kappa(s_2)e^{-s_2}ds_2ds_1+4\int_\infty^0 2\kappa^2(s_1)e^{-2s_1}ds_1\Big\}\kappa(s)+2\kappa(s)\int_\infty^s 2\kappa^2(s_1)e^{-2s_1}ds_1+12\kappa^2(s)e^{-s}\int_\infty^0 2\kappa(s_1)e^{-s_1}ds_1-4\kappa^2(s)e^{-s}\int_\infty^s 2\kappa(s_1)e^{-s_1}ds_1\Big]=0.$$

Wir setzen

$$12\int_\infty^0 2\kappa(s_1)e^{-s_1}\int_\infty^{s_1}2\kappa(s_2)e^{-s_2}ds_2ds_1+4\int_\infty^0 2\kappa^2(s_1)e^{-2s_1}ds_1=A,$$

$$12\int_\infty^0 2\kappa(s_1)e^{-s_1}ds_1=B,$$

$$e^{-s}=t,\quad \kappa(s)=\kappa(t),$$

so nimmt die obige Integralgleichung die folgende Form an

$$\mathfrak{J}\Big[6\kappa^3(t)t^2+A\kappa(t)+2\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1+B\kappa^2(t)t-4\kappa^2(t)t\int_t^0 2\kappa(t_1)dt_1\Big]=0,\quad 0\leqq t\leqq 1.\text{[3]} \tag{2}$$

Wir setzen $\kappa(t)=e^{i\varphi(t)}$, so ist es

$$(\kappa(t))'=\kappa(t)i\varphi'(t).$$

Indem wir die beiden Seiten der obigen Formel $(n-1)$-malig differenzieren,

1) K. Koseki. ÜBER DIE KOEFFIZIENTEN DER SCHLICHTEN FUNKTIONEN (III). Math. Journ. Okay. Univ. Vol, II, NO. I, 1962.

2) K. Koseki. a. a. O. S. 34.

bekommen wir

$$\frac{d^n\kappa(t)}{dt^n}=\sum_{j=0}^{n-1}\frac{d^j\kappa(t)}{dt^j}\,i\,{}_{n-1}C_j\frac{d^{n-j}\varphi(t)}{dt^{n-j}},\quad n\geqq 1. \tag{3}$$

Wir setzen $\frac{d^n\kappa(t)}{dt^n}=\kappa(t)a_n$ $(n=0, 1, 2\cdots)$, so folgt aus (3)

$$a_n=\sum_{j=0}^{n-1}a_j\,{}_{n-1}C_j i\varphi^{(n-j)}(t)$$

$$=a_0 i\varphi^{(n)}(t)+\sum_{p_1=1}^{n-1}a_{p_1\,n-1}C_{p_1}i\varphi^{(n-p_1)}(t)$$

$$=a_0 i\varphi^{(n)}(t)+\sum_{p_1=1}^{n-1}\sum_{p_2=0}^{p_1-1}a_{p_2}\,{}_{p_1-1}C_{p_2}i\varphi^{(p_1-p_2)}(t)\,{}_{n-1}C_{p_1}\,i\varphi^{(n-p_1)}(t)$$

$$=a_0\Big(i\varphi^{(n)}(t)+\sum_{p_1=1}^{n-1}{}_{n-1}C_{p_1}i^2\varphi^{(n-p_1)}(t)\varphi^{(p_1)}(t)\Big)$$

$$+\sum_{p_2=1}^{n-2}\sum_{p_1=p_2+1}^{n-1}a_{p_2}\,{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}i^2\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)$$

$$=\cdots\cdots$$

$$\cdots\cdots$$

$$=a_0\Big(i\varphi^{(n)}(t)+\sum_{p_1=1}^{n-1}{}_{n-1}C_{p_1}i^2\varphi^{(n-p_1)}(t)\varphi^{(p_1)}(t)$$

$$+\sum_{p_2=1}^{n-2}\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}i^3\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\varphi^{(p_2)}(t)+\cdots$$

$$+\sum_{p_{n-2}=1}^{2}\sum_{p_{n-3}=p_{n-2}+1}^{3}\cdots\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots$$

$${}_{p_{n-3}-1}C_{p_{n-2}}i^{n-1}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\cdots\varphi^{(p_{n-3}-p_{n-2})}(t)\varphi^{(p_{n-2})}(t)\Big)$$

$$+\sum_{p_{n-1}=1}^{1}\sum_{p_{n-2}=p_{n+1}+1}^{2}\cdots\sum_{p_1=p_2+1}^{n-1}a_1 i^{n-1}\,{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots\;{}_{p_{n-2}-1}C_1\varphi^{(n-p_1)}(t)\cdots$$

$$\varphi^{(p_{n-2}-p_{n-1})}(t)=a_0\Big\{i\varphi^{(n)}(t)+\sum_{j=1}^{n-1}\Big(\sum_{p_j=1}^{n-j}\sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots$$

$${}_{p_{j-1}-1}C_{p_j}i^{j+1}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\cdots\;\varphi^{(p_{j-1}-p_j)}(t)\varphi^{(p_j)}(t)\Big)\Big\}.$$

Anderseits ist es offenbar $a_0=1$. So bekommen wir

$$a_n=i\varphi^{(n)}(t)+\sum_{j=1}^{n-1}\Big(\sum_{p_j=1}^{n-j}\sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots$$

$${}_{p_{j-1}-1}C_{p_j}i^{j+1}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\cdots\varphi^{(p_{j-1}-p_j)}(t)\varphi^{(p_j)}(t)\Big) \tag{4}$$

---

3) Da $\lim_{t\to 0}\kappa(t)$ existiert, so kann $t=0$ eingeschlossen werden.

für $n \geqq 2$.

Aus (4) folget es ohne weiteres

$$\left.\begin{aligned}
&\frac{d^n\kappa^3(t)}{dt^n} = \kappa^3(t)\Big\{i3\varphi^{(n)}(t)+\sum_{j=1}^{n-1}\Big(\sum_{p_j=1}^{n-j}\sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots\sum_{p_1=p_2+1}^{n-1}3^{j+1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots \\
&{}_{p_{j-1}-1}C_{p_j}i^{j+1}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\cdots\varphi^{(p_{j-1}-p_j)}(t)\varphi^{(p_j)}(t)\Big)\Big\},\quad n\geqq 2,\\
&\frac{d^{n-1}\kappa^3(t)}{dt^{n-1}} = \kappa^3(t)\Big\{i3\varphi^{(n-1)}(t)+\sum_{j=1}^{n-2}\Big(\sum_{p_j=1}^{n-j-1}\sum_{p_{j-1}=p_j+1}^{n-j}\cdots\sum_{p_1=p_2+1}^{n-2}3^{j+1}{}_{n-2}C_{p_1}\,{}_{p_1-1}C_{p_2}\\
&\cdots{}_{p_{j-1}-1}C_{p_j}i^{j+1}\varphi^{(n-1-p_1)}(t)\varphi^{(p_1-p_2)}(t)\cdots\varphi^{(p_{j-1}-p_j)}(t)\varphi^{(p_j)}(t)\Big)\Big\}\quad n\geqq 3,\\
&\frac{d^{n-2}\kappa^3(t)}{dt^{n-2}} = \kappa^3(t)\Big\{i3\varphi^{(n-2)}(t)+\sum_{j=1}^{n-3}\Big(\sum_{p_j=1}^{n-j-2}\sum_{p_{j-1}=p_j+1}^{n-j-1}\cdots\\
&\sum_{p_1=p_2+1}^{n-3}3^{j+1}{}_{n-3}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots\ {}_{p_{j-1}-1}C_{p_j}i^{j+1}\varphi^{(n-2-p_1)}(t)\varphi^{(p_1-p_2)}(t)\cdots\\
&\varphi^{(p_{j-1}-p_j)}(t)\varphi^{(p_j)}(t)\Big)\Big\},\quad n\geqq 4.
\end{aligned}\right\}\tag{5}$$

Es ist nun

$$\frac{d\left(\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1\right)}{dt} = i\kappa(t)\varphi'(t)\int_t^0 2\kappa^2(t_1)t_1dt_1 - 2\kappa^3(t)t.$$

Daher ist

$$\begin{aligned}
&\frac{d^n\left(\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1\right)}{dt^n} = i\sum_{p_1=0}^{n-1}\varphi^{(n-p_1)}(t)_{n-1}C_{p_1}\frac{d^{p_1}\left(\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1\right)}{dt^{p_1}}\\
&-2\sum_{p_1=0}^{n-1}(\kappa^3(t))^{(n-p_1-1)}{}_{n-1}C_{p_1}t^{(p_1)} = i\varphi^{(n)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1 + i\sum_{p_1=1}^{n-1}\varphi^{(n-p_1)}(t)\\
&{}_{n-1}C_{p_1}\frac{d^{p_1}\left(\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1\right)}{dt^{p_1}} - 2(\kappa^3(t))^{n-1}t - 2(\kappa^3(t))^{n-2}{}_{n-1}C_1 = i\varphi^{(n)}(t)\kappa(t)\\
&\int_t^0 2\kappa^2(t_1)t_1dt_1 + i\sum_{p_1=1}^{n-1}\varphi^{(n-p_1)}(t)_{n-1}C_{p_1}\Big[i\varphi^{(p_1)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1 + i\\
&\sum_{p_2=1}^{p_1-1}\varphi^{(p_1-p_2)}(t)_{p_1-1}C_{p_2}\frac{d^{p_2}\left(\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1\right)}{dt^{p_2}} - 2(\kappa^3(t))^{(p_1-1)}t - 2(\kappa^3(t))^{(p_1-2)}{}_{p_1-1}C_1\Big]\\
&-2(\kappa^3(t))^{(n-1)}t - 2(\kappa^3(t))^{(n-2)}{}_{n-1}C_1 = i\varphi^{(n)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1 + i^2\\
&\sum_{p_1=1}^{n-1}{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)\varphi^{(p_1)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1 - 2(\kappa^3(t))^{(n-1)}t - 2(\kappa^3(t))^{(n-2)}{}_{n-1}C_1\\
&-2\sum_{p_1=1}^{n-1}i\,{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)\Big\{(\kappa^3(t))^{(p_1-1)}t + (\kappa^3(t))^{(p_1-2)}{}_{p_1-1}C_1\Big\}
\end{aligned}$$

$$+\sum_{p_2=1}^{n-2}\sum_{p_1=p_2+1}^{n-1} i^2{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\,\frac{d^{p_2}\left(\kappa(t)\int_t^0 2\kappa^2(t_1)\,t_1\,dt_1\right)}{dt^{p_2}}$$

$$=i\varphi^{(n)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1+i^2\sum_{p_1=1}^{n-1}{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)\varphi^{(p_1)}(t)\kappa(t)$$

$$\int_t^0 2\kappa^2(t_1)t_1dt_1-2(\kappa^3(t))^{n-1}t-2(\kappa^3(t))^{(n-2)}{}_{n-1}C_1-2\sum_{p_1=1}^{n-1} i\,{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)$$

$$\left\{(\kappa^3(t))^{(p_1-1)}t+(\kappa^3(t))^{(p_1-2)}{}_{p_1-1}C_1\right\}+\sum_{p_2=1}^{n-2}\sum_{p_1=p_2+1}^{n-1} i^2{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)$$

$$\Bigg[i\varphi^{(p_2)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1+i\sum_{p_3=1}^{p_2-1}\varphi^{(p_2-p_3)}(t){}_{p_2-1}C_{p_3}\,\frac{d^{p_3}\left(\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1\right)}{dt^{p_3}}$$

$$-2(\kappa^3(t))^{(p_2-1)}t-2(\kappa^3(t))^{(p_2-2)}{}_{p_2-1}C_1\Bigg]=i\varphi^{(n)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1+i^2$$

$$\sum_{p_1=1}^{n-1}{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)\varphi^{(p_1)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1+i^3$$

$$\sum_{p_2=1}^{n-2}\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\,\varphi^{(p_2)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1-2(\kappa^3(t))^{(n-1)}t$$

$$-2(\kappa^3(t))^{(n-2)}{}_{n-1}C_1-2\sum_{p_1=1}^{n-1} i\,{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)\left\{(\kappa^3(t))^{(p_1-1)}t+(\kappa^3(t))^{(p_1-2)}{}_{p_1-1}C_1\right\}$$

$$-2\sum_{p_2=1}^{n-2}\sum_{p_1=p_2+1}^{n-1} i^2{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\left\{(\kappa^3(t))^{(p_2-1)}t+(\kappa^3(t))^{(p_2-2)}{}_{p_2-1}C_1\right\}$$

$$+\sum_{p_3=1}^{n-3}\sum_{p_2=p_3+1}^{n-2}\sum_{p_1=p_2+1}^{n-1} i^3{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\,{}_{p_2-1}C_{p_3}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\varphi^{(p_2-p_3)}(t)$$

$$\frac{d^{p_3}\left(\kappa(t))\int_t^0 2\kappa^2(t_1)t_1dt_1\right)}{dt^{p_3}}\;.$$

Indem wir dieses Verfahren wiederholen, bekommen wir

$$\frac{d^n\left(\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1\right)}{dt^n}=i\varphi^{(n)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1+i^2$$

$$\sum_{p_1=1}^{n-1}{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)\varphi^{(p_1)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1+i^3$$

$$\sum_{p_2=1}^{n-2}\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\varphi^{(p_2)}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1+\cdots+$$

$$i^n\sum_{p_{n-1}=1}^{1}\sum_{p_{n-2}=p_{n-1}+1}^{2}\cdots\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots{}_{p_{n-2}-1}C_{p_{n-1}}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\cdots$$

$$\varphi^{(p_{n-2}-p_{n-1})}(t)\varphi^{(p_{n-1})}(t)\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1-2(\kappa^3(t))^{(n-1)}t-2(\kappa^3(t))^{(n-2)}{}_{n-1}C_1-2$$

$$\sum_{p_1=1}^{n-1} i\ {}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)\Big\{(\kappa^3(t))^{(p_1-1)}t+(\kappa^3(t))^{(p_1-2)}{}_{p_1-1}C_1\Big\}-\cdots$$

$$-i^{n-1}\sum_{p_{n-1}=1}^{1}\ \sum_{p_{n-2}=p_{n-1}+1}^{2}\cdots\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots{}_{p_{n-2}-1}C_{p_{n-1}}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\cdots$$

$$\varphi^{(p_{n-2}-p_{n-1})}(t)\Big\{(\kappa^3(t))'t\Big\}. \tag{6}$$

Aus (5) gewinnen wir

$$\frac{d^n\Big(\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1\Big)}{dt^n}\Bigg|_{t=0}=-2(\kappa^3(0))^{(n-2)}{}_{n-1}C_1-2\sum_{j=1}^{n-2}\Bigg(\sum_{p_j=2}^{n-j}\ \sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots$$

$$\sum_{p_1=p_2+1}^{n-1} i^j{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots{}_{p_{j-1}-1}C_{p_j}\varphi^{(n-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\ \varphi^{(p_{j-1}-p_j)}(0)(\kappa^3(0))^{(p_j-2)}$$

(für $n\geqq 3$). (7)

$$=-2{}_{n-1}C_1\kappa^3(0)\Bigg\{i3\varphi^{(n-2)}(0)+\sum_{j=1}^{n-3}\Bigg(\sum_{p_j=1}^{n-j-2}\ \sum_{p_{j-1}=p_j+1}^{n-j-1}\cdots\sum_{p_1=p_2+1}^{n-3}3^{j+1}{}_{n-3}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots$$

$${}_{p_{j-1}-1}C_{p_j}i^{j+1}\varphi^{(n-2-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\ \varphi^{(p_{j-1}-p_j)}(0)\varphi^{(p_j)}(0)\Bigg)\Bigg\}-2\kappa^3(0)$$

$$\sum_{j=1}^{n-4}\Bigg(\sum_{p_j=4}^{n-j}\ \sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots\sum_{p_1=p_2+1}^{n-1} i^j{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots\ {}_{p_{j-1}-1}C_{p_j}\varphi^{(n-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots$$

$$\varphi^{(p_{j-1}-p_j)}(0)\Bigg[i3\varphi^{(p_j-2)}(0)+\sum_{s=1}^{p_j-3}\Bigg(\sum_{q_s=1}^{p_j-s-2}\ \sum_{q_{s-1}=q_s+1}^{p_j-s-1}\cdots\sum_{q_1=q_2+1}^{p_j-3}3^{s+1}{}_{p_j-3}C_{q_1}\,{}_{q_1-1}C_{q_2}\cdots$$

$${}_{q_{s-1}-1}C_{q_s}i^{s+1}\varphi^{(p_j-2-q_1)}(0)\varphi^{(q_1-q_2)}(0)\cdots\ \varphi^{(q_{s-1}-q_s)}(0)\varphi^{(q_s)}(0)\Bigg]\Bigg)-2\kappa^3(0)\sum_{j=1}^{n-2}\Bigg(\sum_{p_j=2}^{2}\ \sum_{p_{j-1}=3}^{n-j+1}\cdots$$

$$\sum_{p_1=p_2+1}^{n-1} i^j{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots\ {}_{p_{j-1}-1}C_2\varphi^{(n-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\ \varphi^{(p_{j-1}-2)}(0)\Bigg)-2\kappa^3(0)$$

$$\sum_{j=1}^{n-3}\Bigg(\sum_{p_j=3}^{3}\ \sum_{p_{j-1}=4}^{n-j+1}\cdots\sum_{p_1=p_2+1}^{n-1} i^j{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots\ {}_{p_{j-1}-1}C_3\varphi^{(n-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots$$

$$\varphi^{(p_{j-1}-3)}3i\varphi'(0)\Bigg)\quad (\text{für } n\geqq 5). \tag{8}$$

Wir wollen nun $\dfrac{d^n\Big(\kappa^2(t)t\int_t^0 2\kappa(t_1)dt_1\Big)}{dt^n}$ ausrechnen.

$$\frac{d^n\Big(\kappa^2(t)t\int_t^0 2\kappa(t_1)dt_1\Big)}{dt^n}=\frac{d^n\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1}{dt^n}t+{}_nC_1\frac{d^{n-1}\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1}{dt^{n-1}}. \tag{9}$$

Es ist aber

$$\frac{d\Big(\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1\Big)}{dt}=2\kappa^2(t)i\varphi'(t)\int_t^0 2\kappa(t_1)dt_1-2\kappa^3(t).$$

