ON THE REPRESENTATIONS OF THE GENERALIZED SYMMETRIC GROUP II

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Introduction. We have determined [**11**] the ordinary irreducible characters of the generalized symmetric group $S(n, m)$. As a sequel of [**11**], we shall investigate in this paper some modular properties of irreducibe representations of $S(n, m)$. In §§1 and 2 we shall give, as preliminaries, some general results concerning the induced representations of a group. Some of them are well known. In §§3 and 4, using these results we shall obtain necessary and sufficient conditions that two irreducible representations of $S(n, m)$ belong to the same block and determine the defect groups of the blocks of $S(n, m)$. We shall also deal with the modular irreducible representations of $S(n, m)$.

1. Let $\mathfrak{G}$ be a group of finite order g and let $\mathfrak{H}$ be an invariant subgroup of $\mathfrak{G}$. We consider the representations of $\mathfrak{G}$ in the field K of the g-th roots of unity. Then every absolutely irreducible representation of $\mathfrak{G}$ can be written with coefficients in K. We denote by $\Gamma(\mathfrak{G})$ the group ring of $\mathfrak{G}$ over K and by $\Delta(\mathfrak{G})$ its center. Similarly we define $\Gamma(\mathfrak{H})$ and $\Delta(\mathfrak{H})$ of $\mathfrak{H}$. Let $e_1, e_2, \dots, e_m$ be the primitive idempotents of $\Delta(\mathfrak{H})$ such that $\sum e_\kappa = 1$. As is well known, $\Gamma(\mathfrak{H})$ is semisimple and is a direct sum of m simple two-sided ideals $\Gamma(\mathfrak{H})e_\kappa$ which themselves are matric algebras over K:

$$(1.1) \qquad \Gamma(\mathfrak{H}) = \Gamma(\mathfrak{H})e_1 + \Gamma(\mathfrak{H})e_2 + \dots + \Gamma(\mathfrak{H})e_m.$$

Let $H \to V_\kappa(H)$ be the irreducible representation of $\mathfrak{H}$ defined by a simple left-ideal contained in $\Gamma(\mathfrak{H})e_\kappa$ and let ζ_κ be its character. Evidently $Ge_1G^{-1}, Ge_2G^{-1}, \dots, Ge_mG^{-1}$ is a permutation of $e_1, e_2, \dots, e_m$. The totality of elements G of $\mathfrak{G}$ which satisfy $Ge_\kappa G^{-1} = e_\kappa$ for a fixed idempotent e_κ constitutes a subgroup $\mathfrak{G}_\kappa$ of $\mathfrak{G}$. We say that $\mathfrak{G}_\kappa$ is a subgroup of $\mathfrak{G}$ corresponding to e_κ.

Lemma 1. *$\zeta_\kappa(G^{-1}HG) = \zeta_\kappa(H)$ for all $H \in \mathfrak{H}$ if and only if G belongs to $\mathfrak{G}_\kappa$.*

Proof. Let G be an arbitrary element of $\mathfrak{G}$. The simple two-sided ideal $\Gamma(\mathfrak{H})Ge_\kappa G^{-1} = G\Gamma(\mathfrak{H})e_\kappa G^{-1}$ of $\Gamma(\mathfrak{H})$ determines the irreducible representation $H \to V_\kappa(G^{-1}HG)$ of $\mathfrak{H}$. This shows that the lemma is true.

Let $(\mathfrak{G} : \mathfrak{G}_\kappa) = s_\kappa$ and let Q_i $(i = 1, 2, \ldots, s_\kappa)$ be a complete residue system of $\mathfrak{G}$ (mod $\mathfrak{G}_\kappa$):

$$\mathfrak{G} = \mathfrak{G}_\kappa Q_1 + \mathfrak{G}_\kappa Q_2 + \ldots + \mathfrak{G}_\kappa Q_{s_\kappa}, \qquad Q_1 = 1. \tag{1.2}$$

We set

$$Q_i e_\kappa Q_i^{-1} = e_\kappa^{(i)} \qquad (i = 1, 2, \ldots, s_\kappa). \tag{1.3}$$

Let us denote by $\zeta_\kappa^{(i)}$ the character of $\mathfrak{H}$ corresponding to $e_\kappa^{(i)}$. We then say that two characters $\zeta_\kappa^{(i)}$ and $\zeta_\kappa^{(j)}$ of $\mathfrak{H}$ are *associated in* $\mathfrak{G}$. Thus the characters ζ of $\mathfrak{H}$ are distributed in k classes of characters of $\mathfrak{H}$ which are associated in $\mathfrak{G}$, where k is equal to the number of classes of conjugate elements of $\mathfrak{G}$ which are contained in $\mathfrak{H}$. Let $\zeta_1, \zeta_2, \ldots, \zeta_k$ be a complete system of representatives for these classes. We set

$$E_\kappa = e_\kappa^{(1)} + e_\kappa^{(2)} + \ldots + e_\kappa^{(s_\kappa)}. \tag{1.4}$$

We then have $GE_\kappa G^{-1} = E_\kappa$ for all $G \in \mathfrak{G}$ and hence E_κ is an idempotent of $\Lambda(\mathfrak{G})$. Since $\sum E_\kappa = 1$, we have

$$\Gamma(\mathfrak{G}) = \Gamma(\mathfrak{G})E_1 + \Gamma(\mathfrak{G})E_2 + \ldots + \Gamma(\mathfrak{G})E_k, \tag{1.5}$$

where the two-sided ideals $\Gamma(\mathfrak{G})E_\kappa$ can be represented as direct sums since the E_κ are not primitive in $\Lambda(\mathfrak{G})$ in general. The irreducible representation of $\mathfrak{G}$ defined by a simple left-ideal contained in $\Gamma(\mathfrak{G})E_\kappa$ is called the *representation of* $\mathfrak{G}$ *determined by* ζ_κ. We say that the representations of $\mathfrak{G}$ determined by ζ_κ are *conjugate with respect to* $\mathfrak{H}$.

It follows from $Ge_\kappa G^{-1} = e_\kappa$ for all $G \in \mathfrak{G}_\kappa$ that e_κ is an idempotent of $\Lambda(\mathfrak{G}_\kappa)$. We shall first investigate the structure of the two-sided ideal $\Gamma(\mathfrak{G}_\kappa)e_\kappa$ of $\Gamma(\mathfrak{G}_\kappa)$ by the same way as in [7]. Since $G\Gamma(\mathfrak{H})e_\kappa G^{-1} = \Gamma(\mathfrak{H})e_\kappa$ for all $G \in \mathfrak{G}_\kappa$, the mapping $a \to GaG^{-1}$, $(a \in \Gamma(\mathfrak{H})e_\kappa)$ is an automorphism of the simple algebra $\Gamma(\mathfrak{H})e_\kappa$ and hence there exists an element $m(G)$ of $\Gamma(\mathfrak{H})e_\kappa$ such that

$$GaG^{-1} = m(G)am(G)^{-1} \qquad (a \in \Gamma(\mathfrak{H})e_\kappa). \tag{1.6}$$

The element $m(G)$ is determined by $G \in \mathfrak{G}_\kappa$ uniquely apart from a factor belonging to K. We can put

$$m(H) = He_\kappa \qquad (H \in \mathfrak{H}).$$

Let $1, S_\sigma, \ldots, S_\rho$ be a complete residue system of $\mathfrak{G}_\kappa$ (mod $\mathfrak{H}$):

$$\mathfrak{G} = \mathfrak{H} + \mathfrak{H}S_\sigma + \ldots + \mathfrak{H}S_\rho. \tag{1.7}$$

We determine an element $m(S_\sigma)$ of $\Gamma(\mathfrak{H})e_\kappa$ by (1. 6) for every S_σ and define for any $G = HS_\sigma \in \mathfrak{G}_\kappa$

$$m(G) = m(H)m(S_\sigma).$$

It is easily seen that

$$m(S_\sigma)m(H) = m(S_\sigma H) \qquad (H \in \mathfrak{H}),$$
$$m(H)m(G) = m(HG) \qquad (G \in \mathfrak{G}_\kappa).$$

Then we can show that the factor set $c(G_1, G_2)$ arising from the relations

$$m(G_1)m(G_2) = c(G_1, G_2)m(G_1G_2) \qquad (G_1, G_2 \in \mathfrak{G}_\kappa) \tag{1.8}$$

is essentially a factor set of $\mathfrak{G}_\kappa/\mathfrak{H}$, that is,

$$c(HS_\sigma, H'S_\tau) = c(S_\sigma, S_\tau). \tag{1.9}$$

Hence we may denote $c(HS_\sigma, H'S_\tau)$ by $c(\sigma, \tau)$, $\sigma, \tau \in \mathfrak{G}_\kappa/\mathfrak{H}$.

If we set

$$a_\sigma = m(S_\sigma)^{-1}S_\sigma, \tag{1.10}$$

then we have

$$a_\sigma a_\tau = c(\sigma, \tau)^{-1}a_{\sigma\tau}. \tag{1.11}$$

It follows from (1.11) that the totality of elements of the form

$$\sum_{\sigma \in \mathfrak{G}_\kappa/\mathfrak{H}} x_\sigma a_\sigma \qquad (x_\sigma \in K)$$

constitutes an algebra $\mathfrak{A}_\kappa$ over K isomorphic to the generalized group ring $\Gamma(\mathfrak{G}_\kappa/\mathfrak{H}, c(\sigma, \tau)^{-1})$ with a factor set $c(\sigma, \tau)^{-1}$. Moreover we see that

$Ha_\sigma = a_\sigma H$ for any $H \in \mathfrak{H}$ so that we have

$$\Gamma(\mathfrak{G}_\kappa)e_\kappa = \Gamma(\mathfrak{H})e_\kappa \times \mathfrak{A}_\kappa, \tag{1.12}$$

where

$$\mathfrak{A}_\kappa \cong \Gamma(\mathfrak{G}_\kappa/\mathfrak{H},\ c(\sigma, \tau)^{-1}). \tag{1.13}$$

If the semisimple algebra $\mathfrak{A}_\kappa$ is a direct sum of u_κ simple two-sided ideals Δ_l, then (1.12) implies that $\Gamma(\mathfrak{G}_\kappa)e_\kappa$ is also a direct sum of u_κ simple two-sided ideals $\Gamma(\mathfrak{H})e_\kappa \times \Delta_l$. Let $\sigma \to W_l(\sigma)$, $(\sigma \in \mathfrak{G}_\kappa/\mathfrak{H})$ be the irreducible projective representation of $\mathfrak{A}_\kappa$ defined by a simple left-ideal contained in Δ_l.

Since

$$\begin{aligned} S_\sigma(Ha_\tau) &= (S_\sigma H S_\sigma^{-1})S_\sigma a_\tau \\ &= m(S_\sigma)\, Hm(S_\sigma)^{-1}S_\sigma a_\tau = (m(S_\sigma)H)a_\sigma a_\tau, \end{aligned}$$

$$G \to V_\kappa(m(G)) \times W_l(\sigma) \qquad (G \in \mathfrak{H}S_\sigma) \tag{1.14}$$

is the irreducible representation of $\mathfrak{G}$ defined by a simple left-ideal contained in $\Gamma(\mathfrak{H})e_\kappa \times \Delta_l$. The representation (1.14) of $\mathfrak{G}_\kappa$ determined by V_κ of $\mathfrak{H}$ and W_l of $\mathfrak{G}_\kappa/\mathfrak{H}$ will be denoted by $Z_{\kappa l}$:

$$Z_{\kappa l}(G) = V_\kappa(m(G)) \times W_l(\sigma). \tag{1.15}$$

We set $Z_{\kappa l}(G) = 0$ if G does not belong to $\mathfrak{G}_\kappa$ so that $Z_{\kappa l}(G)$ is defined for all elements of $\mathfrak{G}$ and define