Daraus folgt ohne weiteres

$$\frac{d^n\left(\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1\right)}{dt^n}=2i\sum_{p_1=0}^{n-1}\varphi^{(n-p_1)}(t)_{n-1}C_{p_1}\frac{d^{p_1}\left(\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1\right)}{dt^{p_1}}-2(\kappa^3(t))^{(n-1)}$$

$$=2i\varphi^{(n)}(t)\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1+2i\sum_{p_1=1}^{n-1}\varphi^{(n-p_1)}(t)_{n-1}C_{p_1}\frac{d^{p_1}\left(\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1\right)}{dt^{p_1}}$$

$$-2(\kappa^3(t))^{(n-1)}=2i\varphi^{(n)}(t)\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1+2i\sum_{p_1=1}^{n-1}{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)$$

$$\left[2i\varphi^{(p_1)}(t)\kappa^3(t)\int_t^0 2\kappa(t_1)dt_1+2i\sum_{p_2=1}^{p_1-1}{}_{p_1-1}C_{p_2}\varphi^{(p_1-p_2)}(t)\frac{d^{p_2}\left(\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1\right)}{dt^{p_2}}\right.$$

$$\left.-2(\kappa^3(t))^{(p_1-1)}\right]-2(\kappa^3(t))^{(n-1)}$$

$$=2i\varphi^{(n)}(t)\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1+(2i)^2\sum_{p_1=1}^{n-1}{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)\varphi^{(p_1)}(t)\kappa^2(t)$$

$$\int_t^0 2\kappa(t_1)dt_1-2(\kappa^3(t))^{(n-1)}-2\sum_{p_1=1}^{n-1}2i\;{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)(\kappa^3(t))^{(p_1-1)}+(2i)^2$$

$$\sum_{p_2=1}^{n-2}\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\;{}_{p_1-1}C_{p_2}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\frac{d^{p_2}\left(\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1\right)}{dt^{p_2}}$$

$$=2i\varphi^{(n)}(t)\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1+\sum_{p_1=1}^{n-1}(2i)^2{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)\varphi^{(p_1)}(t)\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1+\cdots$$

$$+\sum_{p_{n-1}=1}^{1}\sum_{p_{n-2}=p_{n-1}+1}^{2}\cdots\sum_{p_1=p_2+1}^{n-1}(2i)^n{}_{n-1}C_{p_1}\;{}_{p_1-1}C_{p_2}\cdots{}_{p_{n-2}-1}C_{p_{n-1}}\varphi^{(n-p_1)}(t)\varphi^{(p_1-p_2)}(t)\cdots$$

$$\varphi^{(p_{n-2}-p_{n-1})}(t)\varphi^{(p_{n-1})}(t)\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1-2(\kappa^3(t))^{(n-1)}-2$$

$$\sum_{p_1=1}^{n-1}2i\;{}_{n-1}C_{p_1}\varphi^{(n-p_1)}(t)(\kappa^3(t))^{(p_1-1)}-\cdots\quad-2\sum_{p_{n-1}=1}^{1}\sum_{p_{n-2}=p_{n-1}+1}^{2}\cdots$$

$$\sum_{p_1=p_2+1}^{n-1}(2i)^{n-1}{}_{n-1}C_{p_1}\;{}_{p_1-1}C_{p_2}\cdots{}_{p_{n-2}-1}C_{p_{n-1}}\varphi^{(n-p_1)}(t)\cdots\varphi^{(p_{n-2}-p_{n-1})}(t)(\kappa^3(t))'. \tag{10}$$

Daher ist es

$$\left.\frac{d^n\left(\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1\right)}{dt^n}\right|_{t=0}=-2(\kappa^3(0))^{(n-1)}-2\sum_{j=1}^{n-1}\left(\sum_{p_j=1}^{n-j}\sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots\right.$$

$$\sum_{p_1=p_2+1}^{n-1}(2i)^j{}_{n-1}C_{p_1}\;{}_{p_1-1}C_{p_2}\cdots{}_{p_{j-1}-1}C_{p_j}\varphi^{(n-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots$$

$$\left.\varphi^{(p_{j-1}-p_j)}(0)\,(\kappa^3(0))^{(p_j-1)}\right)\qquad(\text{für } n\geqq 2). \tag{11}$$

Aus (9) und (11) gewinnen wir

$$\frac{d^n\left(\kappa^2(t))t\int_t^0 2\kappa(t_1)dt_1\right)}{dt^n}\Bigg|_{t=0} = {}_nC_1\Bigg[-2(\kappa^3(0))^{(n-2)} - 2\sum_{j=1}^{n-2}\Bigg(\sum_{p_j=1}^{n-j-1}\sum_{p_{j-1}=p_j+1}^{n-j}\cdots$$

$$\sum_{p_1=p_2+1}^{n-2}(2i)^j{}_{n-2}C_{p_1\ p_1-1}C_{p_2}\cdots\ \cdots{}_{p_{j-1}-1}C_{p_j}\varphi^{(n-1-p_1)}(0)\,\varphi^{(p_1-p_2)}(0)\cdots$$

$$\varphi^{(p_{j-1}-p_j)}(0)\,(\kappa^3(0))^{(p_j-1)}\Bigg)\Bigg] \quad (\text{für } n\geqq 3) \tag{12}$$

$$= -2{}_nC_1\kappa^3(0)\Bigg\{i3\varphi^{(n-2)}(0) + \sum_{j=1}^{n-3}\Bigg(\sum_{p_j=1}^{n-j-2}\sum_{p_{j-1}=p_j+1}^{n-j-1}\cdots\sum_{p_1=p_2+1}^{n-3}3^{j+1}{}_{n-3}C_{p_1\ p_1-1}C_{p_2}\cdots$$

$${}_{p_{j-1}-1}C_{p_j}i^{j+1}\varphi^{(n-2-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\ \varphi^{(p_{j-1}-p_j)}(0)\varphi^{(p_j)}(0)\Bigg)\Bigg\} - 2{}_nC_1\kappa^3(0)\sum_{j=1}^{n-4}\Bigg(\sum_{p_j=3}^{n-j-1}$$

$$\sum_{p_{j-1}=p_j+1}^{n-j}\cdots\sum_{p_1=p_2+1}^{n-2}(2i)^j{}_{n-2}C_{p_1\ p_1-1}C_{p_2}\cdots\ {}_{p_{j-1}-1}C_{p_j}\varphi^{(n-1-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots$$

$$\varphi^{(p_{j-1}-p_j)}(0)\Bigg[i3\varphi^{(p_j-1)}(0) + \sum_{s=1}^{p_j-2}\Bigg(\sum_{q_s=1}^{p_j-1-s}\sum_{q_{s-1}=q_s+1}^{p_j-s}\cdots\sum_{q_1=q_2+1}^{p_j-2}3^{s+1}{}_{p_j-2}C_{q_1\ q_1-1}C_{q_2}$$

$$\cdots{}_{q_{s-1}-1}C_{q_s}i^{s+1}\varphi^{(p_j-1-q_1)}(0)\varphi^{(q_1-q_2)}(0)\cdots\ \varphi^{(q_{s-1}-q_s)}(0)\varphi^{(q_s)}(0)\Bigg)\Bigg]\Bigg) - 2{}_nC_1\kappa^3(0)$$

$$\sum_{j=1}^{n-2}\Bigg(\sum_{p_j=1}^{1}\sum_{p_{j-1}=2}^{n-j}\cdots\sum_{p_1=p_2+1}^{n-2}(2i)^j{}_{n-2}C_{p_1\ p_1-1}C_{p_2}\cdots\ {}_{p_{j-1}-1}C_{p_j}\varphi^{(n-1-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots$$

$$\varphi^{(p_{j-1}-1)}(0)\Bigg) - 2{}_nC_1\kappa^3(0)\sum_{j=1}^{n-3}\Bigg(\sum_{p_j=2}^{2}\sum_{p_{j-1}=3}^{n-j}\cdots\sum_{p_1=p_2+1}^{n-2}(2i)^j{}_{n-2}C_{p_1\ p_1-1}C_{p_2}\cdots$$

$${}_{p_{j-1}-1}C_{p_j}\varphi^{(n-1-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\ \varphi^{(p_{j-1}-p_j)}3i\varphi'(0)\Bigg) \quad (\text{für } n\geqq 5). \tag{13}$$

Wir setzen die (4), (5), (8) und (13) in (2) ein, so ergibt es sich

$$\Im 12{}_nC_2\kappa^3(0)\Bigg\{i3\varphi^{(n-2)}(0) + \sum_{j=1}^{n-3}\Bigg(\sum_{p_j=1}^{n-j-2}\sum_{p_{j-1}=p_j+1}^{n-j-1}\cdots\sum_{p_1=p_2+1}^{n-3}3^{j+1}{}_{n-3}C_{p_1\ p_1-1}C_{p_2}\cdots$$

$${}_{p_{j-1}-1}C_{p_j}i^{j+1}\varphi^{(n-2-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\ \varphi^{(p_{j-1}-p_j)}(0)\varphi^{(p_j)}(0)\Bigg)\Bigg\} + \Im\kappa(0)A$$

$$\Bigg\{i\varphi^{(n)}(0) + \sum_{j=1}^{n-1}\Bigg(\sum_{p_j=1}^{n-j}\sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1\ p_1-1}C_{p_2}\cdots$$

$${}_{p_{j-1}-1}C_{p_j}i^{j+1}\varphi^{(n-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\ \varphi^{(p_{j-1}-p_j)}(0)\varphi^{(p_j)}(0)\Bigg\} - 4{}_{n-1}C_1\Im\kappa^3(0)$$

$$\Bigg\{i3\varphi^{(n-2)}(0) + \sum_{j=1}^{n-3}\Bigg(\sum_{p_j=1}^{n-j-2}\sum_{p_{j-1}=p_j+1}^{n-j-1}\cdots\sum_{p_1=p_2+1}^{n-3}3^{j+1}{}_{n-3}C_{p_1\ p_1-1}C_{p_2}\cdots$$

$${}_{p_{j-1}-1}C_{p_j}i^{j+1}\varphi^{(n-2-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\ \varphi^{(p_{j-1}-p_j)}(0)\varphi^{(p_j)}(0)\Bigg)\Bigg\} - 4\Im\kappa^3(0)$$

$$\sum_{j=1}^{n-4}\Bigg(\sum_{p_j=4}^{n-j}\sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots\sum_{p_1=p_2+1}^{n-1}i^j{}_{n-1}C_{p_1\ p_1-1}C_{p_2}\ \cdots{}_{p_{j-1}-1}C_{p_j}\varphi^{(n-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots$$

$$\varphi^{(p_{j-1}-p_j)}(0)\Big[i3\varphi^{(p_j-2)}(0)+\sum_{s=1}^{p_j-3}\Big(\sum_{q_s=1}^{p_j-s-2}\ \sum_{q_{s-1}=q_s+1}^{p_j-s-1}\cdots\sum_{q_1=q_2+1}^{p_j-3}3^{s+1}{}_{p_j-3}C_{q_1}\,{}_{q_1-1}C_{q_2}\cdots$$

$${}_{q_{s-1}-1}C_{q_s}i^{s+1}\varphi^{(p_j-2-q_1)}(0)\varphi^{(q_1-q_2)}(0)\cdots\ \varphi^{(q_{s-1}-q_s)}(0)\varphi^{(q_s)}(0)\Big]\Big)-4\Im\kappa^3(0)\sum_{j=1}^{n-2}\Big(\sum_{p_j=2}^{2}\ \sum_{p_{j-1}=3}^{n-j+1}$$

$$\cdots\sum_{p_1=p_2+1}^{n-1}i^j{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots\ {}_{p_{j-1}-1}C_{p_j}\varphi^{(n-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\varphi^{(p_{j-1}-p_j)}(0)\Big)$$

$$-4\Im\kappa^3(0)\sum_{j=1}^{n-3}\Big(\sum_{p_j=3}^{3}\ \sum_{p_{j-1}=4}^{n-j+1}\cdots\sum_{p_j=p_2+1}^{n-1}i^j{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots\ {}_{p_{j-1}-1}C_{p_j}\varphi^{(n-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots$$

$$\varphi^{(p_{j-1}-p_j)}(0)\,3i\varphi'(0)\Big)+{}_nC_1\Im B\kappa^2(0)\Big\{i2\varphi^{(n-1)}(0)+\sum_{j=1}^{n-2}\Big(\sum_{p_j=1}^{n-j-1}\ \sum_{p_{j-1}=p_j+1}^{n-j}\cdots$$

$$\sum_{p_1=p_2+1}^{n-2}2^{j+1}{}_{n-2}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots\ {}_{p_{j-1}-1}C_{p_j}\,i^{j+1}\varphi^{(n-1-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots$$

$$\varphi^{(p_{j-1}-p_j)}(0)\varphi^{(p_j)}(0)\Big)\Big\}+8{}_nC_1\Im\kappa^3(0)\Big\{i3\varphi^{(n-2)}(0)+\sum_{j=1}^{n-3}\Big(\sum_{p_j=1}^{n-j-2}\ \sum_{p_{j-1}=p_j+1}^{n-j-1}\cdots$$

$$\sum_{p_1=p_2+1}^{n-3}3^{j+1}{}_{n-3}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots\ {}_{p_{j-1}-1}C_{p_j}\,i^{j+1}\varphi^{(n-2-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots$$

$$\varphi^{(p_{j-1}-p_j)}(0)\,\varphi^{(p_j)}(0)\Big)\Big\}+8{}_nC_1\Im\kappa^3(0)\sum_{j=1}^{n-4}\Big(\sum_{p_j=3}^{n-j-1}\ \sum_{p_{j-1}=p_j+1}^{n-j}\cdots\sum_{p_1=p_2+1}^{n-2}(2i)^j{}_{n-2}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots$$

$${}_{p_{j-1}-1}C_{p_j}\varphi^{(n-1-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\ \varphi^{(p_{j-1}-p_j)}(0)\Big[3i\varphi^{(p_j-1)}(0)+\sum_{s=1}^{p_j-2}\Big(\sum_{q_s=1}^{p_j-1-s}\ \sum_{q_{s-1}=q_s+1}^{p_j-s}\cdots$$

$$\sum_{q_1=q_2+1}^{p_j-2}3^{s+1}{}_{p_j-2}C_{q_1}\,{}_{q_1-1}C_{q_2}\cdots{}_{q_{s-1}-1}C_{q_s}\,i^{s+1}\varphi^{(p_j-1-q_1)}(0)\varphi^{(q_1-q_2)}(0)\cdots\varphi^{(q_{s-1}-q_s)}(0)\varphi^{(q_s)}(0)\Big)\Big]$$

$$+8{}_nC_1\Im\kappa^3(0)\sum_{j=1}^{n-2}\Big(\sum_{p_j=1}^{1}\ \sum_{p_{j-1}=2}^{n-j}\cdots\ \sum_{p_1=p_2+1}^{n-2}(2i)^j{}_{n-2}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots$$

$${}_{p_{j-1}-1}C_{p_j}\varphi^{(n-1-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\varphi^{(p_{j-1}-p_j)}(0)\Big)+8{}_nC_1\Im\kappa^3(0)\sum_{j=1}^{n-3}\Big(\sum_{p_j=2}^{2}\ \sum_{p_{j-1}=3}^{n-j}\cdots$$

$$\sum_{p_1=p_2+1}^{n-2}(2i)^j{}_{n-2}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots\ {}_{p_{j-1}-1}C_{p_j}\varphi^{(n-1-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots$$

$$\varphi^{(p_{j-1}-p_j)}(0)3i\varphi'(0)\Big)=0. \qquad (14)$$

Diese Formel gilt für $n\geqq 5$. Wenn $n=4$ ist, muss es der Formel (14) an dem Glied $-4\Im\kappa^3(0)\sum_{j=1}^{n-4}\Big(\sum_{p_j=4}^{n-j}\ \sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots\ \sum_{p_1=p_2+1}^{n-1}i^j{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots$

$${}_{p_{j-1}-1}C_{p_j}\varphi^{(n-p_1)}(0)\varphi^{(p_1-p_2)}(0)\cdots\ \varphi^{(p_{j-1}-p_j)}(0)\Big[i3\varphi^{(p_j-2)}(0)+\sum_{s=1}^{p_j-3}\Big(\sum_{q_s=1}^{p_j-s-1}\cdots$$

$$\sum_{q_1=q_2+1}^{p_j-3}3^{s+1}{}_{p_j-3}C_{q_1}\,{}_{q_1-1}C_{q_2}\cdots\ {}_{q_{s-1}-1}C_{q_s}\,i^{s+1}\varphi^{(p_j-2-q_1)}(0)\varphi^{(q_1-q_2)}(0)\cdots$$

$$\varphi^{(q_{s-1}-q_s)}(0)\varphi^{(q_s)}(0)\Big)\Big]\Big)$$

und dem $8_n C_1 \Im \kappa^3(0) \sum_{j=1}^{n-4} \Big(\sum_{p_j=3}^{n-j-1} \sum_{p_{j-1}=p_j+1}^{n-j} \cdots \sum_{p_1=p_2+1}^{n-2} (2i)^j {}_{n-2}C_{p_1} {}_{p_1-1}C_{p_2} \cdots$

$_{p_{j-1}-1}C_{p_j} \varphi^{(n-1-p_1)}(0) \varphi^{(p_1-p_2)}(0) \cdots \varphi^{(p_{j-1}-p_j)}(0) \Big[3i\varphi^{(p_j-1)}(0) + \sum_{s=1}^{p_j-2} \Big(\sum_{q_s=1}^{p_j-1-s} \sum_{q_{s-1}=q_s+1}^{p_1-s} \cdots$

$\sum_{q_1=q_2+1}^{p_j-2} 3^{s+1} {}_{p_j-2}C_{q_1} {}_{q_1-1}C_{q_2} \cdots {}_{q_{s-1}-1}C_{q_s} i^{s+1} \varphi^{(p_j-1-q_1)}(0) \varphi^{(q_1-q_2)}(0) \cdots$

$\varphi^{(q_{s-1}-q_s)}(0) \varphi^{(q_s)}(0) \Big) \Big] \Big)$ fehlen. Wenn $n=3$ ist, besteht es die folgende Formel

$$\Im 12_n C_2 \kappa^3(0) i 3 \varphi^{(n-2)}(0) + \Im \kappa(0) A \Big\{ i\varphi^{(n)}(0) + \sum_{j=1}^{n-1} \Big(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \cdots$$

$$\sum_{p_1=p_2+1}^{n-1} {}_{n-1}C_{p_1} {}_{p_1-1}C_{p_2} \cdots {}_{p_{j-1}-1}C_{p_j} i^{j+1} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \cdots \varphi^{(p_{j-1}-p_j)}(0) \varphi^{(p_j)}(0) \Big) \Big\}$$

$$-4_{n-1} C_1 \Im \kappa^3(0) i 3 \varphi^{(n-2)}(0) - 4 \Im \kappa^3(0) i \, {}_{n-2}C_2 \varphi^{(n-2)}(0) + {}_n C_1 \Im B \kappa^2(0) \Big(i 2 \varphi^{(n-1)}(0) +$$

$$2^2 i^2 \varphi'(0) \Big) + 8_n C_1 \Im \kappa^3(0) i 3 \varphi^{(n-2)}(0) + 8_n C_1 \Im \kappa^3(0) 2 i \varphi'(0) = 0. \tag{15}$$

Wenn $n=2$ ist, so besteht es die Formel

$$\Im 12 \kappa^3(0) + \Im \kappa(0) A \Big(i\varphi''(0) + i^2 \varphi'(0) \Big) - 4_{n-1} C_1 \Im \kappa^3(0) + {}_n C_1 \Im B \kappa^2(0) i 2 \varphi'(0)$$
$$+ 8_n C_1 \Im \kappa^3(0) = 0. \tag{16}$$

Wir setzen nun $\varphi^{(k)}(0) = a_{k,0} + a_{k,1}\varphi'(0) + \cdots + a_{k,k}\varphi'(0)^k = \sum_{j=0}^{k} a_{k,j} \varphi'(0)^j$, [4] so ergibt es sich aus (14), (15) und (16)

$$a_{n,n} = -\Im \kappa(0) A \sum_{j=1}^{n-1} \Big(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \cdots \sum_{p_1=p_2+1}^{n-1} {}_{n-1}C_{p_1} {}_{p_1-1}C_{p_2} \cdots$$

$$_{p_{j-1}-1}C_{p_j} i^{j+1} a_{n-p_1, n-p_1} a_{p_1-p_2, p_1-p_2} \cdots a_{p_{j-1}-p_j, p_{j-1}-p_j} a_{p_j, p_j} / \Re(\kappa(0)A) \Big). \tag{17}$$

Diese Formel gilt für $n \geqq 2$. Wenn j gerade ist, ist es $\Im(\kappa(0)Ai^{j+1}) = -(1)^{\frac{j}{2}} \Re(\kappa(0)A)$. Wenn j ungerade ist, ist es $\Im(\kappa(0)Ai^{j+1}) = (-1)^{\frac{j+1}{2}} \Im(\kappa(0)A) = 0$. Der Einfachheit halber bezeichnen wir von jetzt an die $\kappa(0)$ nur mit κ. Dann ergibt es sich aus (17)

$$a_{n,n} = -\sum_{j=1}^{n-1}{}' (-1)^{\frac{j}{2}} \Big(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \cdots \sum_{p_1=p_2+1}^{n-1} {}_{n-1}C_{p_1} {}_{p_1-1}C_{p_2} \cdots$$

$$_{p_{j-1}-1}C_{p_j} a_{n-p_1, n-p_1} a_{p_1-p_2, p_1-p_2} \cdots a_{p_{j-1}-p_j, p_{j-1}-p_j} a_{p_j, p_j} \Big) = -\sum_{j=1}^{n-1}{}' (-1)^{\frac{j}{2}}$$

$$\Big(\sum_{p_1=j}^{n-1} \sum_{p_2=j-1}^{p_1-1} \cdots \sum_{p_j=1}^{p_{j-1}-1} {}_{n-1}C_{p_1} {}_{p_1-1}C_{p_2} \cdots {}_{p_{j-1}-1}C_{p_j} a_{n-p_1, n-p_1} a_{p_1-p_2, p_1-p_2} \cdots$$

$$a_{p_{j-1}-p_j, p_{j-1}-p_j} a_{p_j, p_j} \Big), \tag{18}$$

---

[4] Diese Darstellung ist nicht notwendig eindeutig.

wo die Summe $\sum_{j=1}^{n-1}{}'$ über alle positiven geraden ganzen Zahlen bis 1 zu $n-1$ zu erstrecken ist.

Wir setzen nun in der Formel (18)

$$\left.\begin{aligned} n-p_1&=r_1\\ p_1-p_2&=r_2\\ &\vdots\\ p_{j-1}-p_j&=r_j\\ p_j&=r_{j+1},\end{aligned}\right\}$$

so gewinnen wir aus (18)

$$a_{n,n}=-\sum_{j=1}^{n-1}{}'(-1)^{\frac{j}{2}}\sum_{r_1=1}\sum_{r_2=1}\cdots\sum_{\substack{r_{j+1}=1\\(r_1+r_2+\cdots+r_{j+1}=n)}}\frac{(n-1)!}{(r_1-1)!(r_2-1)!\cdots(r_{j+1}-1)!}a_{r_1,r_1}a_{r_2,r_2}\cdots$$

$$a_{r_{j+1},r_{j+1}}\prod_{k=1}^{j}\frac{1}{n-r_1-\cdots-r_k}, \tag{19}$$

wo die Summe $\sum_{r_1=1}\sum_{r_2=1}\cdots\sum_{\substack{r_{j+1}=1\\(r_1+\cdots+r_{j+1}=n)}}$ über alle Systeme von positiven ganzen Zahlen $r_1, r_2\cdots, r_{j+1}$ zu erstrecken ist, derart dass $r_1+r_2+\cdots+r_{j+1}=n$ ist.

Anderseits folgt es aus (2)

$$\Re\left[18\kappa^3(t)t^2+A\kappa(t)+2\kappa(t)\int_t^0 2\kappa^2(t_1)t_1dt_1+2\kappa^2(t)tB-8\kappa^2(t)t\int_t^0 2\kappa(t_1)dt_1\right]\varphi'(t)$$
$$+\Im\left[16\kappa^3(t)t+B\kappa^2(t)-4\kappa^2(t)\int_t^0 2\kappa(t_1)dt_1\right]=0.$$

Daher ist für $t=0$

$$\varphi'(0)=-\frac{\Im B\kappa^2(0)}{\Re A\kappa(0)}=-\frac{\Im B\kappa^2}{\Re A\kappa}. \tag{20}$$

Da $\varphi'(0)=\varphi'(0)$ ist, können wir setzen $a_{1,1}=1$ und $a_{1,0}=0$.

Erstens behandeln wir den Fall, wo das Suffix n von $a_{n,n}$ ungerade ist. Wenn $j=n-1$ ist, so müssen alle $r_1, r_2\cdots, r_{j+1}$ gleich 1 sein, da $r_1+r_2+\cdots+r_{j+1}=n$ ist. Daher ist

$$\frac{(n-1)!}{(r_1-1)!(r_2-1)!\cdots(r_{j+1}-1)!}a_{r_1,r_1}a_{r_2,r_2}\cdots a_{r_{j+1},r_{j+1}}\prod_{k=1}^{j}\frac{1}{n-r_1-\cdots-r_k}$$
$$=(n-1)!\prod_{k=1}^{n-1}\frac{1}{n-k},$$

wenn $j=n-1$ ist. Daraus und aus (19) folgt es ohne weiteres

$$a_{n,n}=-(-1)^{\frac{n-1}{2}}(n-1)!\prod_{k=1}^{n-1}\frac{1}{n-k}-\sum_{j=1}^{n-2}{}'(-1)^{\frac{j}{2}}\sum_{r_1=1}\sum_{r_2=1}\cdots$$
$$\sum_{\substack{r_{j+1}=1\\(r_1+\cdots+r_{j+1}=n)}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!}a_{r_1,r_1}\cdots a_{r_{j+1},r_{j+1}}\prod_{k=1}^{j}\frac{1}{n-r_1-\cdots-r_k}. \tag{21}$$

Wenn $j \leqq n-2$ ist, so muss mindestens eine unter $r_1, r_2 \cdots, r_{j+1}$ grösser als 1 sein. Wir nehmen nun an, dass $r_{j+1} > 1$ ist. Es ist dann

$$(-1)^{\frac{j}{2}} \sum_{\substack{r_1=1\, r_2=1 \\ (r_1+r_2+\cdots+r_{j+1}=n,\, r_{j+1}>1)}} \sum \cdots \sum_{r_j=1} \sum_{r_{j+1}>1} \frac{(n-1)!}{(r_1-1)!\,(r_2-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1} \cdots$$

$$a_{r_{j+1}, r_{j+1}} \prod_{k=1}^{j} \frac{1}{n-r_1-\cdots-r_k} = (-1)^{\frac{j}{2}} \sum_{\substack{r_1=1\, r_2=1 \\ (r_1+\cdots+r_j=n-r_{j+1})}} \sum \cdots$$

$$\sum_{r_j=1} \frac{(n-1)!}{(r_1-1)!\,(r_2-1)! \cdots (r_j-1)!} a_{r_1, r_1} a_{r_2, r_2} \cdots a_{r_j, r_j} \times$$

$$-\sum_{j_2=1}^{r_{j+1}-1}{}' (-1)^{\frac{j_2}{2}} \sum_{\substack{s_1=1\, s_2=1 \\ (s_1+s_2+\cdots+s_{j_2+1}=r_{j+1})}} \sum \cdots \sum_{s_{j_2+1}=1} \frac{1}{(s_1-1)! \cdots (s_{j_2+1}-1)!} a_{s_1, s_1} a_{s_2, s_2} \cdots$$

$$a_{s_{j_2+1}, s_{j_2+1}} \prod_{k_2=1}^{j_2} \frac{1}{r_{j+1}-s_1-\cdots-s_{k_2}} \prod_{k_1=1}^{j} \frac{1}{n-r_1-\cdots-r_{k_1}}.$$

Daher ist es

$$-\sum_{j=1}^{n-2}{}'(-1)^{\frac{j}{2}} \sum_{\substack{r_1=1\, r_2=1 \\ (r_1+r_2+\cdots+r_{j+1}=n,\, r_{j+1}>1)}} \sum \cdots \sum_{r_j=1} \sum_{r_{j+1}>1} \frac{(n-1)!}{(r_1-1)!\,(r_2-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1} \cdots$$

$$a_{r_{j+1}, r_{j+1}} \prod_{k=1}^{j} \frac{1}{n-r_1-\cdots-r_k} = (-1)^2 \sum_{j_2=2}^{n-1}{}'(-1)^{\frac{j_2}{2}} \sum_{\substack{r_1=1\, r_2=1 \\ (r_1+\cdots\cdots+r_{j_2+1}=n)}} \sum \cdots \sum_{r_{j_2+1}=1}$$

$$\frac{(n-1)!}{(r_1-1)!\,(r_2-1)! \cdots (r_{j_2+1}-1)!} a_{r_1, r_1} a_{r_2, r_2} \cdots a_{r_{j_2+1}, r_{j_2+1}} \sum_{j_1=1}^{j_2-1}{}' \prod_{k_1=1}^{j_1} \frac{1}{n-r_1-\cdots-r_{k_1}}$$

$$\prod_{k_2=j_1+1}^{j_2} \frac{1}{r_{k_2+1}+\cdots+r_{j_2+1}}.\text{[5]} \tag{22}$$

Wir nehmen zweitens an, dass in der Formel (21) $r_j > 1$ und $r_{j+1} = 1$ ist. Es ist dann

$$(-1)^{\frac{j}{2}} \sum_{\substack{r_1=1\, r_2=1 \\ (r_1+r_2+\cdots+r_j+r_{j+1}=n,\, r_j>1,\, r_{j+1}=1)}} \sum \cdots \sum_{r_j>1} \sum_{r_{j+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1} a_{r_2, r_2} \cdots$$

$$a_{r_{j+1}, r_{j+1}} \prod_{k=1}^{j} \frac{1}{n-r_1-\cdots-r_k} = (-1)^{\frac{j}{2}} \sum_{\substack{r_1=1\, r_2=1 \\ (r_1+\cdots+r_{j-1}=n-1-r_j)}} \sum \cdots \sum_{r_{j-1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j-1}-1)!} a_{r_1, r_1}$$

$$\cdots a_{r_{j-1}, r_{j-1}} \times (-1) \sum_{j_2=1}^{r_j-1}{}'(-1)^{\frac{j_2}{2}} \sum_{\substack{s_1=1\, s_2=1 \\ (s_1+s_2+\cdots+s_{j_2+1}=r_j)}} \sum \cdots \sum_{s_{j_2+1}=1} \frac{1}{(s_1-1)! \cdots (s_{j_2+1}-1)!} a_{s_1, s_1} a_{s_2, s_2} \cdots$$

---

[5] Das Suffix j_2 beginnt in der Tat mit 4. Wenn $j_2 = 2$ ist, so ist $\sum_{j_1=1}^{j_2-1}{}' = 0$.

$$a_{s_{j_2+1}, s_{j_2+1}} \prod_{k_1=1}^{j} \frac{1}{n-r_1\cdots -r_{k_1}} \prod_{k_2=1}^{j_2} \frac{1}{r_j-s_1-\cdots-s_{k_2}}.$$

Daher ist es

$$-\sum_{j=1}^{n-2}{}'(-1)^{\frac{j}{2}} \underset{(r_1+r_2+\cdots+r_{j+1}=n,\ r_j>1,\ r_{j+1}=1)}{\sum_{r_1=1}\sum_{r_2=1}\cdots\sum_{r_j>1}\sum_{r_{j+1}=1}} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!} a_{r_1,r_1}a_{r_2,r_2}\cdots$$

$$a_{r_{j+1},r_{j+1}} \prod_{k=1}^{j} \frac{1}{n-r_1-\cdots-r_k} = (-1)^2 \sum_{j_2=2}^{n-1}{}'(-1)^{\frac{j_2}{2}} \underset{(r_1+r_2+\cdots+r_{j_2}=n-1)}{\sum_{r_1=1}\sum_{r_2=1}\cdots\sum_{r_{j_2}=1}}$$

$$\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2}-1)!} a_{r_1,r_1}a_{r_2,r_2}\cdots a_{r_{j_2},r_{j_2}} \sum_{j_1=1}^{j_2-1}{}' \prod_{k_1=1}^{j_1-1} \frac{1}{1+r_{k_1+1}+\cdots+r_{j_2}}$$

$$\prod_{k_2=j_1}^{j_2-1} \frac{1}{r_{k_2+1}+\cdots+r_{j_2}}. \tag{23}$$

Wir nehmen drittens an, dass in der Formel (21) $r_{j-1}>1$ und $r_j=r_{j+1}=1$ ist. Es ist dann

$$(-1)^{\frac{j}{2}} \underset{(r_1+r_2+\cdots+r_{j+1}=n,\ r_{j-1}>1,\ r_j=r_{j+1}=1)}{\sum_{r_1=1}\sum_{r_2=1}\cdots\sum_{r_{j-1}>1}\sum_{r_j=1}\sum_{r_{j+1}=1}} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!} a_{r_1,r_1}a_{r_2,r_2}\cdots$$

$$a_{r_{j+1},r_{j+1}} \prod_{k=1}^{j} \frac{1}{n-r_1-\cdots-r_k} = (-)^{\frac{j}{2}} \underset{(r_1+r_2+\cdots+r_{j-2}=n-2-r_{j-1})}{\sum_{r_1=1}\sum_{r_2=1}\cdots\sum_{r_{j-2}=1}} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j-2}-1)!} a_{r_1,r_1}\cdots$$

$$a_{r_{j-2},r_{j-2}} \times (-1) \sum_{j_2=1}^{r_{j-1}-1}{}' (-1)^{\frac{j_2}{2}} \underset{(s_1+\cdots+s_{j_2+1}=r_{j-1})}{\sum_{s_1=1}\cdots\sum_{s_{j_2+1}=1}} \frac{1}{(s_1-1)!\cdots(s_{j_2+1}-1)!} a_{s_1,s_1}\cdots$$

$$a_{s_{j_2+1},s_{j_2+1}} \prod_{k_1=1}^{j} \frac{1}{n-r_1-\cdots-r_{k_1}} \prod_{k_2=1}^{j_2} \frac{1}{r_{j-1}-s_1-\cdots-s_{k_2}}.$$

Daher ist es

$$-\sum_{j=2}^{n-2}{}'(-1)^{\frac{j}{2}} \underset{(r_1+r_2+\cdots+r_{j+1}=n,\ r_{j-1}>1,\ r_j=r_{j+1}=1)}{\sum_{r_1=1}\cdots\sum_{r_{j-1}>1}\sum_{r_j=1}\sum_{r_{j+1}=1}} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!} a_{r_1,r_1}\cdots$$

$$a_{r_{j+1},r_{j+1}} \prod_{k_1=1}^{j} \frac{1}{n-r_1-\cdots-r_k} = (-1)^2 \sum_{j_2=4}^{n-1}{}'(-1)^{\frac{j_2}{2}} \underset{(r_1+r_2+\cdots+r_{j_2-1}=n-2)}{\sum_{r_1=1}\sum_{r_2=1}\cdots\sum_{r_{j_2-1}=1}}$$

$$\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2-1}-1)!} a_{r_1,r_1}\cdots a_{r_{j_2-1},r_{j_2-1}} \sum_{j_1=2}^{j_2-1}{}' \alpha_2^{1,1} \prod_{k_2=j_1-1}^{j_2-2} \frac{1}{r_{k_2+1}+\cdots+r_{j_2-1}}, \tag{24}$$

wo

$$\alpha_2^{1,1} = \begin{cases} \dfrac{1}{2!} & (\text{wenn } j_1=2 \text{ ist}) \\ \dfrac{1}{2!} \displaystyle\prod_{k_1=1}^{j_1-2} \frac{1}{2+r_{k_1+1}+\cdots+r_{j_2-1}} & (\text{wenn } j_1>2 \text{ ist}) \end{cases} \quad \text{ist.}$$

Dieses Verfahren wiederholen wir, so erhalten wir endlich

$$(-1)^{\frac{j}{2}}\sum_{r_1>1}\sum_{r_2=1}\cdots\sum_{r_{j+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!}a_{r_1,r_1}\cdots a_{r_{j+1},r_{j+1}}\prod_{k=1}^{j}$$
$$(r_1+r_2+\cdots+r_{j+1}=n,\ r_2=\cdots=r_{j+1}=1,\ r_1>1)$$

$$\frac{1}{n-r_1-\cdots-r_k}=(-1)^{\frac{j}{2}}\sum_{r_1=n-j}^{n-j}\frac{(n-1)!}{(r_1-1)!}\times(-1)\sum_{j_2=1}^{r_1-1}{}'(-1)^{\frac{j_2}{2}}\sum_{s_1=1}\cdots$$
$$(s_1+\cdots+s_{j_2+1}=r_1)$$

$$\sum_{s_{j_2+1}=1}\frac{(r_1-1)!}{(s_1-1)!\cdots(s_{j_2+1}-1)!}a_{s_1,s_1}\cdots a_{s_{j_2+1},s_{j_2+1}}\prod_{k_1=1}^{j}\frac{1}{n-r_1-\cdots-r_{k_1}}\prod_{k_2=1}^{j_2}$$
$$\frac{1}{r_1-s_1-\cdots-s_{k_2}}.$$

Daher ist es

$$-\sum_{j=n-3}^{n-2}{}'(-1)^{\frac{j}{2}}\sum_{r_1>1}\sum_{r_2=1}\cdots\sum_{r_{j+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!}a_{r_1,r_1}\cdots$$
$$(r_1+\cdots+r_{j+1}=n,\ r_1>1,\ r_2=\cdots\cdots=r_{j+1}=1)$$

$$a_{r_{j+1},r_{j+1}}\prod_{k=1}^{j}\frac{1}{n-r_1-\cdots-r_k}=(-1)^2\sum_{j_2=n-2}^{n-1}{}'(-1)^{\frac{j_2}{2}}\sum_{r_1=1}\sum_{r_2=1}\sum_{r_3=1}$$
$$(r_1+r_2+r_3=3)$$

$$\frac{(n-1)!}{(r_1-1)!\,(r_2-1)!\,(r_3-1)!}a_{r_1,r_1}a_{r_2,r_2}a_{r_3,r_3}\frac{1}{(n-3)!}\frac{1}{(r_2+r_3)r_3}. \tag{25}$$