$$\tilde{Z}_{\kappa l}(G) = (Z_{\kappa l}(Q_u G Q_v^{-1})) \tag{1.16}$$

(u row index, v column index). This is a representation of $\mathfrak{G}$ and is called the representation of $\mathfrak{G}$ induced by $Z_{\kappa l}$ of $\mathfrak{G}_\kappa$. As is well known [5], $\tilde{Z}_{\kappa l}$ is irreducible and every irreducible representation of $\mathfrak{G}$ determined by ζ_κ is obtained in this form. Let us denote by $\tilde{X}_{\kappa l}$ the character of $\tilde{Z}_{\kappa l}$. Let K_ν be a class of conjugate elements of $\mathfrak{G}$ and let g_ν be the number of elements in K_ν. The character $\tilde{X}_{\kappa l}$ of $\mathfrak{G}$ determines a character $\tilde{\omega}_{\kappa l}$ of $\Lambda(\mathfrak{G})$ which is given by

$$\tilde{\omega}_{\kappa l}(K_\nu) = g_\nu \tilde{X}_{\kappa l}(G_\nu)/\tilde{z}_{\kappa l}, \tag{1.17}$$

where G_ν is an element in K_ν and $\tilde{z}_{\kappa l}$ is the degree of $\tilde{Z}_{\kappa l}$. As is well known, we have

(1.18) $$\tilde{Z}_{\kappa t}(K_\nu) = \tilde{\omega}_{\kappa t}(K_\nu) I,$$

where I is the unit matrix of degree $\tilde{z}_{\kappa t}$. Similarly let $\omega_{\kappa t}$ be the character of $\Lambda(\mathfrak{G}_\kappa)$ determined by the character $\chi_{\kappa t}$ of $Z_{\kappa t}$. Then

(1.19) $$Z_{\kappa t}(K_\sigma') = \omega_{\kappa t}(K_\sigma') I,$$

where I is the unit matrix of degree $z_{\kappa t} = \chi_{\kappa t}(1)$. It follows from (1.16), (1.18), and (1.19) that

(1.20) $$\tilde{\omega}_{\kappa t}(K_\nu) = \sum_\sigma{}' \omega_{\kappa t}(K_\sigma'),$$

where K_σ' ranges over all classes of $\mathfrak{G}_\kappa$ which belong to K_ν. If K_ν does not contain elements of $\mathfrak{G}_\kappa$, then

(1.21) $$\tilde{\omega}_{\kappa t}(K_\nu) = 0.$$

2. Let p be a prime and let p^a be the highest power of p dividing g, so that

(2.1) $$g = p^a g', \qquad (g', p) = 1.$$

Let $\mathfrak{o}$ be the ring of integers of K and let $\mathfrak{p}$ be a fixed prime ideal divisor of p in $\mathfrak{o}$. We denote by $\mathfrak{o}^*$ the ring of $\mathfrak{p}$-integers of K and by K^* the residue class field of $\mathfrak{o}^*$ (mod $\mathfrak{p}$). We consider the representations of $\mathfrak{G}$ in K^*. We denote by $\Gamma^*(\mathfrak{G})$ the group ring of $\mathfrak{G}$ over K^* and by $\Lambda^*(\mathfrak{G})$ its center.

Throughout §2 we shall assume that *the order h of $\mathfrak{H}$ is prime to p*. Then we can set $e_\kappa^* = e_\kappa$ (mod $\mathfrak{p}$). Since the e_κ^* are the primitive idempotent of $\Lambda^*(\mathfrak{H})$ such that $\sum' e_\kappa^* = 1$, $\Gamma^*(\mathfrak{H})$ is a direct sum of m simple two-sided ideals $\Gamma^*(\mathfrak{H})e_\kappa^*$ which themselves are matric algebras over K^*:

(2.2) $$\Gamma^*(\mathfrak{H}) = \Gamma^*(\mathfrak{H})e_1^* + \Gamma^*(\mathfrak{H})e_2^* + \ldots + \Gamma^*(\mathfrak{H})e_m^*.$$

Let $H \to V_\kappa^*(H)$ be the irreducible representation of $\mathfrak{H}$ defined by a simple left-ideal contained in $\Gamma^*(\mathfrak{H})e_\kappa^*$. The matrix $V_\kappa^*(H)$ is obtained by replacing every coefficient in $V_\kappa(H)$ with coefficients in $\mathfrak{o}^*$ by its residue class.

Evidently $Ge_\kappa^* G^{-1} = e_\kappa^*$ if and only if G belongs to the subgroup

$\mathfrak{G}_\kappa$ of $\mathfrak{G}$ defined in §1. Hence if we set $E_\kappa^* = E_\kappa \pmod{\mathfrak{p}}$, we have

(2.3) $$\Gamma^*(\mathfrak{G}) = \Gamma^*(\mathfrak{G})E_1^* + \Gamma^*(\mathfrak{G})E_2^* + \ldots + \Gamma^*(\mathfrak{G})E_k^*.$$

By the similar way as in §1, we have

(2.4) $$\Gamma^*(\mathfrak{G}_\kappa)e_\kappa^* = \Gamma^*(\mathfrak{H})e_\kappa^* \times \mathfrak{A}_\kappa^*,$$

where $\mathfrak{A}_\kappa^*$ is an algebra over K^* isomorphic to the generalized group ring $\Gamma^*(\mathfrak{G}_\kappa/\mathfrak{H},\ c^*(\sigma, \tau)^{-1})$ if we set $c^*(\sigma, \tau) = c(\sigma, \tau) \pmod{\mathfrak{p}}$. Hence we have

Lemma 2. *Let $\sigma \to U_\rho(\sigma)$ be the indecomposable constituents of the regular representation of $\mathfrak{A}_\kappa^*$. All indecomposable constituents of the regular representation of $\Gamma^*(\mathfrak{G}_\kappa)$ defined by the indecomposable left-ideals contained in $\Gamma^*(\mathfrak{G}_\kappa)e_\kappa^*$ are given by*

(2.5) $$G \to V_\kappa^*(m(G)) \times U_\rho(\sigma) \qquad (G \in \mathfrak{H}S_\sigma).$$

We set

(2.6) $$U_{\kappa\rho}(G) = V_\kappa^*(m(G)) \times U_\rho(\sigma).$$

Let $\sigma \to F_\rho(\sigma)$ be the irreducible projective representation of $\mathfrak{A}_\kappa^*$ which appears as first constituent in U_ρ. Then (2.4) implies that

(2.7) $$G \to F_{\kappa\rho}(G) = V_\kappa^*(m(G)) \times F_\rho(\sigma) \qquad (G \in \mathfrak{H}S_\sigma)$$

is the modular irreducible representation of $\mathfrak{G}_\kappa$ corresponding to $U_{\kappa\rho}$ in the above sence.