Aus (21), (22), (23), (24) und (25) gewinnen wir sofort

$$a_{n,n}=-(-1)^{\frac{n-1}{2}}\frac{(n-1)!}{(n-1)!}+(-1)^2\sum_{j_2=2}^{n-1}{}'(-1)^{\frac{j_2}{2}}\sum_{r_1=1}\cdots\sum_{r_{j_2+1}=1}$$
$$(r_1+\cdots\cdots+r_{j_2+1}=n)$$

$$\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2+1}-1)!}a_{r_1,r_1}\cdots a_{r_{j_2+1},r_{j_2+1}}\sum_{j_1=1}^{j_2-1}{}'\prod_{k_1=1}^{j_1}\frac{1}{r_{k_1+1}+\cdots+r_{j_2+1}}\prod_{k_2=j_1+1}^{j_2}$$

$$\frac{1}{r_{k_2+1}+\cdots+r_{j_2+1}}+(-1)^2\sum_{p_1=1}^{n-3}\Bigg(\sum_{j_2=p_1+2}^{n-1}{}'(-1)^{\frac{j_2}{2}}\sum_{r_1=1}\cdots\sum_{r_{j_2-p_1+1}=1}$$
$$(r_1+r_2+\cdots+r_{j_2-p_1+1}=n-p_1)$$

$$\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2-p_1+1}-1)!}a_{r_1,r_1}\cdots a_{r_{j_2-p_1+1},r_{j_2-p_1+1}}\sum_{j_1=p_1}^{j_2-1}{}'\alpha_{p_1}^{1,1}$$

$$\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots+r_{j_2-p_1+1}}=-(-1)^{\frac{n-1}{2}}\frac{(n-1)!}{(n-1)!}+(-1)^2\sum_{p_1=0}^{n-3}\Bigg(\sum_{j_2=q_1}^{n-1}{}'(-1)^{\frac{j_2}{2}}$$

$$\sum_{r_1=1}\sum_{r_2=1}\cdots\sum_{r_{j_2-p_1+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2-p_1+1}-1)!}a_{r_1,r_1}\cdots a_{r_{j_2-p_1+1},r_{j_2-p_1+1}}$$
$$(r_1+r_2+\cdots+r_{j_2-p_1+1}=n-p_1)$$

$$\sum_{j_1=q_1}^{j_2-1}{}'\alpha_{p_1}^{1,1}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots+r_{j_2-p_1+1}}, \tag{26}$$

wo $q_1=\text{Max.}\,(p_1+2,\,4)$ und $q_1'=\text{Max.}\,(p_1,\,1)$ und

$$\alpha_{p_1}^{1,1}=\begin{cases}\dfrac{1}{p_1!} & (\text{wenn } j_1=p_1 \text{ ist})\\[2mm] \dfrac{1}{p_1!}\prod\limits_{k_1=1}^{j_1-p_1}\dfrac{1}{p_1+r_{k_1+1}+\cdots\ +r_{j_2-p_1+1}} & (\text{wenn } j_1>p_1 \text{ ist})\end{cases}$$

Wenn in der Formel (26) $j_2=n-1$ ist, so müssen alle $r_1, r_2 \cdots\ , r_{j_2-p_1+1}$ gleich 1 sein, da $r_1+\cdots\ +r_{j_2-p_1+1}=r_1+r_2+\cdots+r_{n-p_1}=n-p_1$ ist. Es ist aber

$$\prod_{k_1=1}^{j_1-p_1}\frac{1}{p_1+r_{k_1+1}+\cdots\ +r_{j_2-p_1+1}}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots\ +r_{j_2-p_1+1}}$$
$$=\frac{(p_1+j_2-j_1)!}{j_2!}\,\frac{1}{(j_2-j_1)!}$$
$$=\frac{(j_2-j_1+p_1)(j_2-j_1+p_1-1)\cdots\ (j_2-j_1+1)}{j_2!},\quad \text{wenn } j_1>p_1 \text{ ist.} \tag{27}$$

Daher ist

$$\frac{1}{p_1!}\prod_{k_1=1}^{j_1-p_1}\frac{1}{p_1+r_{k_1+1}+\cdots\ +r_{j_2-p_1+1}}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots\ +r_{j_2-p_1+1}}$$
$$=\frac{1}{j_2!}\,{}_{j_2-j_1+p_1}C_{p_1}\,.$$

Wenn $j_1=p_1$ ist, ist es

$$\frac{1}{p_1!}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots\ +r_{j_2-p_1+1}}=\frac{1}{p_1!}\,\frac{1}{(j_2-p_1)!}=\frac{1}{j_2!}\,{}_{j_2}C_{p_1}\,. \tag{28}$$

Aus (26), (27) und (28) folgt es sofort

$$a_{n,n}=-(-1)^{\frac{n-1}{2}}\frac{(n-1)!}{(n-1)!}+(-1)^2(-1)^{\frac{n-1}{2}}\sum_{p_1=0}^{n-3}\ \sideset{}{'}\sum_{j_1=q_1'}^{n-2}(n-1)!\ \beta_{p_1}^{1,1}\beta_{p_1}^{1,2}+(-1)^2$$
$$\sum_{p_1=0}^{n-3}\Bigg(\sideset{}{'}\sum_{j_2=q_1}^{n-2}(-1)^{\frac{j_2}{2}}\underset{(r_1+\cdots\ +r_{j_2-p_1+1}=n-p_1)}{\sum_{r_1=1}\cdots\ \sum_{r_{j_2-p_1+1}=1}}\frac{(n-1)!}{(r_1-1)!\cdots\ (r_{j_2-p_1+1}-1)!}a_{r_1,r_1}\cdots$$
$$a_{r_{j_2-p_1+1},r_{j_2-p_1+1}}\sideset{}{'}\sum_{j_1=q_1}^{j_2-1}\alpha_{p_1}^{1,1}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots\ +r_{j_2-p_1+1}}=-(-1)^{\frac{n-1}{2}}\frac{(n-1)!}{(n-1)!}$$
$$+(-1)^2(-1)^{\frac{n-1}{2}}\sum_{p_1=0}^{n-3}\ \sideset{}{'}\sum_{j_1=q_1}^{n-2}{}_{n-1-j_1+p_1}C_{p_1}+(-1)^2\sum_{p_1=0}^{n-3}\Bigg(\sideset{}{'}\sum_{j_2=q_1}^{n-2}(-1)^{\frac{j_2}{2}}\underset{(r_1+\cdots+r_{j_2-p_1+1}=n-p_1)}{\sum_{r_1=1}\cdots\ \sum_{r_{j_2-p_1+1}=1}}$$
$$\frac{(n-1)!}{(r_1-1)!\cdots\ (r_{j_2-p_1+1}-1)!}a_{r_1,r_1}\cdots\ a_{r_{j_2-p_1+1},r_{j_2-p_1+1}}\sideset{}{'}\sum_{j_1=q_1}^{j_2-1}\alpha_{p_1}^{1,1}$$

$$\prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1}+\cdots+r_{j_2-p_1+1}}, \tag{29}$$

wo

$$\beta_{p_1}^{1,1}=\begin{cases} \dfrac{1}{p_1!} & (\text{wenn } j_1=p_1 \text{ ist}) \\ \dfrac{1}{p_1!}\dfrac{(n-1-j_1+p_1)!}{(n-1)!} & (\text{wenn } j_1>p_1 \text{ ist}) \end{cases} \quad \text{und } \beta_{p_1}^{1,2}=\frac{1}{(n-1-j_1)!} \text{ ist.}$$

Wenn in der Formel (26) $j_2<n-1$ ist, so muss mindestens eine unter $r_1, r_2\cdots, r_{j_2-p_1+1}$ grösser als 1 sein. Wir nehmen erstens an, dass in der Formel (29) $r_{j_2-p_1+1}>1$ ist. Es ist dann

$$(-1)^{\frac{j_2}{2}}\sum_{r_1=1}\cdots\sum_{\substack{r_{j_2-p_1+1}>1 \\ (r_1+\cdots+r_{j_2-p_1+1}=n-p_1,\, r_{j_2-p_1+1}>1)}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2-p_1+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_2-p_1+1},r_{j_2-p_1+1}}\sum_{j_1=q_1}^{j_2-1}{}' \alpha_{p_1}^{1,1}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots+r_{j_2-p_1+1}}=(-1)^{\frac{j_2}{2}}$$

$$\sum_{r_1=1}\cdots\sum_{\substack{r_{j_2-p_1}=1 \\ (r_1+\cdots+r_{j_2-p_1}=n-p_1-r_{j_2-p_1+1})}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2-p_1}-1)!}a_{r_1,r_1}\cdots\ a_{r_{j_2-p_1},r_{j_2-p_1}}\times(-1)$$

$$\sum_{j_3=1}^{r_{j_2-p_1+1}-1}{}'(-1)^{\frac{j_3}{2}}\sum_{s_1=1}\sum_{s_2=1}\cdots\sum_{\substack{s_{j_3+1}=1 \\ (s_1+s_2+\cdots+s_{j_3+1}=r_{j_2-p_1+1})}}\frac{1}{(s_1-1)!\cdots(s_{j_3+1}-1)!}a_{s_1,s_1}\cdots\ a_{s_{j_3+1},s_{j_3+1}}$$

$$\prod_{k_3=1}^{j_3}\frac{1}{r_{j_2-p_1+1}-s_1-\cdots-s_{k_3}}\sum_{j_1=q_1}^{j_2-1}{}'\alpha_{p_1}^{1,1}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots+r_{j_2-p_1+1}}.$$

Daraus ergibt es sich ohne weiteres

$$\sum_{j_2=q_1}^{n-2}{}'(-1)^{\frac{j_2}{2}}\sum_{r_1=1}\cdots\sum_{\substack{r_{j_2-p_1+1}>1 \\ (r_1+\cdots+r_{j_2-p_1+1}=n-p_1,\, r_{j_2-p_1+1}>1)}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2-p_1+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_2-p_1+1},r_{j_2-p_1+1}}\sum_{j_1=q_1}^{j_2-1}{}'\alpha_{p_1}^{1,1}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots+r_{j_2-p_1+1}}=-\sum_{j_3=q_1+2}^{n-1}{}'(-1)^{\frac{j_3}{2}}$$

$$\sum_{r_1=1}\cdots\sum_{\substack{r_{j_3-p_1+1}=1 \\ (r_1+\cdots+r_{j_3-p_1+1}=n-p_1)}}\frac{(n-1)!}{(r_1-1)\cdots(r_{j_3-p_1+1}-1)!}a_{r_1,r_1}\cdots\ a_{r_{j_3-p_1+1},r_{j_3-p_1+1}}\sum_{j_2=q_1}^{j_3-1}{}'\sum_{j_1=q_1}^{j_2-1}{}'$$

$$\alpha_{p_1,0}^{2,1}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots+r_{j_3-p_1+1}}\prod_{k_3=j_2-p_1+1}^{j_3-p_1}\frac{1}{r_{k_3+1}+\cdots+r_{j_3-p_1+1}}, \tag{30}$$

$$\text{wo } \alpha^{2,1}_{p_1,0}=\begin{cases}\dfrac{1}{p_1!} & (\text{wenn } j_1=p_1 \text{ ist})\\ \dfrac{1}{p_1!}\prod_{k_1=1}^{j_1-p_1}\dfrac{1}{p_1+r_{k_1+1}+\cdots+r_{j_3-p_1+1}} & (\text{wenn } j_1>p_1 \text{ ist})\end{cases} \text{ ist.}$$

Wir nehmen zweitens an, dass in der Formel (29) $r_{j_2-p_1}>1$ und $r_{j_2-p_1+1}=1$ ist. Es ist dann

$$(-1)^{\frac{j_2}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_2-p_1+1}=n-p_1,\, r_{j_2-p_1}>1,\, r_{j_2-p_1+1}=1)}}\cdots\sum_{r_{j_2-p_1}>1}\sum_{r_{j_2-p_1+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2-p_1+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_2-p_1+1},r_{j_2-p_1+1}}\sum_{j_1=q_1}^{j_2-1}{}'\alpha^{1,1}_{p_1}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots+r_{j_2-p_1+1}}=(-1)^{\frac{j_2}{2}}$$

$$\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_2-p_1-1}=n-p_1-1-r_{j_2-p_1})}}\cdots\sum_{r_{j_2-p_1-1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2-p_1-1}-1)!}a_{r_1,r_1}\cdots a_{r_{j_2-p_1-1},r_{j_2-p_1-1}}\times(-1)$$

$$\sum_{j_3=1}^{r_{j_2-p_1}-1}{}'(-1)^{\frac{j_3}{2}}\sum_{\substack{s_1=1\\(s_1+\cdots+s_{j_3+1}=r_{j_2-p_1})}}\cdots\sum_{s_{j_3+1}=1}\frac{1}{(s_1-1)!\cdots(s_{j_3+1}-1)!}a_{s_1,s_1}\cdots a_{s_{j_3+1},s_{j_3+1}}\prod_{k_3=1}^{j_3}$$

$$\frac{1}{r_{j_2-p_1}-s_1-\cdots-s_{k_3}}\sum_{j_1=q_1}^{j_2-1}{}'\alpha^{1,1}_{p_1}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots+r_{j_2-p_1+1}}.$$

Daher ist es

$$\sum_{j_2=q_1}^{n-2}{}'(-1)^{\frac{j_2}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_2-p_1+1}=n-p_1,\ r_{j_2-p_1}>1,\ r_{j_2-p_1+1}=1)}}\cdots\sum_{r_{j_2-p_1}>1}\sum_{r_{j_2-p_1+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2-p_1+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_2-p_1+1},r_{j_2-p_1+1}}\sum_{j_1=q_1}^{j_2-1}{}'\alpha^{1,1}_{p_1}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots+r_{j_2-p_1+1}}=-\sum_{j_3=q_1+2}^{n-1}{}'(-1)^{\frac{j_3}{2}}$$

$$\sum_{\substack{r_1=1\\(r_1+r_2+\cdots+r_{j_3-p_1}=n-p_1-1)}}\sum_{r_2=1}\cdots\sum_{r_{j_3-p_1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3-p_1}-1)!}a_{r_1,r_1}\cdots a_{r_{j_3-p_1},r_{j_3-p_1}}\sum_{j_2=q_1}^{j_3-1}{}'\sum_{j_1=q_1}^{j_2-1}{}'$$

$$\alpha^{2,1}_{p_1,1}\alpha^{2,2}_{p_1,1}\prod_{k_3=j_2-p_1}^{j_3-p_1-1}\frac{1}{r_{k_3+1}+\cdots\cdots+r_{j_3-p_1}},\tag{31}$$

$$\alpha^{2,1}_{p_1,1}=\begin{cases}\dfrac{1}{p_1!} & (\text{wenn } j_1=p_1 \text{ ist})\\ \dfrac{1}{p_1!}\prod_{k_1=1}^{j_1-p_1}\dfrac{1}{p_1+1+r_{k_1+1}+\cdots+r_{j_3-p_1}} & (\text{wenn } j_1>p_1 \text{ ist})\end{cases} \text{ und}$$

$$\alpha^{2,2}_{p_1,1}=\prod_{k_2=j_1-p_1+1}^{j_2-p_1-1}\frac{1}{1+r_{k_2+1}+\cdots+r_{j_3-p_1}} \text{ ist.}$$

Dieses Verfahren setzen wir fort, so gewinnen wir

$$\sum_{j_2=q_1}^{n-2}{}'(-1)^{\frac{j_2}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_2-p_1+1}=n-1)}}\cdots\sum_{r_{j_2-p_1+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2-p_1+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_2-p_1+1},r_{j_2-p_1+1}}\sum_{j_1=q_1}^{j_2-1}{}',\alpha_{p_1}^{1,1}\prod_{k_2=j_1-p_1+1}^{j_2-p_1}\frac{1}{r_{k_2+1}+\cdots+r_{j_2-p_1+1}}$$

$$=-\sum_{p_2=0}^{n-3}\sum_{j_3=q_2}^{n-1}{}'(-1)^{\frac{j_3}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_3-p_1-p_2+1}=n-p_1-p_2)}}\cdots\sum_{r_{j_3-p_1-p_2+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3-p_1-p_2+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_3-p_1-p_2+1},r_{j_3-p_1-p_2+1}}\sum_{j_2=q_2}^{j_3-1}{}',\sum_{j_1=q_1}^{j_2-1}{}',\alpha_{p_1,p_2}^{2,1}\alpha_{p_1,p_2}^{2,2}\alpha_{p_1,p_2}^{2,3}, \tag{32}$$

wo

$$\alpha_{p_1,p_2}^{2,1}=\begin{cases}\dfrac{1}{p_1!} & (\text{wenn } j_1=p_1 \text{ ist})\\[2mm] \dfrac{1}{p_1!}\prod\limits_{k_1=1}^{j_1-p_1}\dfrac{1}{p_1+p_2+r_{k_1+1}+\cdots+r_{j_3-p_1-p_2+1}} & (\text{wenn } j_1>p_1 \text{ und } j_1-p_1+1\leqq j_2-p_1-p_2+1 \text{ ist})\\[2mm] \dfrac{1}{p_1!}\dfrac{(p_1+j_2-j_1)!}{(p_1+p_2)!}\prod\limits_{k_1=1}^{j_2-p_1-p_2}\dfrac{1}{p_1+p_2+r_{k_1+1}+\cdots+r_{j_3-p_1-p_2+1}} & (\text{wenn } j_1>p_1 \text{ und } j_1-p_1+1>j_2-p_1-p_2+1 \text{ ist})\end{cases}$$ und

$$\alpha_{p_1,p_2}^{2,2}=\begin{cases}\dfrac{1}{(j_2-j_1)!} & (\text{wenn } j_2-p_1-p_2+1\leqq j_1-p_1+1 \text{ ist})\\[2mm] \dfrac{1}{p_2!}\prod\limits_{k_2=j_1-p_1+1}^{j_2-p_1-p_2}\dfrac{1}{p_2+r_{k_2+1}+\cdots+r_{j_3-p_1-p_2+1}} & (\text{wenn } j_2-p_1-p_2+1>j_1-p_1+1 \text{ ist})\end{cases}$$ und

$$\alpha_{p_1,p_2}^{2,3}=\prod_{k_3=j_2-p_1-p_2+1}^{j_3-p_1-p_2}\frac{1}{r_{k_3+1}+\cdots+r_{j_3-p_1-p_2+1}}$$ und

$q_2=\mathrm{Max.}(4,\ p_1+4,\ p_1+p_2+2)$ und
$q_2'=\mathrm{Max.}(4,\ p_1+2,\ p_1+p_2)$ ist.

Aus (32) und (29) folgt es

$$a_{n,n}=-(-)^{\frac{n-1}{2}}\frac{(n-1)!}{(n-1)!}+(-1)^2(-1)^{\frac{n-1}{2}}\sum_{p_1=0}^{n-3}\sum_{j_1=q_1'}^{n-2}{}'(n-1)!\,\beta_{p_1}^{1,1}\beta_{p_1}^{1,2}+(-1)^3$$

$$\sum_{p_1=0}^{n-3}\sum_{p_2=0}^{n-3}\sum_{j_3=q_2}^{n-1}{}'(-1)^{\frac{j_3}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_3-p_1-p_2+1}=n-p_1-p_2)}}\sum_{r_2=1}\cdots\sum_{r_{j_3-p_1-p_2+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3-p_1-p_2+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_3-p_1-p_2+1},r_{j_3-p_1-p_2+1}}\sum_{j_2=q_2'}^{j_3-1}{}'\sum_{j_1=q_1'}^{j_2-1}{}'\alpha_{p_1,p_2}^{2,1}\alpha_{p_1,p_2}^{2,2}\alpha_{p_1,p_2}^{2,3}. \tag{33}$$

Wenn $j_3=n-1$ ist, so müssen alle $r_1, r_2, \cdots, r_{j_3-p_1-p_2+1}$ gleich 1 sein, da $r_1+r_2+\cdots+r_{j_3-p_1-p_2+1}=n-p_1-p_2$ ist. Daher ist für $j_3=n-1$

$$\alpha^{2,1}_{p_1,p_2}=\beta^{2,1}_{p_1,p_2}=\begin{cases}\dfrac{1}{p_1!} & (\text{wenn } j_1=p_1 \text{ ist})\\[2mm] \dfrac{1}{p_1!}\dfrac{(p_1+j_3-j_1)!}{(n-1)!} & (\text{wenn } j_1>p_1 \text{ und } j_1-p_1+1\leqq j_2-p_1-p_2+1 \text{ ist})\\[2mm] \dfrac{1}{p_1!}\dfrac{(p_1+j_2-j_1)!}{(p_1+p_2)!}\dfrac{(j_3-j_2+p_1+p_2)!}{(n-1)!} & (\text{wenn } j_1>p_1 \text{ und } j_1-p_1+1>j_2-p_1-p_2+1 \text{ ist})\end{cases}$$

$$\alpha^{2,2}_{p_1,p_2}=\beta^{2,2}_{p_1,p_2}=\begin{cases}\dfrac{1}{(j_2-j_1)!} & (\text{wenn } j_2-p_1-p_2+1\leqq j_1-p_1+1 \text{ ist})\\[2mm] \dfrac{1}{p_2!}\dfrac{(p_2+j_3-j_2)!}{(n-1-j_1)!} & (\text{wenn } j_2-p_1-p_2+1>j_1-p_1+1 \text{ ist})\end{cases}$$

$$\alpha^{2,3}_{p_1,p_2}=\beta^{2,3}_{p_1,p_2}=\frac{1}{(j_3-j_2)!}\,. \tag{34}$$

Aus (33) und (34) folgt es ohne weiteres

$$a_{n,n}=-(-1)^{\frac{n-1}{2}}\frac{(n-1)!}{(n-1)!}+(-1)^2(-1)^{\frac{n-1}{2}}\sum_{p_1=0}^{n-3}\sideset{}{'}\sum_{j_1=q_1'}^{n-2}(n-1)!\;\beta^{1,1}_{p_1}\beta^{1,2}_{p_1}$$

$$+(-1)^3(-1)^{\frac{n-1}{2}}\sum_{p_1=0}^{n-3}\sum_{p_2=0}^{n-3}\sideset{}{'}\sum_{j_2=q_2'}^{n-2}\sideset{}{'}\sum_{j_1=q_1'}^{j_2-1}(n-1)!\;\beta^{2,1}_{p_1,p_2}\beta^{2,2}_{p_1,p_2}\beta^{2,3}_{p_1,p_2}+(-1)^3$$

$$\sum_{p_1=0}^{n-3}\sum_{p_2=0}^{n-3}\sideset{}{'}\sum_{j_3=q_2}^{n-2}(-1)^{\frac{j_3}{2}}\underset{(r_1+r_2+\cdots+r_{j_3-p_1-p_2+1}=n-p_1-p_2)}{\sum_{r_1=1}\sum_{r_2=1}\cdots\sum_{r_{j_3-p_1-p_2+1}=1}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3-p_1-p_2+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_3-p_1-p_2+1},r_{j_3-p_1-p_2+1}}\sideset{}{'}\sum_{j_2=q_2'}^{j_3-1}\sideset{}{'}\sum_{j_1=q_1'}^{j_2-1}\alpha^{2,1}_{p_1,p_2}\alpha^{2,2}_{p_1,p_2}\alpha^{2,3}_{p_1,p_2}\,.$$

Dieses Verfahren setzen wir fort, so bekommen wir

$$a_{n,n}=-(-1)^{\frac{n-1}{2}}\frac{(n-1)!}{(n-1)!}+(-1)^2(-1)^{\frac{n-1}{2}}\sum_{p_1=0}^{n-3}\sideset{}{'}\sum_{j_1=q_1'}^{n-2}(n-1)!\;\beta^{1,1}_{p_1}\beta^{1,2}_{p_1}+\cdots$$

$$+(-1)^{s+1}(-1)^{\frac{n-1}{2}}\sum_{p_1=0}^{n-3}\sum_{p_2=0}^{n-3}\cdots\sum_{p_s=0}^{n-3}\sideset{}{'}\sum_{j_s=q'_s}^{n-2}\sideset{}{'}\sum_{j_{s-1}=q'_{s-1}}^{j_s-1}\cdots\sideset{}{'}\sum_{j_1=q'_1}^{j_2-1}(n-1)!\;\beta^{s,1}_{p_1,p_2,\dots,p_s}$$

$$\beta^{s,2}_{p_1,p_2,\dots,p_s}\cdots\beta^{s,s+1}_{p_1,p_2,\dots,p_s}+(-1)^{s+1}\sum_{p_1=0}^{n-3}\sum_{p_2=0}^{n-3}\cdots\sum_{p_s=0}^{n-3}\sideset{}{'}\sum_{j_{s+1}=q_s}^{n-2}(-1)^{\frac{j_{s+1}}{2}}\sum_{r_1=1}\sum_{r_2=1}\cdots$$

$$\underset{(r_1+r_2+\cdots+r_{j_{s+1}-p_1-\cdots-p_s+1}=n-p_1-\cdots-p_s)}{\sum_{r_{j_{s+1}-p_1-\cdots-p_s+1}=1}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_{s+1}-p_1-\cdots-p_s+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_{s+1}-p_1-\cdots-p_s+1},r_{j_{s+1}-p_1-\cdots-p_s+1}}\sideset{}{'}\sum_{j_s=q_s}^{j_{s+1}-1}\sideset{}{'}\sum_{j_{s-1}=q_{s-1}}^{j_s-1}\cdots\sideset{}{'}\sum_{j_1=q_1}^{j_2-1}\alpha^{s,1}_{p_1,p_2,\dots,p_s}\alpha^{s,2}_{p_1,p_2,\dots,p_s}\cdots\alpha^{s,s+1}_{p_1,p_2,\dots,p_s}, \tag{35}$$

wo $q'_s=\text{Max.}\,(q_{s-1},\,p_1+p_2+\cdots+p_s)$ und $q_s=\text{Max.}\,(2s,\,p_1+2+2(s-1),\,p_1+p_2+2(s-2),\,\dots,\,p_1+\cdots+p_{s-1}+4,\,p_1+p_2+\cdots+p_s+2)$ ist und $\alpha^{s,j}_{p_1,p_2,\dots,p_s}$ und $\beta^{s,j}_{p_1,p_2,\dots,p_s}$ die folgenden Sinne haben, Für die Zahlen j_1-p_1+1, $j_2-p_1-p_2+1$, $\cdots$, $j_s-p_1-p_2-\cdots-p_s+1$ definieren wir eine Ordnung. Zwei Zahlen $j_i-p_1-\cdots-p_i+1$ und $j_l-p_1-\cdots-p_l+1$ heissen wohl geordnet, wenn $j_i-p_1-\cdots-p_i+1\leqq j_l-p_1-\cdots-p_l+1$ für $i<l$ ist. Wenn $j_i-p_1-\cdots-p_i+1>j_l-p_1-\cdots-p_l+1$ für $i<l$ ist, so heissen $j_i-p_1-\cdots-p_i+1$ und $j_l-p_1-\cdots-p_l+1$ umgekehrt geordnet.

Wenn j_1-p_1+1 und $j_2-p_1-p_2+1$ wohl geordnet sind, so setzen wir $f_1^1=1$. Wenn j_1-p_1+1 und $j_2-p_1-p_2+1$ umgekehrt geordnet sind, so setzen wir $f_1^1=\dfrac{(p_1+j_2-j_1)!}{(p_1+p_2)!}$

Wenn $j_3-p_1-p_2-p_3+1$ und das Minimum von j_1-p_1+1 und $j_2-p_1-p_2+1$ wohl geordnet sind, so setzen wir $f_2^1=1$. Wenn $j_3-p_1-p_2-p_3+1$ und das Minimum von j_1-p_1+1 und $j_2-p_1-p_2+1$ umgekehrt geordnet sind, so setzen wir $f_2^1=\dfrac{(p_1+j_3-j_1)!}{(p_1+p_2+p_3)!}$ oder $f_2^1=\dfrac{(p_1+p_2+j_3-j_2)!}{(p_1+p_2+p_3)!}$, jenachdem das Minimum von j_1-p_1+1 und $j_2-p_1-p_2+1$ mit j_1-p_1+1 oder mit $j_2-p_1-p_2+1$ übereinstimmt.

Im allgemeinen setzen wir Min. $(j_1-p_1+1,\ j_2-p_1-p_2+1,\cdots,\ j_{i-1}-p_1-p_2-\cdots-p_{i-1}+1)=j_l-p_1-\cdots-p_l+1$. Wenn $j_i-p_1-\cdots-p_i+1$ und $j_l-p_1-\cdots-p_l+1$ wohl geordnet sind, so setzen wir $f_i^1=1$. Wenn $j_i-p_1-\cdots-p_i+1$ und $j_l-p_1-\cdots-p_l+1$ umgekehrt geordnet sind, so setzen wir $f_i^1=\dfrac{(j_i-j_l+p_1+p_2+\cdots+p_l)!}{(p_1+p_2+\cdots+p_{i+1})!}$. Es besteht dann

$$\alpha^{s,1}_{p_1,p_2,\cdots,p_s}=\begin{cases}\dfrac{1}{p_1!} & (\text{wenn } j_1=p_1 \text{ ist})\\[2ex] \dfrac{1}{p_1!}\displaystyle\prod_{i=1}^{s-1}f_i^1\prod_{k_1=1}^{\delta_s^1}\frac{1}{p_1+p_2+\cdots+p_s+r_{k_1+1}+\cdots+r_{j_{s+1}-p_1-\cdots-p_s+1}} & \\ & (\text{wenn } j_1>p_1 \text{ ist})\end{cases} \tag{36}$$

wo $\delta_s^1+1=$ Min. $(j_1-p_1+1, j_2-p_1-p_2+1, \cdots j_s-p_1-\cdots-p_s+1)$ ist.

Ebenfalls setzen wir Min. $(j_i-p_1-\cdots-p_i+1, \cdots, j_{l-1}-p_1-\cdots-p_{l-1}+1)=j_m-p_1-\cdots-p_m+1$, wo $l-1\geqq i$ ist. Wenn $j_l-p_1-\cdots-p_l+1$ und $j_m-p_1-\cdots-p_m+1$ wohl geordnet sind, setzen wir $f_{l-1}^i=1$. Wenn $j_l-p_1-\cdots-p_l+1$ und $j_m-p_1-\cdots-p_m+1$ umgekehrt geordnet sind, so setzen wir $f_{l-1}^i=\dfrac{(j_l-j_m+p_i+p_{i+1}+\cdots+p_m)!}{(p_i+p_{i+1}+\cdots+p_l)!}$. Es besteht dann, für $i=2,\cdots.s$,

$$\alpha^{s,i}_{p_1,p_2,\cdots,p_s}=\begin{cases}\dfrac{1}{(j_i-j_{i-1})!} & (\text{wenn } j_i-p_1-\cdots-p_i+1\leqq j_{i-1}-p_1-\cdots-p_{i-1}+1 \text{ ist}).\\[2ex] \dfrac{1}{p_i!}\displaystyle\prod_{j=i}^{s-1}f_j^i\prod_{k_i=j_{i-1}-p_1-\cdots-p_{i-1}+1}^{\delta_i^s} & \\ \dfrac{1}{p_i+p_{i+1}+\cdots+p_s+r_{k_i+1}+\cdots+r_{j_{s+1}-p_1-\cdots-p_s+1}} & \end{cases}$$

$$\left\{\begin{array}{l}\text{(wenn jede von } j_i-p_1-\cdots-p_i+1,\ j_{i+1}-p_1-\cdots-p_{i+1}+1,\\ \cdots,\ j_s-p_1-p_2-\cdots-p_s+1 \text{ grösser als } j_{i-1}-p_1-\cdots-p_{i-1}+\\ 1 \text{ ist).}\\ \dfrac{1}{p_i!}\prod_{j=i}^{l-1} f_j^i \dfrac{(j_l-j_m+p_i+p_{i+1}+\cdots+p_m)!}{(j_l-j_{l-1})!},\ \text{(wenn für } i\leqq l\\ \leqq s \text{ es } j_l-p_1-\cdots-p_l+1\leqq j_{l-1}-p_1-\cdots-p_{l-1}+1 \text{ besteht}\\ \text{und wenn jede von } j_i-p_1-\cdots-p_i+1,\ j_{i+1}-p_1-\cdots-p_{i+1}\\ +1,\ \cdots,\ j_{l-1}-p_1-\cdots-p_{l-1}+1 \text{ grösser als } j_{l-1}-p_1-\cdots-\\ p_{l-1}+1 \text{ ist)}\end{array}\right. \tag{37}$$

wo $\delta_i^s+1=\text{Min.}\,(j_i-p_1-\cdots-p_i+1, \cdots, j_s-p_1-\cdots-p_s+1)$ und $\text{Min.}\,(j_i-p_1-\cdots-p_i+1, \cdots, j_{l-1}-p_1-\cdots-p_{l-1}+1)=j_m-p_1-\cdots-p_m+1$ ist.

$$\alpha^{s,s+1}_{p_1,p_2,\cdots,p_s}=\prod_{k_{s+1}=j_s-p_1-\cdots-p_s+1}^{j_{s+1}-p_1-\cdots-p_s}\frac{1}{r_{k_{s+1}+1}+\cdots+r_{j_{s+1}-p_1-\cdots-p_s+1}}. \tag{38}$$

$$\beta^{s,1}_{p_1,p_2,\cdots,p_s}=\begin{cases}\dfrac{1}{p_1!} & \text{(wenn } j_1=p_1 \text{ ist)}\\ \dfrac{1}{p_1!}\prod_{i=1}^{s-1} f_i^1\dfrac{(n-1-j_m+p_1+\cdots+p_m)!}{(n-1)!}, & \text{(wenn } j_1>p_1 \text{ ist)}\end{cases} \tag{39}$$

wo $\text{Min.}\,(j_1-p_1+1, j_2-p_1-p_2+1, \cdots, j_s-p_1-\cdots-p_s+1)=j_m-p_1-\cdots-p_m+1$ ist.

$$\beta^{s,i}_{p_1,p_2,\cdots,p_s}=\left\{\begin{array}{l}\dfrac{1}{(j_i-j_{i-1})!}\ \text{(wenn } j_i-p_1-p_2-\cdots-p_i+1\leqq j_{i-1}-p_1-\cdots\\ \qquad -p_{i-1}+1 \text{ ist)}\\ \dfrac{1}{p_i!}\prod_{j=i}^{s-1} f_j^i\dfrac{(n-1-j_m+p_i+p_{i+1}+\cdots+p_m)!}{(n-1-j_{i-1})!}\ \text{(wenn jede von}\\ j_i-p_1-\cdots-p_i+1,\ j_{i+1}-p_1-\cdots-p_{i+1}+1,\ \cdots,\ j_s-p_1-\cdots-\\ p_s+1 \text{ grösser als } j_{i-1}-p_1-\cdots-p_{i-1}+1 \text{ ist), wo Min.}\,(j_i\\ -p_1-\cdots-p_i+1,\ \cdots,\ j_s-p_1-\cdots-p_s+1)=j_m-p_1-\cdots-p_m\\ +1 \text{ ist,}\\ \dfrac{1}{p_i!}\prod_{j=i}^{l-1} f_j^i\dfrac{(j_l-j_m+p_i+\cdots+p_m)!}{(j_l-j_{l-1})!},\ \text{(wenn für } i\leqq l\leqq s\\ \text{es } j_l-p_1-\cdots-p_l+1\leqq j_{l-1}-p_1-\cdots-p_{l-1}+1 \text{ besteht und}\\ \text{wenn jede von } j_i-p_1-\cdots-p_i+1,\ j_{i+1}-p_1-\cdots-p_{i+1}+1,\ \cdots\\ j_{l-1}-p_1-\cdots-p_{l-1}+1 \text{ grösser als } j_{i-1}-p_1-p_2-\cdots-p_{i-1}+\\ 1 \text{ ist),}\end{array}\right. \tag{40}$$

wo $\text{Min.}\,(j_i-p_1-\cdots-p_i+1, \cdots, j_{l-1}-p_1-\cdots-p_{l-1}+1)=j_m-p_1-\cdots-p_m+1$ ist. Diese Formel gilt für $i=2, 3, \cdots, s$.

$$\beta^{s,s+1}_{p_1,p_2,\cdots,p_s}=\frac{1}{(n-1-j_s)!}\,. \tag{41}$$

Dieses Verfahren setzen wir fort, so gewinnen wir endlich

$$a_{n,n}=-(-1)^{\frac{n-1}{2}}\frac{(n-1)!}{(n-1)!}+\sum_{s=1}^{\frac{n-1}{2}}(-1)^{s+1}(-1)^{\frac{n-1}{2}}\sum_{p_1=0}^{n-3}\sum_{p_2=0}^{n-3}\cdots\sum_{p_s=0}^{n-3}\sum_{j_s=q_s}^{n-2}{}'\sum_{j_{s-1}=q'_{s-1}}^{j_s-1}{}'\cdots$$

$$\sum_{j=q_1}^{j_2-1}{}'(n-1)!\,\beta^{s,1}_{p_1,p_2,\cdots,p_s}\beta^{s,2}_{p_1,p_2,\cdots,p_s}\cdots\beta^{s,s+1}_{p_1,p_2,\cdots,p_s}. \tag{42}$$

Dabei wird $\beta^{s,i}_{p_1,p_2,\cdots,p_s}$ durch (39), (40) und (41) gegeben.