If $\mathfrak{A}_\kappa^*$ is a direct sum of v_κ indecomposable two-sided ideals Σ_λ, then $\Gamma^*(\mathfrak{G}_\kappa)e_\kappa^*$ is also a direct sum of v_κ indecomposable two-sided ideals $\Gamma^*(\mathfrak{H})e_\kappa^* \times \Sigma_\lambda$. Let B_λ' be the blocks of $\mathfrak{A}_\kappa^*$ determined by Σ_κ. To every block B_λ' there corresponds uniquely a block B_λ of $\mathfrak{G}_\kappa$ determined by $\Gamma^*(\mathfrak{H})e_\kappa^* \times \Sigma_\lambda$. The v_κ blocks B_λ of $\mathfrak{G}_\kappa$ determined by $\Gamma^*(\mathfrak{G}_\kappa)e_\kappa^*$ will be called the *blocks of* $\mathfrak{G}_\kappa$ *determined by* ζ_κ. We have by (1.12) and (2.4)

Lemma 3. *Two irreducible representations $Z_{\kappa i}$ and $Z_{\kappa j}$ of $\mathfrak{G}_\kappa$ determined by ζ_κ belong to the same block if and only if two projective representations W_i and W_j of $\mathfrak{G}_\kappa/\mathfrak{H}$ corresponding to $Z_{\kappa i}$ and $Z_{\kappa j}$ belong to the same block of $\mathfrak{A}_\kappa^*$.*

Lemma 4. *Let B' be the block of $\mathfrak{A}_\kappa^*$ and let B be the block of $\mathfrak{G}_\kappa$ determined by B'. The decomposition matrix of B is the same as that of B'.*

Proof. Replacing every coefficient in the projective representation W_i of $\mathfrak{A}_\kappa$ with coefficients in $\mathfrak{o}^*$ by its residue class, we obtain a modular representation W_i^* of $\mathfrak{A}_\kappa^*$. Let $d_{i\rho}^{(\kappa)}$ be the multiplicity of F_ρ as irreducible constituent of W_i^*:

$$W_i^* \leftrightarrow \sum_\rho d_{i\rho}^{(\kappa)} F_\rho. \tag{2.8}$$

According to (1.14) and (2.7)

$$Z_{\kappa i}^* = V_\kappa^* \times W_i^* \leftrightarrow \sum_\rho d_{i\rho}^{(\kappa)} F_{\kappa\rho}. \tag{2.9}$$

This proves the lemma.

Lemma 4 implies that the Cartan matrix of B is the same as that of B'. Furthermore we see by (1.15) and Lemma 3 that B and B' have the same defect[1]. The representation $\widetilde{F}_{\kappa\rho}$ of $\mathfrak{G}$ induced by $F_{\kappa\rho}$ of $\mathfrak{G}_\kappa$ is irreducible and every irreducible representation of $\mathfrak{G}$ determined by ζ_κ is obtained in this form [5]. According to (2.9)

$$\widetilde{Z}_{\kappa i}^* \leftrightarrow \sum_\rho d_{i\rho}^{(\kappa)} \widetilde{F}_{\kappa\rho}, \tag{2.10}$$

where $\widetilde{Z}_{\kappa i}^*$ denotes a modular representation of $\mathfrak{G}$ obtained by replacing every coefficient in $\widetilde{Z}_{\kappa i}$ of $\mathfrak{G}$ with coefficients in $\mathfrak{o}^*$ by its residue class. We see by (1.5) and (2.3) that two irreducible representations of $\mathfrak{G}$ belonging to the same block must be conjugate with respect to $\mathfrak{H}$. Now (2.10) implies that two irreducible representations $\widetilde{Z}_{\kappa i}$ and $\widetilde{Z}_{\kappa j}$ of $\mathfrak{G}$ belong to the same block if and only if $Z_{\kappa i}$ and $Z_{\kappa j}$ of $\mathfrak{G}_\kappa$ belong to the same block. Consequently there exists a (1—1) correspondence between the blocks B of $\mathfrak{G}_\kappa$ determined by ζ_κ and the blocks $\widetilde{B}$ of $\mathfrak{G}$ determined by $\Gamma^*(\mathfrak{G})E_\kappa^*$. Moreover we have by (2.10)

Lemma 5. *Let B be a block of $\mathfrak{G}_\kappa$ determined by ζ_κ of $\mathfrak{H}$ and let $\widetilde{B}$ be the block of $\mathfrak{G}$ corresponding to B. The decomposition matrix of $\widetilde{B}$ is the same as that of B.*

Lemma 5 implies that the Cartan matrix of $\widetilde{B}$ is the same as that of B. Hence we have

1) See [1]. Cf. also [3], [4], and [12].

Lemma 6. *Let $U_{\kappa\rho}$ be the indecomposable constituent of the regular representation of $\mathfrak{G}_\kappa$ defined by an indecomposable left-ideal contained in $\Gamma^*(\mathfrak{G}_\kappa)e_\kappa^*$. The representation $\widetilde{U}_{\kappa\rho}$ of $\mathfrak{G}$ induced by $U_{\kappa\rho}$ is the indecomposable constituent of the regular representation of $\mathfrak{G}$ defined by an indecomposable left-ideal contained in $\Gamma^*(\mathfrak{G})E_\kappa^*$.*

Since every representation of $\mathfrak{G}$ induced by $Z_{\kappa t}$ in B of $\mathfrak{G}_\kappa$ constitutes a block $\widetilde{B}$ of $\mathfrak{G}$, $\widetilde{B}$ and B have the same defect. Now we prove

Lemma 7. *Let B be a block of $\mathfrak{G}_\kappa$ determined by ζ_κ of $\mathfrak{H}$ and let $\widetilde{B}$ be the block of $\mathfrak{G}$ corresponding to B. The defect group[1] of $\widetilde{B}$ is the same as that of B.*