Wir behandeln nun den Fall, wo das Suffix n von $a_{n,n}$ gerade ist. Nach (16) ist $a_{2,2}=-\Im(\kappa Ai^2)/\Re(\kappa A)=\dfrac{\Im(\kappa A)}{\Re(\kappa A)}=0$, da $\Im(\kappa A)=0$ ist.

Wir nehmen an, dass $a_{p,p}=0$ ist für jede gerade positive ganze Zahl p, wo $2\leqq p<n$ ist. In der Formel (19) ist es

$$r_1+r_2+\cdots+r_{j+1}=n,\ j=\text{gerade Zahl}.$$

Daher ist mindestens eine unter $r_1,\ r_2,\ \cdots,\ r_{j+1}$ eine gerade Zahl. Folglich ist

$$a_{n,n}=0\,. \tag{43}$$

Diese Formel gilt für alle geraden Zahlen n.

Wir wollen nun die Koeffizient $a_{n,\,n-1}$ von $\varphi^{(n)}(0)=\sum_{j=0}^{n}a_{n,j}\varphi'(0)^j$ berechnen. Es ergibt sich aus (14) und (15)

$$\Im\kappa A\Big\{ia_{n,n-1}+\sum_{j=1}^{n-1}\Big(\sum_{p_j=1}^{n-j}\ \sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots{}_{p_{j-1}-1}C_{p_j}i^{j+1}$$

$$\sum_{\substack{k_1=0\\(k_1+k_2+\cdots+k_{j+1}=n-1)}}\sum_{k_2=0}\cdots\sum_{k_{j+1}=0}a_{n-p_1,k_1}a_{p_1-p_2,k_2}\cdots a_{p_{j-1}-p_j,k_j}a_{p_j,k_{j+1}}\Big)\Big\}+{}_nC_1\Im B\kappa^2\Big\{i2a_{n-1,n-1}+$$

$$\sum_{j=1}^{n-2}\Big(\sum_{p_j=1}^{n-j-1}\ \sum_{p_{j-1}=p_j+1}^{n-j}\cdots\sum_{p_1=p_2+1}^{n-2}2^{j+1}{}_{n-2}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots{}_{p_{j-1}-1}C_{p_j}i^{j+1}a_{n-1-p_1,n-1-p_1}$$

$$a_{p_1-p_2,p_1-p_2}\cdots a_{p_{j-1}-p_j,p_{j-1}-p_j}a_{p_j,p_j}\Big)\Big\}=0,\quad(\text{für } n\leqq 3)$$

wo $\sum\limits_{\substack{k_1=0\\(k_1+k_2+\cdots+k_{j+1}=n-1)}}\sum\limits_{k_2=0}\cdots\sum\limits_{k_{j+1}=0}$ über alle Systeme von 0 und positiven ganzen Zahlen $k_1,k_2,\cdots,k_{j+1}$ zu erstrecken ist, derart dass $k_1+k_2+\cdots+k_{j+1}=n-1$ ist.

Daraus folgt ohne weiteres

$$a_{n,n-1}=-\Im\kappa A\sum_{j=1}^{n-1}\Big(\sum_{p_j=1}^{n-j}\ \sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots{}_{p_{j-1}-1}C_{p_j}i^{j+1}$$

$$\sum_{k_1=0}\sum_{k_2=0}\cdots\sum_{\substack{k_{j+1}=0\\(k_1+k_2+\cdots+k_{j+1}=n-1)}} a_{n-p_1,k_1}a_{p_1-p_2,k_2}\cdots a_{p_{j-1}-p_j,k_j}a_{p_j,k_{j+1}}\Big)\Big/\Re(\kappa A)-{}_nC_1\Im B\kappa^2$$

$$\Big\{i2a_{n-1,n-1}+\sum_{j=1}^{n-2}\Big(\sum_{p_j=1}^{n-j-1}\sum_{p_{j-1}=p_j+1}^{n-j}\cdots\sum_{p_1=p_2+1}^{n-2}2^{j+1}{}_{n-2}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots{}_{p_{j-1}-1}C_{p_j}i^{j+1}$$

$$a_{n-1-p_1,n-1-p_1}a_{p_1-p_2,p_1-p_2}\cdots a_{p_{j-1}-p_j,p_{j-1}-p_j}a_{p_j,p_j}\Big)\Big\}\Big/\Re(A\kappa). \tag{44}$$

Es ist aber

$$-\Im\kappa A\sum_{j=1}^{n-1}\Big(\sum_{p_j=1}^{n-j}\sum_{p_{j-1}=p_j+1}^{n-j+1}\cdots\sum_{p_1=p_2+1}^{n-1}{}_{n-1}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots{}_{p_{j-1}-1}C_{p_j}i^{j+1}$$

$$\sum_{k_1=0}\sum_{k_2=0}\cdots\sum_{\substack{k_{j+1}=0\\(k_1+k_2+\cdots+k_{j+1}=n-1)}} a_{n-p_1,k_1}a_{p_1-p_2,k_2}\cdots a_{p_{j-1}-p_j,k_j}a_{p_j,k_{j+1}}\Big)\Big/\Re(\kappa A)$$

$$=\sum_{j=1}^{n-1}{}'(-1)^{\frac{j}{2}}\sum_{r_1=1}\sum_{r_2=1}\cdots\sum_{\substack{r_{j+1}=1\\(r_1+r_2+\cdots+r_{j+1}=n)}}\frac{(n-1)!}{(r_1-1)!\,(r_2-1)!\cdots(r_{j+1}-1)!}\sum_{s_1=0}^{j+1}a_{r_1,r_1}\cdots$$

$$a_{r_{s_1-1},r_{s_1}-1}a_{r_{s_1},r_{s_1}-1}a_{r_{s_1+1},r_{s_1+1}}\cdots a_{r_{j+1},r_{j+1}}\prod_{k=1}^{j}\frac{1}{n-r_1-\cdots-r_k}. \tag{45}$$

Ebenfalls ist es

$${}_nC_1\Im B\kappa^2\Big\{2i\,a_{n-1,n-1}+\sum_{j=1}^{n-2}\Big(\sum_{p_j=1}^{n-j-1}\sum_{p_{j-1}=p_j+1}^{n-j}\cdots\sum_{p_1=p_2+1}^{n-2}2^{j+1}i^{j+1}{}_{n-2}C_{p_1}\,{}_{p_1-1}C_{p_2}\cdots$$

$${}_{p_{j-1}-1}C_{p_j}\,a_{n-1-p_1,\,n-1-p_1}\,a_{p_1-p_2,\,p_1-p_2}\cdots\cdots a_{p_{j-1}-p_j,\,p_{j-1}-p_j}a_{p_j,p_j}\Big)\Big\}\Big/\Re(\kappa A)$$

$$=2{}_nC_1\Re(B\kappa^2)a_{n-1,\,n-1}\Big/\Re(\kappa A)+{}_nC_1\sum_{j=1}^{n-2}\Im(B\kappa^2i^{j+1})\sum_{r_1=1}\sum_{r_2=1}\cdots\sum_{\substack{r_{j+1}=1\\(r_1+r_2+\cdots+r_{j+1}=n-1)}}$$

$$\frac{(n-2)!\,2^{j+1}}{(r_1-1)!\,(r_2-1)!\cdots(r_{j+1}-1)!}a_{r_1,r_1}\cdots a_{r_{j+1},\,r_{j+1}}$$

$$\prod_{k=1}^{j}\frac{1}{n-1-r_1-\cdots-r_k}\Big/\Re(\kappa A). \tag{46}$$

Aus (44), (45) und (46) bekommen wir

$$a_{n,\,n-1}=-\sum_{j=1}^{n-1}{}'(-1)^{\frac{j}{2}}\sum_{r_1=1}\cdots\sum_{\substack{r_{j+1}=1\\(r_1+\cdots+r_{j+1}=n)}}\frac{(n-1)!}{(r_1-1)!\,(r_2-1)!\cdots(r_{j+1}-1)!}$$

$$\sum_{s_1=0}^{j+1}a_{r_1,r_1}\cdots a_{r_{s_1},r_{s_1}-1}\cdots a_{r_{j+1},r_{j+1}}\prod_{k=1}^{j}\frac{1}{n-r_1-\cdots-r_k}$$

$$-2{}_nC_1\Re(B\kappa^2)a_{n-1,\,n-1}\Big/\Re(\kappa A)-{}_nC_1\sum_{j=1}^{n-2}\Im(B\kappa^2i^{j+1})\sum_{r_1=1}\cdots\sum_{\substack{r_{j+1}=1\\(r_1+\cdots+r_{j+1}=n-1)}}$$

$$\frac{(n-2)!\,2^{j+1}}{(r_1-1)!\cdots(r_{j+1}-1)!}a_{r_1,r_1}\cdots\quad a_{r_{j+1},r_{j+1}}\prod_{k=1}^{j}\frac{1}{n-1-r_1-\cdots-r_k}\Big/\Re(\kappa A).\quad (\text{für } n\geqq 3) \tag{47}$$

Wir wollen nun das erste Glied von der rechten Seite der obigen Formel berechnen. Es ist

$$\underset{(r_1+\cdots+r_{j+1}=n)}{\sum_{r_1=1}\cdots\sum_{r_{j+1}=1}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!}a_{r_1,r_1-1}\,a_{r_2,r_2}\cdots\quad a_{r_{j+1},r_{j+1}}\prod_{k=1}^{j}$$

$$\frac{1}{n-r_1-\cdots-r_k}=\underset{(r_2+r_3+\cdots+r_{j+1}=n-2)}{\sum_{r_2=1}\sum_{r_3=1}\cdots\sum_{r_{j+1}=1}}\frac{(n-1)!}{(r_2-1)!\cdots(r_{j+1}-1)!}a_{r_2,r_2}\,a_{r_3,r_3}\cdots$$

$$a_{r_{j+1},r_{j+1}}\frac{1}{n-2}\prod_{k=2}^{j}\frac{1}{n-2-r_2-\cdots-r_k}a_{2,1}+\underset{(r_1+r_2+\cdots+r_{j+1}=n,\ r_1\geqq 3)}{\sum_{r_1=3}\sum_{r_2=1}\cdots\sum_{r_{j+1}=1}}$$

$$\frac{(n-1)!}{(r_1-1)!\,(r_2-1)!\cdots(r_{j+1}-1)!}a_{r_1,r_1-1}\,a_{r_2,r_2}\cdots\quad a_{r_{j+1},r_{j+1}}\prod_{k=1}^{j}\frac{1}{n-r_1-\cdots-r_k}.$$

Das zweite Glied von der rechten Seite der obigen Formel

$$=\underset{(r_1+\cdots+r_{j+1}=n,\ r_1\geqq 3)}{\sum_{r_1=3}\sum_{r_2=1}\cdots\sum_{r_{j+1}=1}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!}a_{r_2,r_2}\,a_{r_3,r_3}\cdots\quad a_{r_{j+1},r_{j+1}}\prod_{k_1=1}^{j}$$

$$\frac{1}{n-r_1-\cdots-r_{k_1}}\times-\left[\sum_{j_2=1}^{r_1-1}{}'(-1)^{\frac{j_2}{2}}\underset{(s_1+\cdots+s_{j_2+1}=r_1)}{\sum_{s_1=1}\cdots\sum_{s_{j_2+1}=1}}\frac{(r_1-1)!}{(s_1-1)!\cdots(s_{j_2+1}-1)!}\right.$$

$$\sum_{h_2=1}^{j_2+1}a_{s_1,s_1}\cdots\quad a_{s_{h_2},s_{h_2}-1}\cdots\quad a_{s_{j_2+1},s_{j_2+1}}\prod_{k_2=1}^{j_2}\frac{1}{s_{k_2+1}+\cdots+s_{j_2+1}}{}^{6)}$$

$$+2_{r_1}C_1\Re(B\kappa^2)a_{r_1-1,r_1-1}\Big/\Re(\kappa A)+{}_{r_1}C_1\sum_{j_2=1}^{r_1-2}\Im(B\kappa^2 i^{j_2+1})\underset{(s_1+\cdots+s_{j_2+1}=r_1-1)}{\sum_{s_1=1}\cdots\sum_{s_{j_2+1}=1}}$$

$$\frac{(r_1-2)!\,2^{j_2+1}}{(s_1-1)!\cdots(s_{j_2+1}-1)!}a_{s_1,s_1}\cdots\quad a_{s_{j_2+1},s_{j_2+1}}\prod_{k_2=1}^{j_2}\frac{1}{s_{k_2+1}+\cdots+s_{j_2+1}}{}^{7)}\Big/\Re(\kappa A).$$

Es ist aber

$$-\sum_{j=1}^{n-1}{}'(-1)^{\frac{j}{2}}\underset{(r_1+r_2+\cdots+r_{j+1}=n,\ r_1\geqq 3)}{\sum_{r_1=3}\sum_{r_2=1}\cdots\sum_{r_{j+1}=1}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!}a_{r_2,r_2}\cdots$$

---

6) und 7). Die Formeln 6) und 7) sind in der Tat voneinander verschieden, da in 6) $s_1+\cdots+s_{j_2+1}=r_1$ und in 7) $s_1+\cdots+s_{j_2+1}=r_1-1$ ist.

$$a_{r_{j+1},\, r_{j+1}} \times -\sum_{j_2=1}^{r_1-1}{}'(-1)^{\frac{j_2}{2}} \sum_{s_1=1} \cdots \sum_{\substack{s_{j_2+1}=1 \\ (s_1+\cdots+s_{j_2+1}=r_1)}} \frac{(r_1-1)}{(s_1-1)!\ \cdots (s_{j_2+1}-1)!} \sum_{h_2=1}^{j_2+1} a_{s_1,\, s_1} \cdots$$

$$a_{s_{h_2},\, s_{h_2}-1} \cdots\ a_{s_{j_2+1},\, s_{j_2+1}} \prod_{k_2=1}^{j_2} \frac{1}{s_{k_2+1}+\cdots+s_{j_2+1}} \prod_{k_1=1}^{j_1} \frac{1}{n-r_1-\cdots-r_{k_1}}$$

$$=(-1)^2 \sum_{j_2=2}^{n-1}{}'(-1)^{\frac{j_2}{2}} \sum_{r_1=1} \cdots \sum_{\substack{r_{j_2+1}=1 \\ (r_1+\cdots+r_{j_2+1}=n)}} \frac{(n-1)!}{(r_1-1)!\cdots (r_{j_2+1}-1)!} \sum_{j_1=1}^{j_2-1}{}' \sum_{h_2=1}^{j_2-j_1+1} a_{r_1, r_1} \cdots$$

$$a_{r_{j_1+1},\, r_{j_1+1}} \cdots\ a_{r_{j_1+h_2},\, r_{j_1+h_2}-1} \cdots\ a_{r_{j_2+1},\, r_{j_2+1}} \prod_{k_1=1}^{j_1} \frac{1}{r_{k_1}+\cdots+r_{j_1}}$$

$$\prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}}. \tag{48}$$

Es ist aber

$$-\sum_{j=1}^{n-1}{}'(-1)^{\frac{j}{2}} \sum_{r_1=3} \sum_{r_2=1} \cdots \sum_{\substack{r_{j+1}=1 \\ (r_1+\cdots+r_{j+1}=n,\ r_1\geqq 3)}} \frac{(n-1)!}{(r_1-1)!\cdots (r_{j+1}-1)!} \prod_{k_1=1}^{j} \frac{1}{n-r_1-\cdots-r_{k_1}} \times$$

$$-2_{r_1} C_1 \Re(B\kappa^2) a_{r_1-1,\, r_1-1} \Big/ \Re(\kappa A) = (-1)^3 \sum_{j_2=2}^{n-2}{}' (-1)^{\frac{j_2}{2}} \sum_{r_1=1} \sum_{j_2=1} \cdots \sum_{\substack{r_{j_2+1}=1 \\ (r_1+r_2+\cdots+r_{j_2+1}=n-1)}}$$

$$\frac{(n-1)!}{(r_1-1)!\cdots (r_{j_2+1}-1)!} a_{r_1, r_1} \cdots\ a_{r_{j_2+1}, r_{j_2+1}} \sum_{j_1=1}^{j_2-1}{}' \frac{r_{j_1+1}+\cdots+r_{j_2+1}+1}{r_{j_1+1}+\cdots+r_{j_2+1}} 2$$

$$\prod_{k_1=1}^{j_1} \frac{1}{r_{k_1}+\cdots+r_{j_1}} \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}} \Re(B\kappa^2) \Big/ \Re(\kappa A). \tag{49}$$

Es ist aber

$$-\sum_{j=1}^{n-1}{}' (-1)^{\frac{j}{2}} \sum_{r_1=3} \sum_{r_2=1} \cdots \sum_{\substack{r_{j+1}=1 \\ (r_1+r_2+\cdots+r_{j+1}=n,\ r_1\geqq 3)}} \frac{(n-1)!}{(r_1-1)!\ (r_2-1)!\cdots\ (r_{j+1}-1)!} a_{r_2,\, r_2} \cdots$$

$$a_{r_{j+1},\, r_{j+1}} \times -{}_{r_1}C_1 \left[\sum_{j_2=1}^{r_1-2}{}'(-1)^{\frac{j_2}{2}} \Re(B\kappa^2) + \sum_{j_2=1}^{r_1-2}{}''(-1)^{\frac{j_2+1}{2}} \Im(B\kappa^2) \right] \sum_{s_1=1} \cdots \sum_{\substack{s_{j_2+1}=1 \\ (s_1+\cdots+s_{j_2+1}=r_1-1)}}$$

$$\frac{(r_1-2)!\ 2^{j_2+1}}{(s_1-1)!\cdots (s_{j_2+1}-1)!} a_{s_1, s_1} \cdots\ a_{s_{j_2+1}, s_{j_2+1}} \prod_{k_2=1}^{j_2} \frac{1}{s_{k_2+1}+\cdots+s_{j_2+1}}$$

$$\prod_{k_1=1}^{j} \frac{1}{n-r_1-\cdots-r_{k_1}} \Big/ \Re(\kappa A)$$

$$=(-1)^2 \sum_{j_2=2}^{n-2}{}'(-1)^{\frac{j_2}{2}} \sum_{r_1=1} \cdots \sum_{\substack{r_{j_2+1}=1 \\ (r_1+\cdots+r_{j_2+1}=n-1)}} \frac{(n-1)!}{(r_1-1)!\cdots (r_{j_2+1}-1)!} a_{r_1, r_1} \cdots$$

$$a_{r_{j_2+1},r_{j_2+1}} \sum_{j_1=1}^{j_2-1}{}' \frac{r_{j_1+1}+\cdots+r_{j_2+1}+1}{r_{j_1+1}+\cdots+r_{j_2+1}} 2^{j_2-j_1+1} \prod_{k_1=1}^{j_1} \frac{1}{r_{k_1}+\cdots+r_{j_1}}$$

$$\prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}} \Re(B\kappa^2)\Big/\Re(\kappa A)$$

$$+(-1)^2 \sum_{j_2=2}^{n-2}{}'' (-1)^{\frac{j_2+1}{2}} \sum_{r_1=1}\cdots\sum_{\substack{r_{j_2+1}=1 \\ (r_1+\cdots+r_{j_2+1}=n-1)}} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2+1}-1)!} a_{r_1,r_1}\cdots$$

$$a_{r_{j_2+1},r_{j_2+1}} \sum_{j_1=1}^{j_2-1}{}' \frac{r_{j_1+1}+\cdots+r_{j_2+1}+1}{r_{j_1+1}+\cdots+r_{j_2+1}} 2^{j_2-j_1+1} \prod_{k_1=1}^{j_1} \frac{1}{r_{k_1}+\cdots+r_{j_1}}$$

$$\prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}} \Im(B\kappa^2)\Big/\Re(\kappa A), \tag{50}$$

wo die Summe $\sum_{j_2=2}^{n-2}{}''$ über alle positiven ungeraden Zahlen bis 2 zu $n-2$ zu erstrecken ist.

Ebenfalls ist es

$$\sum_{r_1=1}\cdots\sum_{\substack{r_{j+1}=1 \\ (r_1+\cdots+r_{j+1}=n)}} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!} a_{r_1,r_1} a_{r_2,r_2-1}\cdots \quad a_{r_{j+1},r_{j+1}}$$

$$\prod_{k=1}^{j} \frac{1}{n-r_1-\cdots-r_k} = \sum_{r_1=1}\cdots\sum_{\substack{r_j=1 \\ (r_1+\cdots+r_j=n-2)}} \frac{(n-1)!}{(r_1-1)!\cdots(r_j-1)!} a_{r_1,r_1} \ a_{r_2,r_2}\cdots$$

$$a_{r_j,r_j} \frac{1}{n-r_1} \prod_{k=2}^{j} \frac{1}{r_k+\cdots+r_j} a_{2,1} + \sum_{r_1=1} \sum_{r_2=3}\cdots\sum_{\substack{r_{j+1}=1 \\ (r_1+\cdots+r_{j+1}=n,\ r_2\geqq 3)}}$$

$$\frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!} a_{r_1,r_1} a_{r_2,r_2-1}\cdots \quad a_{r_{j+1},r_{j+1}} \prod_{k=1}^{j} \frac{1}{n-r_1-\cdots-r_k}.$$

Es ist nun

$$-\sum_{j=1}^{n-1}{}' (-1)^{\frac{j}{2}} \sum_{r_1=1} \sum_{r_2=3}\cdots\sum_{\substack{r_{j+1}=1 \\ (r_1+\cdots+r_{j+1}=n,\ r_2\geqq 3)}} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!} a_{r_1,r_1} \ a_{r_2,r_2-1}\cdots$$

$$a_{r_{j+1},r_{j+1}} \prod_{k=1}^{j} \frac{1}{n-r_1-\cdots-r_k} = -\sum_{j=1}^{n-1}{}' (-1)^{\frac{j}{2}} \sum_{r_1=1} \sum_{r_2=3}\cdots\sum_{\substack{r_{j+1}=1 \\ (r_1+\cdots+r_{j+1}=n,\ r_2\geqq 3)}}$$

$$\frac{(n-1)!}{(r_1-1)!\cdots(r_{j+1}-1)!} a_{r_1,r_1} \ a_{r_3,r_3}\cdots \quad a_{r_{j+1},r_{j+1}} \times -\left[\sum_{j_2=1}^{r_2-1}{}' (-1)^{\frac{j_2}{2}}\right.$$

$$\sum_{s_1=1}\cdots\sum_{\substack{s_{j_2+1}=1 \\ (s_1+\cdots+s_{j_2+1}=r_2)}} \frac{(r_2-1)!}{(s_1-1)!\cdots(s_{j_2+1}-1)!} \sum_{h_2=1}^{j_2+1} a_{s_1,s_1}\cdots \quad a_{s_{h_2},s_{h_2}-1}\cdots \quad a_{s_{j_2+1},s_{j_2+1}}$$

$$\prod_{k_2=1}^{j_2}\frac{1}{s_{k_2+1}+\cdots+s_{j_2+1}}+2_{r_2}C_1\Re(B\kappa^2)a_{r_2-1,\,r_2-1}\Big/\Re(\kappa A)+{}_{r_2}C_1$$

$$\left\{\sum_{j_2=1}^{r_2-2}{}'(-1)^{\frac{j_2}{2}}\Re(B\kappa^2)+\sum_{j_2=1}^{r_2-2}{}''(-1)^{\frac{j_2+1}{2}}\Im(B\kappa^2)\right\}\sum_{\substack{s_1=1\\(s_1+\cdots+s_{j_2+1}=r_2-1)}}\cdots\sum_{s_{j_2+1}=1}$$

$$\frac{(r_2-2)!\;2^{j_2+1}}{(s_1-1)!\cdots(s_{j_2+1}-1)!}a_{s_1,s_1}\cdots a_{s_{j_2+1},\,s_{j_2+1}}\prod_{k_2=1}^{j_2}\frac{1}{s_{k_2+1}+\cdots+s_{j_2+1}}\Big/\Re(\kappa A)\Big]$$

$$\prod_{k=1}^{j}\frac{1}{n-r_1-\cdots-r_k}$$

$$=(-1)^2\sum_{j_2=2}^{n-1}{}'(-1)^{\frac{j_2}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_2+1}=n)}}\cdots\sum_{r_{j_2+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2+1}-1)!}\sum_{j_1=1}^{j_2-1}{}'\sum_{h_2=1}^{j_2-j_1+1}a_{r_1,r_1}\cdots$$

$$a_{r_{j_1+1},r_{j_1+1}}\cdots a_{r_{j_1+h_2},r_{j_1+h_2-1}}\cdots a_{r_{j_2+1},r_{j_2+1}}\frac{1}{n-r_1}\prod_{k_1=2}^{j_1}\frac{1}{r_{k_1}+\cdots+r_{j_1}}$$

$$\prod_{k_2=1}^{j_2-j_1}\frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}}$$

$$+(-1)^3\sum_{j_2=2}^{n-2}{}'(-1)^{\frac{j_2}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_2+1}=n-1)}}\cdots\sum_{r_{j_2+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_2+1},r_{j_2+1}}\sum_{j_1=1}^{j_2-1}{}'\frac{r_{j_1+1}+\cdots+r_{j_2+1}+1}{r_{j_1+1}+\cdots+r_{j_2+1}}2\frac{1}{n-r_1}\prod_{k_1=2}^{j_1}\frac{1}{r_{k_1}+\cdots+r_{j_1}}\prod_{k_2=1}^{j_2-j_1}$$

$$\frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}}\Re(B\kappa^2)\Big/\Re(\kappa A)$$

$$+(-1)^2\sum_{j_2=2}^{n-2}{}'(-1)^{\frac{j_2}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_2+1}=n-1)}}\sum_{r_2=1}\cdots\sum_{r_{j_2+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_2+1},r_{j_2+1}}\sum_{j_1=1}^{j_2-1}{}'\frac{r_{j_1+1}+\cdots+r_{j_2+1}+1}{r_{j_1+1}+\cdots+r_{j_2+1}}2^{j_2-j_1+1}\frac{1}{n-r_1}\prod_{k_1=2}^{j_1}\frac{1}{r_{k_1}+\cdots+r_{j_1}}$$

$$\prod_{k_2=1}^{j_2-j_1}\frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}}\Re(B\kappa^2)\Big/\Re(\kappa A)$$

$$+(-1)^2\sum_{j_2=2}^{n-2}{}''(-1)^{\frac{j_2+1}{2}}\sum_{\substack{r_1=1\\(r_1+r_2+\cdots+r_{j_2+1}=n-1)}}\sum_{r_2=1}\cdots\sum_{r_{j_2+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_2+1},r_{j_2+1}}\sum_{j_1=1}^{j_2-1}{}'\frac{r_{j_1+1}+\cdots+r_{j_2+1}+1}{r_{j_1+1}+\cdots+r_{j_2+1}}2^{j_2-j_1+1}\frac{1}{n-r_1}\prod_{k_1=2}^{j_1}\frac{1}{r_{k_1}+\cdots+r_{j_1}}$$

$$\prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}} \Im(B\kappa^2)\Big/\Re(\kappa A). \tag{51}$$

Dieses Verfahren setzen wir fort, so gewinnen wir aus (48), (49), (50) und (51)

das erste Glied von der rechten Seite der Formel (47)

$$= -\sum_{j_1=1}^{n-2}{}'(-1)^{\frac{j_1}{2}} \sum_{r_1=1}\cdots\sum_{r_{j_1}=1} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j_1}-1)!} a_{r_1,r_1}\cdots a_{r_{j_1},r_{j_1}} \sum_{p=1}^{j_1+1}\alpha_p\beta_p a_{2,1}$$
$$(r_1+\cdots+r_{j_1}=n-2)$$

$$+(-1)^2\sum_{j_2=2}^{n-1}{}'(-1)^{\frac{j_2}{2}} \sum_{r_1=1}\cdots\sum_{r_{j_2+1}=1} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2+1}-1)!} \sum_{j_1=1}^{j_2-1}{}' \sum_{h_2=1}^{j_2-j_1+1} a_{r_1,r_1}\cdots$$
$$(r_1+\cdots+r_{j_2+1}=n)$$

$$a_{r_{j_1},r_{j_1}} a_{r_{j_1+1},r_{j_1+1}}\cdots a_{r_{j_1+h_2},r_{j_1+h_2}-1}\cdots a_{r_{j_2+1},r_{j_2+1}} \sum_{p=1}^{j_1+1}\alpha_p\beta_p$$

$$\prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}}$$

$$+(-1)^3\sum_{j_2=2}^{n-2}{}'(-1)^{\frac{j_2}{2}} \sum_{r_1=1}\cdots\sum_{r_{j_2+1}=1} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2+1}-1)!} a_{r_1,r_1}\cdots a_{r_{j_2+1},r_{j_2+1}}$$
$$(r_1+\cdots+r_{j_2+1}=n-1)$$

$$\sum_{j_1=1}^{j_2-1}{}' \frac{r_{j_1+1}+\cdots+r_{j_2+1}+1}{r_{j_1+1}+\cdots+r_{j_2+1}}\, 2\sum_{p=1}^{j_1+1}\alpha_p\beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}} \Re(B\kappa^2)\Big/\Re(\kappa A)$$

$$+(-1)^2\sum_{j_2=2}^{n-2}{}'(-1)^{\frac{j_2}{2}} \sum_{r_1=1}\cdots\sum_{r_{j_2+1}=1} \frac{(n-1)}{(r_1-1)!\cdots(r_{j_2+1}-1)!} a_{r_1,r_1}\cdots a_{r_{j_2+1},r_{j_2+1}}$$
$$(r_1+\cdots+r_{j_2+1}=n-1)$$

$$\sum_{j_1=1}^{j_2-1}{}' \frac{r_{j_1+1}+\cdots+r_{j_2+1}+1}{r_{j_1+1}+\cdots+r_{j_2+1}}\, 2^{j_2-j_1+1}\sum_{p=1}^{j_1+1}\alpha_p\beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}} \Re(B\kappa^2)\Big/\Re(\kappa A)$$

$$+(-1)^2\sum_{j_2=2}^{n-2}{}''(-1)^{\frac{j_2+1}{2}} \sum_{r_1=1}\cdots\sum_{r_{j_2+1}=1} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2+1}-1)!} a_{r_1,r_1}\cdots a_{r_{j_2+1},r_{j_2+1}}$$
$$(r_1+\cdots+r_{j_2+1}=n-1)$$

$$\sum_{j_1=1}^{j_2-1}{}' \frac{r_{j_1+1}+\cdots+r_{j_2+1}+1}{r_{j_1+1}+\cdots+r_{j_2+1}}\, 2^{j_2-j_1+1}\sum_{p=1}^{j_1+1}\alpha_p\beta_p$$

$$\prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}} \Im(B\kappa^2)\Big/\Re(\kappa A), \tag{52}$$

wo
$$\alpha_p=\begin{cases}\displaystyle\prod_{k_1=p}^{j_1}\frac{1}{r_{k_1}+\cdots+r_{j_1}} & (\text{wenn } p<j_1+1 \text{ ist})\\ 1 & (\text{wenn } p=j_1+1 \text{ ist})\end{cases}$$
und

$$\beta_p=\begin{cases}1 & \text{(wenn } p=1 \text{ ist)}\\ \prod_{\kappa_1=1}^{p-1}\frac{1}{n-r_1-\cdots-r_{\kappa_1}} & \text{(wenn } p>1 \text{ ist)}\end{cases} \quad \text{ist.}$$

Wir wollen nun das zweite Glied von der rechten Seite der Formel (52) behandeln, und wir nehmen erstens an, dass $h_2=1$ ist.

$$\sum_{j_2=2}^{n-1}{}'(-1)^{\frac{j_2}{2}}\sum_{r_1=1}\cdots\sum_{\substack{r_{j_2+1}=1\\(r_1+\cdots+r_{j_2+1}=n)}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2+1}-1)!}\sum_{j_1=1}^{j_2-1}{}'a_{r_1,r_1}\cdots$$

$$a_{r_{j_1},r_{j_1}}a_{r_{j_1+1},r_{j_1+1}-1}\cdots a_{r_{j_2+1},r_{j_2+1}}\sum_{p=1}^{j_1+1}\alpha_p\beta_p\prod_{k_2=1}^{j_2-j_1}\frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}}$$

$$=\sum_{j_2=2}^{n-2}{}'(-1)^{\frac{j_2}{2}}\sum_{r_1=1}\cdots\sum_{r_{j_1}=1}\sum_{r_{j_1+2}=1}\cdots\sum_{\substack{r_{j_2+1}=1\\(r_1+\cdots+r_{j_1}+r_{j_1+2}+\cdots+r_{j_2+1}=n-2)}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_1}-1)!(r_{j_1+2}-1)!\cdots(r_{j_2+1}-1)!}$$

$$\sum_{j_1=1}^{j_2-1}{}'a_{r_1,r_1}\cdots a_{r_{j_1},r_{j_1}}a_{r_{j_1+2},r_{j_1+2}}\cdots a_{r_{j_2+1},r_{j_2+1}}\sum_{p=1}^{j_1+1}\alpha_p\beta_p\prod_{k_2=1}^{j_2-j_1}\frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}}a_{2,1}$$

$$+\sum_{j_2=2}^{n-2}{}'(-1)^{\frac{j_2}{2}}\sum_{r_1=1}\cdots\sum_{r_{j_1}=1}\sum_{r_{j_1+1}=3}\cdots\sum_{\substack{r_{j_2+1}=1\\(r_1+\cdots+r_{j_2+1}=n,\,r_{j_1+1}\geqq 3)}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_1}-1)!(r_{j_1+1}-1)!\cdots(r_{j_2+1}+1)!}$$

$$\sum_{j_1=1}^{j_2-1}{}'a_{r_1,r_1}\cdots a_{r_{j_1},r_{j_1}},a_{r_{j_1+1};r_{j_1+1}-1}\cdots a_{r_{j_2+1};r_{j_2+1}}\sum_{p=1}^{j_1+1}\alpha_p\beta_p\prod_{k_2=1}^{j_2-j_1}\frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}}.$$

Es ist aber
das erste Glied von der rechten Seite der obigen Formel

$$=\sum_{j_2=2}^{n-2}{}'(-1)^{\frac{j_2}{2}}\sum_{r_1=1}\cdots\sum_{\substack{r_{j_2}=1\\(r_1+\cdots+r_{j_2}=n-2)}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2}-1)!}\sum_{j_1=1}^{j_2-1}{}'a_{r_1,r_1}$$

$$a_{r_{j_1},r_{j_1}}a_{r_{j_1+1},r_{j_1+1}}\cdots a_{r_{j_2},r_{j_2}}\sum_{p=1}^{j_1+1}\alpha_p\beta_p\prod_{k_2=1}^{j_2-j_1}\frac{1}{r_{j_1+k_2}+\cdots+r_{j_2}}a_{2,1}. \tag{53}$$

Das zweite Glied von der rechten Seite der obigen Formel

$$=\sum_{j_2=2}^{n-2}{}'(-1)^{\frac{j_2}{2}}\sum_{r_1=1}\cdots\sum_{r_{j_1+1}=3}\cdots\sum_{\substack{r_{j_2+1}=1\\(r_1+\cdots+r_{j_2+1}=n,\,r_{j_1+1}\geqq 3)}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_1}-1)!(r_{j_1+1}-1)!\cdots(r_{j_2+1}-1)!}$$

$$\sum_{j_1=1}^{j_2-1}{}'a_{r_1,r_1}\cdots a_{r_{j_1},r_{j_1}}a_{r_{j_1+2},r_{j_1+2}}\cdots a_{r_{j_2+1},r_{j_2+1}}\times-\Bigg[\sum_{j_3=1}^{r_{j_1+1}-1}{}'(-1)^{\frac{j_3}{2}}\sum_{s_1=1}\cdots\sum_{\substack{s_{j_3+1}=1\\(s_1+\cdots+s_{j_3+1}=r_{j_1+1})}}$$

$$\frac{(r_{j_1+1}-1)!}{(s_1-1)!\cdots(s_{j_3+1}-1)!}\sum_{h_3=1}^{j_3+1} a_{s_1,r_1}\cdots a_{s_{h_3},s_{h_3}-1}\cdots a_{s_{j_3+1},s_{j_3+1}}\prod_{k_3=1}^{j_3}\frac{1}{s_{k_3+1}+\cdots+s_{j_3+1}}$$

$$+2\,{}_{r_{j_1+1}}C_1\Re(B\kappa^2)a_{r_{j_1+1}-1,r_{j_1+1}-1}\Big/\Re(\kappa A)+{}_{r_{j_1+1}}C_1\Big\{\sum_{j_3=1}^{r_{j_1+1}-2}{}'(-1)^{\frac{j_3}{2}}\Re(B\kappa^2)$$

$$+\sum_{j_3=1}^{r_{j_1+1}-2}{}''(-1)^{\frac{j_3+1}{2}}\Im(B\kappa^2)\Big\}\underset{(s_1+\cdots+s_{j_3+1}=r_{j_1+1}-1)}{\sum_{s_1=1}\cdots\sum_{s_{j_3+1}=1}}\frac{(r_{j_1+1}-2)!\,2^{j_3+1}}{(s_1-1)!\cdots(s_{j_3+1}-1)!}a_{s_1,s_1}\cdots a_{s_{j_3+1},s_{j_3+1}}$$

$$\prod_{k_3=1}^{j_3}\frac{1}{s_{k_3+1}+\cdots+s_{j_3+1}}\Big/\Re(\kappa A)\Big]\sum_{p=1}^{j_1+1}\alpha_p\beta_p\prod_{k_2=1}^{j_2-j_1}\frac{1}{r_{j_1+k_2+1}+\cdots+r_{j_2+1}}$$