Proof. Let $\widetilde{\mathfrak{D}}$ be the defect group of $\widetilde{B}$ and let p^d be its order. Then the order of a defect group $\mathfrak{D}$ of B is also p^d since two blocks have the same defect. There exists [**4** ; **12**] a p-regular class K_ν of $\mathfrak{G}$ such that $\widetilde{\mathfrak{D}}$ is a defect group of K_ν and that for the $\widetilde{\omega}_{\kappa t}$ belonging to the character $\widetilde{\chi}_{\kappa t}$ in $\widetilde{B}$

$$\widetilde{\omega}_{\kappa t}(K_\nu) \not\equiv 0 \pmod{\mathfrak{p}}. \tag{2.11}$$

It follows from (1.20) that there exists a p-regular class K_α' of $\mathfrak{G}_\kappa$ which belongs to K_ν and satisfies

$$\omega_{\kappa t}(K_\alpha') \not\equiv 0 \pmod{\mathfrak{p}}. \tag{2.12}$$

Let $\mathfrak{D}_\alpha'$ be a defect group of K_α'. Evidently $\mathfrak{D}_\alpha' \subseteq \widetilde{\mathfrak{D}}$. On the other hand, (2.12) implies $\mathfrak{D} \subseteq \mathfrak{D}_\alpha'$ [12]. Hence $\mathfrak{D} \subseteq \widetilde{\mathfrak{D}}$ and consequently $\widetilde{\mathfrak{D}} = \mathfrak{D}$, since they have the same order.

3. In this and the following sections we use the same notation as in [**11**]. As was defined in [**11**], the generalized symmetric group $S(n, m)$ consists of all permutations of the mn symbols commutative with

$$(1_1\ 2_1 \ldots m_1)\ (1_2\ 2_2 \ldots m_2) \ldots (1_n\ 2_n \ldots m_n).$$

We set $Q_i = (1_i\ 2_i \ldots m_i)$. Then n cycles Q_i generate an invariant commutative subgroup $\mathfrak{Q}$ of order m^n and $S(n, m)$ is the product of $\mathfrak{Q}$ and the subgroup S_n^* isomorphic to the symmetric group S_n:

$$S(n, m) = S_n^* \mathfrak{Q},$$

1) See [2]. Cf. also [3], [4], and [12].

where $S_n^* \cap \mathfrak{Q} = 1$. Hence we have

$$S(n, m)/\mathfrak{Q} \cong S_n. \tag{3.1}$$

Let $\zeta^{(\alpha_i)}$ be the character of type $(n_0, n_1, \dots, n_{m-1})$ of $\mathfrak{Q}$ and let $\mathfrak{G}^{(\alpha_i)}$ be the subgroup of $S(n, m)$ corresponding to $\zeta^{(\alpha_i)}$. Then

$$\mathfrak{G}^{(\alpha_i)} = S_{(n_i)}^* \mathfrak{Q}, \qquad S_{(n_i)}^* \cap \mathfrak{Q} = 1, \tag{3.2}$$

where $S_{(n_i)}^*$ is the subgroup of S_n^* isomorphic to the direct product of S_{n_i}:

$$S_{(n_i)}^* \cong S_{n_0} \times S_{n_1} \times \dots \times S_{n_{m-1}},$$

whence

$$\mathfrak{G}^{(\alpha_i)} \cong S(n_0, m) \times S(n_1, m) \times \dots \times S(n_{m-1}, m). \tag{3.3}$$

Every (ordinary) irreducible representation of $\mathfrak{G}^{(\alpha_i)}$ determined by $\zeta^{(\alpha_i)}$ of $\mathfrak{Q}$ is given by

$$G = U^* \mathfrak{Q} \rightarrow D(U^*) \times \zeta^{(\alpha_i)}(Q), \tag{3.4}$$

where $U^* \rightarrow D(U^*)$, $(U^* \in S_{(n_i)}^*)$ is an irreducible representation of $S_{(n_i)}^* \cong \mathfrak{G}^{(\alpha_i)}/\mathfrak{Q}$[1]. We denote by $[\alpha]$ the irreducible representation of S_n associated with a diagram $[\alpha]$ of n nodes. Then every irreducible representation $U^* \rightarrow D(U^*)$ of $S_{(n_i)}^*$ is obtained by the Kronecker product representation

$$[\alpha_0] \times [\alpha_1] \times \dots \times [\alpha_{m-1}], \tag{3.5}$$

where $[\alpha_i]$ is an irreducible representation of S_{n_i}.

Lemma 8. *Two representations $[\alpha_0] \times [\alpha_1] \times \dots \times [\alpha_{m-1}]$ and $[\beta_0] \times [\beta_1] \times \dots \times [\beta_{m-1}]$ of $S_{(n_i)}^*$ belong to the same block if and only if $[\alpha_i]$ and $[\beta_i]$ have the same p-core for $i = 0, 1, \dots, m-1$.*

Proof. These representations of $S_{(n_i)}^*$ belong to the same block if and only if $[\alpha_i]$ and $[\beta_j]$ of S_{n_i} belong to the same block [3] and the condition for this is that $[\alpha_i]$ and $[\beta_i]$ have the same p-core [3; 8; 13;

1) This shows that the algebra $\mathfrak{A}_k$ in (1.12) is isomorphic to the group ring of $S_{(n_i)}^*$ over K in our case.

14].

As was shown in §1, the representation of $S(n, m)$ induced by the representation (3.4) of $\mathfrak{G}^{(\alpha_i)}$ is irreducible and every irreducible representation of $S(n, m)$ determined by $\zeta^{(\alpha_i)}$ is obtained in this form. Hence the irreducible representations of $S(n, m)$ determined by $\zeta^{(\alpha_i)}$ are in (1 — 1) correspondence with star diagrams

$$[\alpha]_m^* = [\alpha_0] \cdot [\alpha_1] \cdot \ldots \cdot [\alpha_{m-1}] \tag{3.6}$$

of n nodes such that the i-th component $[\alpha_i]$ is a diagram of n_i nodes. We denote by $[\alpha]^*$ the irreducible representation of $S(n, m)$ associated with $[\alpha]_m^*$. We say that the star diagrams (3. 6) are *of type* $(n_0, n_1, \ldots n_{m-1})$. If $[\alpha]_m^*$ is of type $(n_0, n_1, \ldots, n_{m-1})$, then the representation $[\alpha]^*$ is also called the *representation of type* $(n_0, n_1, \ldots, n_{m-1})$. We see that two representations $[\alpha]^*$ and $[\beta]^*$ are conjugate with respect to $\mathfrak{Q}$ if and only if they are of same type.

We shall assume in the remainder of this section that *the order of* $\mathfrak{Q}$ *is prime to* p, that is, $(m, p) = 1$. Then we can apply our general results in §§1 and 2 to $S(n, m)$ and its invariant subgroup $\mathfrak{Q}$ of order m^n.

Let $[\alpha_i^{(0)}]$ be the p-core of $[\alpha_i]$. Then the star diagram

$$[\alpha_0^{(0)}] \cdot [\alpha_1^{(0)}] \cdot \ldots \cdot [\alpha_{m-1}^{(0)}]$$

is called the *p-core* of $[\alpha]_m^*$.

Lemma 9. *Two representations $[\alpha]^*$ and $[\beta]^*$ of same type of $S(n, m)$ with $(m, p) = 1$ belong to the same block if and only if their star diagrams $[\alpha]_m^*$ and $[\beta]_m^*$ have the same p-core.*

Proof. According to Lemma 3, two representations $[\alpha]^*$ and $[\beta]^*$ of same type belong to the same block if and only if the representations $[\alpha_0] \times [\alpha_1] \times \ldots \times [\alpha_{m-1}]$ and $[\beta_0] \times [\beta_1] \times \ldots \times [\beta_{m-1}]$ of $S_{(n_i)}^*$ corresponding to $[\alpha]^*$ and $[\beta]^*$ belong to the same block. Then, by Lemma 8 we see that the lemma is true.

Theorem 1. *Two representations $[\alpha]^*$ and $[\beta]^*$ of $S(n, m)$ with $(m, p) = 1$, associated with $[\alpha]_m^*$ and $[\beta]_m^*$ belong to the same block if and only if $[\alpha_i]$ and $[\beta_i]$ have the same p-core and the same weight for $i = 0, 1, \ldots, m-1$.*

Proof. If $[\alpha]^*$ and $[\beta]^*$ belong to the same block, then they are conjugate with respect to $\mathfrak{Q}$, whence $[\alpha]_m{}^*$ and $[\beta]_m{}^*$ must be of same type. Then Lemma 9 implies that $[\alpha_i]$ and $[\beta_i]$ have the same p-core and hence they have the same weight. The converse is also true by Lemma 9.

Let $[\alpha]_m{}^*$ be a star diagram of type $(n_0, n_1, \ldots, n_{m-1})$ and let $[\alpha_i]$ be the diagram of weight b_i. We set $b=\sum b_i$. The block B of $S(n, m)$ which contains $[\alpha]^*$ is characterized by the p-core of $[\alpha]_m{}^*$ and by a set of m non-negative integers $(b_0, b_1, \ldots, b_{m-1})$. We say that B is a block of weight $b=\sum b_i$.

Let $l(b)$[1] be the number of ordinary irreducible representations in a block of weight b of S_n. According to Theorem 1, we have

Theorem 2. *The number of ordinary irreducible representations in a block of weight $b=\sum b_i$ of $S(n, m)$ with $(m, p)=1$ is independent of the p-core and is given by*

$$l(b_0,\ b_1, \ldots, b_{m-1}) = \prod_{i=0}^{m-1} l(b_i). \tag{3.7}$$

The defect group $\mathfrak{D}_b$ of a block of weight b of S_n was determined by Brauer [3]. $\mathfrak{D}_b$ is a p-Sylow-subgroup of a subgroup $S(b, p)$ of S_n. This, combined with Lemma 7, yields

Theorem 3. *The defect group $\mathfrak{D}^{(b_i)}$ of a block of weight $b=\sum b_i$ of $S(n, m)$ with $(m, p)=1$ is the direct product of defect groups $\mathfrak{D}_{b_i}$ in S_{n_i}:*

$$\mathfrak{D}^{(b_i)} = \mathfrak{D}_{b_0} \times \mathfrak{D}_{b_1} \times \ldots \times \mathfrak{D}_{b_{m-1}}. \tag{3.8}$$

We now consider the modular representations of $S(n, m)$. Every modular irreducible representation of $\mathfrak{G}^{(\alpha_i)}$ determined by $\zeta^{(\alpha_i)}$ is obtained by

$$G = U^*Q \rightarrow F(U^*) \times \zeta^{(\alpha_i)}(Q), \tag{3.9}$$

where $U^* \rightarrow F(U^*)$, $(U^* \in S_{(n_i)}{}^*)$ is a modular irreducible representation of $S_{(n_i)}{}^*$. As was shown by Robinson and Taulbee [16], there exists a (1-1) correspondence between the modular irreducible representations of S_n and the p-regular diagrams of n nodes. Let $[\alpha]$ be a p-regular dia-

1) See [6], [8], and [15]

gram. We shall denote by $\{\alpha\}$ the modular irreducible representation associated with $[\alpha]$. Then every modular irreducible representation $U^* \to F(U^*)$ of $S_{(n_i)}{}^*$ is given by the Kronecker product representation

$$\{\alpha_0\} \times \{\alpha_1\} \times \ldots \times \{\alpha_{m-1}\},$$

where $\{\alpha_i\}$ is a modular irreducible representation of S_{n_i}. As was shown in §2, the representation of $S(n, m)$ induced by the representation (3.9) is irreducible and every modular irreducible representation of $S(n, m)$ determined by $\zeta^{(\alpha_i)}$ is obtained in this form.