$$=-\sum_{j_3=3}^{n-1}{}'(-1)^{\frac{j_3}{2}}\underset{(r_1+\cdots+r_{j_3+1}=n)}{\sum_{r_1=1}\cdots\sum_{r_{j_3+1}=1}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3+1}-1)!}\sum_{j_2=2}^{j_3-1}{}'\sum_{j_1=1}^{j_2-1}{}'a_{r_1,r_1}\cdots$$

$$a_{r_{j_2},r_{j_2}}\sum_{h_3=1}^{j_3-j_2+1}a_{r_{j_2+1},r_{j_2+1}}\cdots a_{r_{j_2+h_3},r_{j_2+h_3}-1}\cdots a_{r_{j_3+1},r_{j_3+1}}\sum_{p=1}^{j_1+1}\alpha_p\beta_p\prod_{k_2=1}^{j_2-j_1}\frac{1}{r_{j_1+k_2}+\cdots+r_{j_2}}$$

$$\prod_{k_3=1}^{j_3-j_2}\frac{1}{r_{j_2+k_3+1}+\cdots+r_{j_3+1}}$$

$$+\sum_{j_3=3}^{n-2}{}'(-1)^{\frac{j_3}{2}}\underset{(r_1+\cdots+r_{j_3+1}=n-1)}{\sum_{r_1=1}\cdots\sum_{r_{j_3+1}=1}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3+1}-1)!}a_{r_1,r_1}\cdots a_{r_{j_3+1},r_{j_3+1}}$$

$$\sum_{j_2=2}^{j_3-1}{}'\sum_{j_1=1}^{j_2-1}{}'\frac{r_{j_2+1}+\cdots+r_{j_3+1}+1}{r_{j_2+1}+\cdots+r_{j_3+1}}\,2\sum_{p=1}^{j_1+1}\alpha_p\beta_p\prod_{k_2=1}^{j_2-j_1}\frac{1}{r_{j_1+k_2}+\cdots+r_{j_2}}$$

$$\prod_{k_3=1}^{j_3-j_2}\frac{1}{r_{j_2+k_3+1}+\cdots+r_{j_3+1}}\Re(B\kappa^2)\Big/\Re(\kappa A)$$

$$-\sum_{j_3=3}^{n-2}{}'(-1)^{\frac{j_3}{2}}\underset{(r_1+\cdots+r_{j_3+1}=n-1)}{\sum_{r_1=1}\cdots\sum_{r_{j_3+1}=1}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3+1}-1)!}a_{r_1,r_1}\cdots a_{r_{j_3+1},r_{j_3+1}}$$

$$\sum_{j_2=2}^{j_3-1}{}'\sum_{j_1=1}^{j_2-1}{}'\frac{r_{j_2+1}+\cdots+r_{j_3+1}+1}{r_{j_2+1}+\cdots+r_{j_3+1}}\,2^{j_3-j_2+1}\sum_{p=1}^{j_1+1}\alpha_p\beta_p\prod_{k_2=1}^{j_2-j_1}\frac{1}{r_{j_1+k_2}+\cdots+r_{j_2}}$$

$$\prod_{k_3=1}^{j_3-j_2}\frac{1}{r_{j_2+k_3+1}+\cdots+r_{j_3+1}}\Re(B\kappa^2)\Big/\Re(\kappa A)$$

$$-\sum_{j_3=3}^{n-2}{}''(-1)^{\frac{j_3+1}{2}}\underset{(r_1+\cdots+r_{j_3+1}=n-1)}{\sum_{r_1=1}\cdots\sum_{r_{j_3+1}=1}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3+1}-1)!}a_{r_1,r_1}\cdots a_{r_{j_3+1},r_{j_3+1}}$$

$$\sum_{j_2=2}^{j_3-1}{}' \sum_{j_1=1}^{j_2-1}{}' \frac{r_{j_2+1}+\cdots+r_{j_3+1}+1}{r_{j_2+1}+\cdots+r_{j_3+1}} 2^{j_3-j_2+1} \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2}+\cdots+r_{j_2}}$$

$$\prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1}+\cdots+r_{j_3+1}} \Im(B\kappa^2)/\Re(\kappa A). \qquad (54)$$

Indem wir ganze analoge Berechnungen für $h_2=2, \cdots, j_2-j_1+1$ ausführen, gewinnen wir aus (52), (53) und (54)

das zweite Glied von der rechten Seite der Formel (52)

$$=(-1)^2 \sum_{j_2=2}^{n-1}{}' (-1)^{\frac{j_2}{2}} \sum_{\substack{r_1=1 \\ (r_1+\cdots+r_{j_2}=n-2)}} \cdots \sum_{r_{j_2}=1} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j_2}-1)!} a_{r_1,r_1}\cdots$$

$$a_{r_{j_2},r_{j_2}} \sum_{j_1=1}^{j_2-1}{}' \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2 a_{2,1} + (-1)^3 \sum_{j_3=3}^{n-1}{}' (-1)^{\frac{j_3}{2}} \sum_{\substack{r_1=1 \\ (r_1+\cdots+r_{j_3+1}=n)}} \cdots \sum_{r_{j_3+1}=1}$$

$$\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3+1}-1)!} \sum_{j_2=2}^{j_3-1}{}' \sum_{j_1=1}^{j_2-1}{}' a_{r_1,r_1}\cdots \quad a_{r_{j_2},r_{j_2}} \sum_{h_3=1}^{j_3-j_2+1} a_{r_{j_2+1},r_{j_2+1}}\cdots$$

$$a_{r_{j_2+h_3},r_{j_2+h_3}-1} \cdots a_{r_{j_3+1},r_{j_3+1}} \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2 \prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1}+\cdots+r_{j_3+1}}$$

$$+(-1)^4 \sum_{j_3=3}^{n-2}{}' (-1)^{\frac{j_3}{2}} \sum_{\substack{r_1=1 \\ (r_1+\cdots+r_{j_3+1}=n-1)}} \cdots \sum_{r_{j_3+1}=1} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3+1}-1)!} a_{r_1,r_1}\cdots$$

$$a_{r_{j_3+1},r_{j_3+1}} \sum_{j_2=2}^{j_3-1}{}' \sum_{j_1=1}^{j_2-1}{}' \frac{r_{j_2+1}+\cdots+r_{j_3+1}+1}{r_{j_2+1}+\cdots+r_{j_3+1}} 2 \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2$$

$$\prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1}+\cdots+r_{j_3+1}} \Re(B\kappa^2)/\Re(\kappa A)$$

$$+(-1)^3 \sum_{j_3=3}^{n-2}{}' (-1)^{\frac{j_3}{2}} \sum_{\substack{r_1=1 \\ (r_1+\cdots+r_{j_3+1}=n-1)}} \cdots \sum_{r_{j_3+1}=1} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3+1}-1)!} a_{r_1,r_1}\cdots$$

$$a_{r_{j_3+1},r_{j_3+1}} \sum_{j_2=2}^{j_3-1}{}' \sum_{j_1=1}^{j_2-1}{}' \frac{r_{j_2+1}+\cdots+r_{j_3+1}+1}{r_{j_2+1}+\cdots+r_{j_3+1}} 2^{j_3-j_2+1} \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2$$

$$\prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1}+\cdots+r_{j_3+1}} \Re(B\kappa^2)/\Re(\kappa A)$$

$$+(-1)^3 \sum_{j_3=3}^{n-2}{}'' (-1)^{\frac{j_3+1}{2}} \sum_{\substack{r_1=1 \\ (r_1+\cdots+r_{j_3+1}=n-1)}} \cdots \sum_{r_{j_3+1}=1} \frac{(n-1)!}{(r_1-1)!\cdots(r_{j_3+1}-1)!} a_{r_1,r_1}\cdots$$

$$a_{r_{j_3+1},r_{j_3+1}}\sum_{j_2=2}^{j_3-1}{}'\sum_{j_1=1}^{j_2-1}{}'\frac{r_{j_2+1}+\cdots+r_{j_3+1}+1}{r_{j_2+1}+\cdots+r_{j_3+1}}2^{j_3-j_2+1}\sum_{p_1=1}^{j_1+1}\alpha^1_{p_1}\beta^1_{p_1}\sum_{p_2=1}^{j_2+1}\alpha^2_{p_2}\beta^2_{p_2}$$

$$\prod_{k_3=1}^{j_3-j_2}\frac{1}{r_{j_2+k_3+1}+\cdots+r_{j_3+1}}\Im(B\kappa^2)/\Re(\kappa A), \tag{55}$$

wo

$$\alpha^1_{p_1}=\begin{cases}\prod_{k_1=p_1}^{j_1}\frac{1}{r_{k_1}+\cdots+r_{j_1}} & (\text{wenn } 1\leqq p_1\leqq j_1 \text{ ist})\\ 1 & (\text{wenn } p_1=j_1+1 \text{ ist})\end{cases}\quad \text{und}$$

$$\beta^1_{p_1}=\begin{cases}1 & (\text{wenn } p_1=1 \text{ ist})\\ \prod_{k_1=1}^{p_1-1}\frac{1}{n-r_1-\cdots-r_{k_1}} & (\text{wenn } 2\leqq p_1\leqq j_1+1 \text{ ist})\end{cases}\quad \text{und}$$

$$\alpha^2_{p_2}=\begin{cases}\prod_{k_2=p_2}^{j_2-j_1}\frac{1}{r_{j_1+k_2}+\cdots+r_{j_2}} & (\text{wenn } 1\leqq p_2\leqq j_2-j_1 \text{ ist})\\ 1 & (\text{wenn } p_2=j_2-j_1+1 \text{ ist})\end{cases}\quad \text{und}$$

$$\beta^2_{p_2}=\begin{cases}1 & (\text{wenn } p_2=1 \text{ ist})\\ \prod_{k_2=1}^{p_2-1}\frac{1}{n-r_1-\cdots-r_{j_1+k_2}} & (\text{wenn } 2\leqq p_2\leqq j_2-j_1+1 \text{ ist})\end{cases}\quad \text{ist.}$$

Dieses Verfahren setzen wir fort, so gewinnen wir endlich

$$a_{n,n-1}=\sum_{l=2}^{n-2}(-1)^l\sum_{j_l=l}^{n-2}{}'(-1)^{\frac{j_l}{2}}\sum_{r_1=1}\cdots\sum_{\substack{r_{j_l}=1\\(r_1+\cdots+r_{j_l}=n-2)}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_l}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_l},r_{j_l}}\sum_{j_{l-1}=1}^{j_l-1}{}'\sum_{j_{l-2}=1}^{j_{l-1}-1}{}'\cdots\sum_{j_2=1}^{j_3-1}{}'\sum_{j_1=1}^{j_2-1}{}'\sum_{p_1=1}^{j_1+1}\alpha^1_{p_1}\beta^1_{p_1}\sum_{p_2=1}^{j_2+1}\alpha^2_{p_2}\beta^2_{p_2}\cdots\sum_{p_l=1}^{j_l+1}\alpha^l_{p_l}\beta^l_{p_l}a_{2,1}$$

$$-\sum_{j_1=1}^{n-2}{}'(-1)^{\frac{j_1}{2}}\sum_{r_1=1}\cdots\sum_{\substack{r_{j_1+1}=1\\(r_1+\cdots+r_{j_1+1}=n-1)}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_1+1}-1)!}a_{r_1,r_1}\cdots a_{r_{j_1+1},r_{j_1+1}}$$

$$\frac{r_1+r_2+\cdots+r_{j_1+1}+1}{r_1+r_2+\cdots+r_{j_1+1}}2^{j_1+1}\prod_{k_1=1}^{j_1}\frac{1}{r_{k_1+1}+\cdots+r_{j_1+1}}\Re(B\kappa^2)/\Re(\kappa A)$$

$$+\sum_{l=2}^{n-2}(-1)^l\sum_{j_l=l}^{n-2}{}'(-1)^{\frac{j_l}{2}}\sum_{r_1=1}\cdots\sum_{\substack{r_{j_l+1}=1\\(r_1+\cdots+r_{j_l+1}=n-1)}}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_l+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_l+1},r_{j_l+1}}\sum_{j_{l-1}=1}^{j_l-1}{}'\sum_{j_{l-2}=1}^{j_{l-1}-1}{}'\cdots\sum_{j_2=1}^{j_3-1}{}'\sum_{j_1=1}^{j_2-1}{}'\frac{r_{j_{l-1}+1}+\cdots+r_{j_l+1}+1}{r_{j_{l-1}+1}+\cdots+r_{j_l+1}}2^{j_l-j_{l-1}+1}$$

$$\sum_{p_1=1}^{j_1+1}\alpha^1_{p_1}\beta^1_{p_1}\sum_{p_2=1}^{j_2+1}\alpha^2_{p_2}\beta^2_{p_2}\cdots\sum_{p_l=1}^{j_l+1}\alpha^l_{p_l}\beta^l_{p_l}\prod_{k_l=1}^{j_l-j_{l-1}}\frac{1}{r_{j_{l-1}+k_l+1}+\cdots+r_{j_l+1}}\Re(B\kappa^2)\Big/\Re(\kappa A)$$

$$-\sum_{j_1=1}^{n-2}{}''(-1)^{\frac{j_1+1}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_1+1}=n-1)}}\cdots\sum_{r_{j_1+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_1+1}-1)!}a_{r_1,r_1}\cdots a_{r_{j_1+1},r_{j_1+1}}$$

$$\frac{r_1+\cdots+r_{j_1+1}+1}{r_1+\cdots+r_{j_1+1}}2^{j_1+1}\prod_{k_1=1}^{j_1}\frac{1}{r_{k_1+1}+\cdots+r_{j_1+1}}\Im(B\kappa^2)\Big/\Re(\kappa A)$$

$$+\sum_{l=2}^{n-2}(-1)^l\sum_{j_l=l}^{n-2}{}'(-1)^{\frac{j_l+1}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_l+1}=n-1)}}\cdots\sum_{r_{j_l+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_l+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_l+1},r_{j_l+1}}\sum_{j_{l-1}=1}^{j_l-1}{}'\sum_{j_{l-2}=1}^{j_{l-1}-1}{}'\cdots\sum_{j_2=1}^{j_3-1}{}'\sum_{j_1=1}^{j_2-1}{}'\frac{r_{j_{l-1}+1}+\cdots+r_{j_l+1}+1}{r_{j_{l-1}+1}+\cdots+r_{j_l+1}}2^{j_l-j_{l-1}+1}$$

$$\sum_{p_1=1}^{j_1+1}\alpha^1_{p_1}\beta^1_{p_1}\sum_{p_2=1}^{j_2+1}\alpha^2_{p_2}\beta^2_{p_2}\cdots\sum_{p_l=1}^{j_l+1}\alpha^l_{p_l}\beta^l_{p_l}\prod_{k_l=1}^{j_l-j_{l-1}}\frac{1}{r_{j_{l-1}+k_l+1}+\cdots+r_{j_l+1}}\Im(B\kappa^2)\Big/\Re(\kappa A)$$

$$+(-1)^2\sum_{j_1=1}^{n-2}{}'(-1)^{\frac{j_1}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_1+1}=n-1)}}\cdots\sum_{r_{j_1+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_1+1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_1+1},r_{j_1+1}}\frac{r_1+r_2+\cdots+r_{j_1+1}+1}{r_1+r_2+\cdots+r_{j_1+1}}2\Re(B\kappa^2)\Big/\Re(\kappa A)\quad+\sum_{l=2}^{n-2}(-1)^{l+1}\sum_{j_l=l}^{n-2}(-1)^{\frac{j_l}{2}}$$

$$\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_l+1}=n-1)}}\cdots\sum_{r_{j_l+1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_l+1}-1)!}a_{r_1,r_1}\cdots\quad a_{r_{j_l+1},r_{j_l+1}}$$

$$\sum_{j_{l-1}=1}^{j_l-1}{}'\sum_{j_{l-2}=1}^{j_{l-1}-1}{}'\cdots\sum_{j_2=1}^{j_3-1}{}'\sum_{j_1=1}^{j_2-1}{}'\frac{r_{j_{l-1}+1}+\cdots+r_{j_l+1}+1}{r_{j_{l-1}+1}+\cdots+r_{j_l+1}}2\sum_{p_1=1}^{j_1+1}\alpha^1_{p_1}\beta^1_{p_1}\sum_{p_2=1}^{j_2+1}\alpha^2_{p_2}\beta^2_{p_2}\cdots$$

$$\sum_{p_l=1}^{j_l+1}\alpha^l_{p_l}\beta^l_{p_l}\prod_{k_l=1}^{j_l-j_{l-1}}\frac{1}{r_{j_{l-1}+k_l+1}+\cdots+r_{j_l+1}}\Re(B\kappa^2)\Big/\Re(\kappa A)$$

$$-\sum_{j_1=1}^{n-2}{}'(-1)^{\frac{j_1}{2}}\sum_{\substack{r_1=1\\(r_1+\cdots+r_{j_1}=n-2)}}\cdots\sum_{r_{j_1}=1}\frac{(n-1)!}{(r_1-1)!\cdots(r_{j_1}-1)!}a_{r_1,r_1}\cdots$$

$$a_{r_{j_1},r_{j_1}}\sum_{p_1=1}^{j_1+1}\alpha^1_{p_1}\beta^1_{p_1}a_{2,1},\tag{56}$$

wo

$$\alpha^1_{p_1}=\begin{cases}\displaystyle\prod_{k_1=p_1}^{j_1}\frac{1}{r_{k_1}+\cdots+r_{j_1}} & (\text{wenn } 1\leqq p_1\leqq j_1 \text{ ist})\\ 1 & (\text{wenn } p_1=j_1+1 \text{ ist})\end{cases}\qquad\text{und}$$

$$\left.\begin{array}{l}
\beta^1_{p_1}=\begin{cases} 1 & (\text{wenn } p_1=1 \text{ ist}) \\ \prod\limits_{k_1=1}^{p_1-1} \dfrac{1}{n-r_1-\cdots-r_{k_1}} & (\text{wenn } 2\leqq p_1\leqq j_1+1 \text{ ist}) \end{cases} \quad \text{und} \\
\alpha^s_{p_s}=\begin{cases} \prod\limits_{k_s=p_s}^{j_s-j_{s-1}} \dfrac{1}{r_{j_{s-1}+k_s}+\cdots+r_{j_s}} & (\text{wenn } 1\leqq p_s\leqq j_s-j_{s-1} \text{ ist}) \\ 1 & (\text{wenn } p_s=j_s-j_{s-1}+1 \text{ ist}) \end{cases} \quad \text{und} \\
\beta^s_{p_s}=\begin{cases} 1 & (\text{wenn } p_s=1) \\ \prod\limits_{k_s=1}^{p_s-1} \dfrac{1}{n-r_1-\cdots-r_{j_{s-1}+k_s}} & (\text{wenn } 2\leqq p_s\leqq j_s-j_{s-1}+1 \text{ ist}) \end{cases} \quad \text{ist.} \\
\qquad\qquad (\text{für } s=2,3,\cdots,n-2)
\end{array}\right\} \quad (57)$$

MATEMATISCHES INSTITUT.
UIVIVERSITÄT ZU OKAYAMA.

(*Eingegangen am 14. November, 1962*)