Let $l^*(b)$[1] be the number of modular irreducible representations in a block of weight b of S_n. We then have

Theorem 4. *The number of modular irreducible representations in a block of weight $b = \sum b_i$ of $S(n, m)$ with $(m, p) = 1$ is independent of the p-core and is given by*

$$l^*(b_0, b_1, \ldots, b_{m-1}) = \prod_{i=0}^{m-1} l^*(b_i). \tag{3.10}$$

A star diagram $[\alpha]_m{}^*$ is called *p-regular* if every component is p-regular. Then, as was shown above, there exists a $(1-1)$ correspondence between the modular irreducible representations of $S(n, m)$ and the p-regular star diagrams of n nodes. We denote by $\{\alpha\}^*$ the modular irreducible representation of $S(n, m)$ associated with a p-regular star diagram $[\alpha]_m{}^*$. Evidently $\{\alpha\}^*$ and $\{\beta\}^*$ of $S(n, m)$ with $(m, p) = 1$ belong to the same block if and only if $[\alpha]_m{}^*$ and $[\beta]_m{}^*$ have the same p-core and the same weight for $i = 0, 1, \ldots, m-1$.

Theorem 5. *Let B be a block of $S(n, m)$ with $(m, p) = 1$, which contains an irreducible representation $[\alpha]^*$ and let B_i be the block of S_{n_i} which contains $[\alpha_i]$. The decomposition matrix of B* is *given by the Kronecker product of the matrices D_i:*

$$D = D_0 \times D_1 \times \ldots \times D_{m-1}, \tag{3.11}$$

where D_i denotes the decomposition matrix of B_i.

4. We consider in this section the representations of $S(n, m)$ in the

1) See [6], [9], [10], and [15].

case where m *is divisible by* p. We first mention the following special results.

Lemma 10. *$S(n, p^e)$, $0 < e$ possesses only one block (for p). The defect group is a p-Sylow-subgroup of $S(n, p^e)$.*

Proof. Since the invariant subgroup $\mathfrak{Q}$ of order p^{en} coincides with its centralizer $C(\mathfrak{Q})$ in $S(n, p^e)$, we see by Lemma 2 [**14**] that the lemma is true.

Lemma 11. *Let $\{\alpha\}$ be the modular irreducible representations of S_n associated with p-regular diagrams $[\alpha]$ of n nodes. Then all modular irreducible representations of $S(n, p^e)$ are given by $\{\alpha\}$.*

Proof. In general if a group $\mathfrak{G}$ contains an invariant subgroup $\mathfrak{H}$ whose order is a power of p, then all elements of $\mathfrak{H}$ are represented by the unit matrix I in each modular irreducible representation of $\mathfrak{G}$. This, combined with (3.1), proves the lemma.

We denote by $a'(n)$ the number of modular irreducible representations of S_n. $S(n, p^e)$ has also $a'(n)$ modular irreducible representations.

Now we consider the general case. Let p^e $(0 < e)$ be the highest power of p dividing m so that

$$m = p^e t, \qquad (t, p) = 1. \tag{4.1}$$

We set

$$Q_i^{p^e} = H_i, \qquad Q_i^t = P_i \qquad (i = 1, 2, \dots, n).$$

Then the H_i generate an invariant subgroup $\mathfrak{H}$ of order t^n. On the other hand, the P_i generate an invariant subgroup $\mathfrak{P}$ of order p^{en}. Moreover $\mathfrak{Q}$ is the direct product of $\mathfrak{P}$ and $\mathfrak{H}$:

$$\mathfrak{Q} = \mathfrak{P} \times \mathfrak{H}. \tag{4.2}$$

Hence

$$S(n, p^e t)/\mathfrak{P} \cong S(n, t), \tag{4.3}$$

$$S(n, p^e t)/\mathfrak{H} \cong S(n, p^e). \tag{4.4}$$

We see by (4.3) that all modular irreducible representations of $S(n, p^e t)$ are given by those of $S(n, t)$[1].

1) See the proof of Lemma 11.

We shall apply our general results in §§1 and 2 to $S(n, p^e t)$ and its invariant subgroup $\mathfrak{H}$ of order t^n. Denote by θ a primitive t-th root of unity. Then

$$(4.5) \qquad H_i \to \theta^{\sigma_i} \qquad (0 \leqq \sigma_i < t), \qquad i = 1, 2, \ldots, n$$

forms an irreducible representation of $\mathfrak{H}$. We denote by $\xi_{(\sigma_i)}$ the character of the representation (4.5). $\xi_{(\sigma_i)}$ is called the character of type $(k_0, k_1, \ldots, k_{t-1})$, if the number of σ_i such that $\sigma_i = s$ is k_s. Evidently two charactetrs $\xi_{(\alpha_i)}$ and $\xi_{(\tau_i)}$ are associated in $S(n, p^e t)$ if and only if they are of same type. Let $\mathfrak{G}_{(\sigma_i)}$ be the subgroup of $S(n, p^e t)$ corresponding to $\xi_{(\sigma_i)}$ of $\mathfrak{H}$. Then we have

$$(4.6) \qquad \mathfrak{G}_{(\sigma_i)} = S_{(k_i)}{}^*\mathfrak{Q}, \qquad S_{(k_i)}{}^* \cap \mathfrak{Q} = 1,$$

where $S_{(k_i)}{}^*$ is the subgroup of $S_n{}^*$ and is isomorphic to the direct product of S_{k_i}:

$$S_{(k_i)}{}^* \cong S_{k_0} \times S_{k_1} \times \ldots \times S_{k_{t-1}}.$$

We set $\mathfrak{T}^{(k_i)} = S_{(k_i)}{}^*\mathfrak{P}$. Then

$$(4.7) \qquad \mathfrak{T}^{(k_i)} \cong S(k_0, p^e) \times S(k_1, p^e) \times \ldots \times S(k_{t-1}, p^e),$$

and

$$(4.8) \qquad \mathfrak{G}_{(\sigma_i)}/\mathfrak{H} \cong \mathfrak{T}^{(k_i)}.$$

It is easily seen that every ordinary (modular) irreducible representation of $\mathfrak{G}_{(\sigma_i)}$ determined by $\xi_{(\sigma_i)}$ is obtained by

$$(4.9) \qquad G = V^*H \to M(V^*) \times \xi_{(\sigma_i)}(H) \qquad (V^* \in \mathfrak{T}^{(k_i)},\ H \in \mathfrak{H}),$$

where $V^* \to M(V^*)$ is an ordinary (modular) irreducible representation of $\mathfrak{T}^{(k_i)}$. Furthermore every irreducible representation of $S(n, p^e t)$ determined by $\xi_{(\sigma_i)}$ is given by the representation induced by the representation (4.9) of $\mathfrak{G}_{(\sigma_i)}$. According to Lemma 10, $\mathfrak{T}^{(k_i)}$ possesses only one block (for p). Hence it follows from Lemma 3 that all ordinary irreducible representations of $\mathfrak{G}_{(\alpha_i)}$ determined by $\xi_{(\sigma_i)}$ constitute a block of $\mathfrak{G}_{(\sigma_i)}$. Since this block is uniquely determined by $\xi_{(\sigma_i)}$, it will be denoted by $B_{(\sigma_i)}$. Let $\widetilde{B}_{(\sigma_i)}$ be the block of $S(n, p^e t)$ determined by

$B_{(\sigma_t)}$. $\widetilde{B}_{(\sigma_t)}$ consists of all irreducible representations of $S(n, p^e t)$ determined by $\xi_{(\sigma_t)}$. Thus we have proved

Therorem 6. *Two ordinary irreducible representations of $S(n, p^e t)$ with $0 < e$, $(t, p) = 1$ belong to the same block if and only if they are conjugate with respect to $\mathfrak{H}$.*

Corollary. *The number of blocks of $S(n, p^e t)$ with $0 < e$, $(t, p) = 1$ is equal to the number of classes of conjugate elements of $S(n, p^e t)$ contained in $\mathfrak{H}$.*

The block $B_{(\sigma_t)}$ of $S(n, p^e t)$ determined by $\xi_{(\sigma_t)}$ of type $(k_0, k_1, \ldots, k_{t-1})$ will be called the *block of type* $(k_0, k_1, \ldots, k_{t-1})$. Let us denote by $a(n, m)$ the number of ordinary irreducible representations of $S(n, m)$. Since the number of ordinary irreducible representations of $\mathfrak{T}^{(k_i)}$ is equal to $\Pi\, a(k_i, p^e)$, we have

Theorem 7. *The number of ordinary irreducible representations in a block of type $(k_0, k_1, \ldots, k_{t-1})$ of $S(n, p^e t)$ with $0 < e$, $(t, p) = 1$ is equal to*

$$r(k_0, k_1, \ldots, k_{t-1}) = \prod_{i=0}^{t-1} a(k_i, p^e). \tag{4.10}$$

Since the number of modular irreducible representations of $\mathfrak{T}^{(k_i)}$ is equal to $\Pi\, a'(k_i)$, we obtain

Theorem 8. *The number of modular irreducible representations in a block of type $(k_0, k_1, \ldots, k_{t-1})$ of $S(n, p^e t)$ with $0 < e$, $(t, p) = 1$ is equal to*

$$r'(k_0, k_1, \ldots, k_{t-1}) = \prod_{i=0}^{t-1} a'(k_i). \tag{4.11}$$

We obtain by Lemmas 7, 10 and (4.8)

Theorem 9. *The defect group of a block of type $(k_0, k_1, \ldots, k_{t-1})$ of $S(n, p^e t)$ with $0 < e$, $(t, p) = 1$ is the direct product of $\mathfrak{P}^{(k_i)}$ in $S(k_i, p^e)$:*

$$\mathfrak{D}_{(k_i)} = \mathfrak{P}^{(k_0)} \times \mathfrak{P}^{(k_1)} \times \ldots \times \mathfrak{P}^{(k_{t-1})}, \tag{4.12}$$

where $\mathfrak{P}^{(k_i)}$ is a p-Sylow-subgroup of $S(k_i, p^e)$.

Let $\zeta^{(\alpha_i)}$ be the character of type $(n_0, n_1, \ldots, n_{m-1})$ of $\mathfrak{Q}$ as before. We set

$$(4.13) \qquad k_i = \sum_{a=0}^{p^e-1} n_{at+i} \qquad (i = 0, 1, \ldots, t-1).$$

We see easily that $\zeta^{(\alpha_i)}$ is the character of $\mathfrak{Q}$ determined by $\xi_{(\sigma_i)}$ of type $(k_0, k_1, \ldots, k_{t-1})$ of $\mathfrak{H}$. Hence two characters $\zeta^{(\alpha_i)}$ and $\zeta^{(\beta_i)}$ of type $(n_0, n_1, \ldots, n_{m-1})$ and $(n_0', n_1', \ldots, n'_{m-1})$ are conjugate with respect to $\mathfrak{H}$ if and only if

$$(4.14) \qquad \sum_{a=0}^{p^e-1} n_{at+i} = \sum_{a=0}^{pe-1} n'_{at+i} \qquad (i = 0, 1, \ldots, t-1).$$

Let $[\alpha]^*$ be the irreducible representation of $S(n, p^e t)$ associated with a star diagram $[\alpha]_m^*$ of type $(n_0, n_1, \ldots, n_{m-1})$. Since $[\alpha]^*$ is the representation determined by $\zeta^{(\alpha_i)}$ of type $(n_0, n_1, \ldots, n_{m-1})$ of $\mathfrak{Q}$, it is determined by $\xi_{(\sigma_i)}$ of type $(k_0, k_1, \ldots, k_{t-1})$ of $\mathfrak{H}$, where k_i is defined by (4.13).

Theorem 10. *Two representations $[\alpha]^*$ and $[\beta]^*$ of $S(n, p^e t)$ with $0 < e$, $(t, p) = 1$, associated with $[\alpha]_m^*$ and $[\beta]_m^*$ of type $(n_0, n_1, \ldots, n_{m-1})$ and $(n_0', n'_1, \ldots, n'_{m-1})$ respectively belong to the same block if and only if the equalities (4.14) hold.*

Proof. Let $[\alpha]^*$ and $[\beta]^*$ be determined by $\zeta^{(\alpha_i)}$ and $\zeta^{(\beta_i)}$ respectively. Then they belong to the same block if and only if $\zeta^{(\alpha_i)}$ and $\zeta^{(\beta_i)}$ are conjugate with respect to $\mathfrak{H}$. This proves the theorem.

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(Received September 22, 1956)