Representation Theory and its Combinatorial Aspects

Date : October 28, 2019 (Mon) – October 31, 2019 (Thu)
Place : Research Institute for Mathematical Sciences,
Kyoto University (Kitashirakawa Oiwake-cho, Sakyo-ku, Kyoto 606-8502, Japan)

Abstracts of Talks (alphabetical order)

1. Sota Asai (RIMS, Kyoto University)

   The wall-chamber structures of the real Grothendieck groups for path algebras

   Let A be a finite-dimensional algebra over a field. Then, the real Grothendieck group $K_0(\operatorname{proj} A) \otimes_{\mathbb{Z}} \mathbb{R}$ is an Eucliean space. For each finite-dimensional A-module M, we define a subset $\Theta_M \subset K_0(\operatorname{proj} A) \otimes_{\mathbb{Z}} \mathbb{R}$ so that $\theta \in \Theta_M$ holds if and only if M is θ-semistable in the sense of King. The subsets Θ_M are rational polyhedral cones, and by regarding them as walls, we have a wall-chamber structure of the real Grothendieck group. In this talk, I will first state general theory of the wall-chamber structure. After that, I will give my results on combinatorial properties of the walls in the case that A is a path algebra of an acyclic quiver.

2. Haruhisa Enomoto (Nagoya University)

   Bruhat inversions in the symmetric group and torsion-free classes over a type A quiver

   For a Dynkin quiver, Gabriel's theorem gives a bijection between indecomposable representations of it and positive roots of the corresponding root system. Using this, every element of the Weyl group gives a subcategory of the module category by taking the inversion set. Under this construction, it was shown by Ingalls-Thomas that "good" elements in the Weyl group are in bijection with "good" subcategories of the module category, namely, torsion-free classes. In this talk, I will restrict our attention to type A case (where everything is explicit), and give an overview of these results. Then I will explain my recent result (arXiv:1908.05446), which gives a bijection between simple objects and Bruhat inversions, and characterize whether a torsion-free class satisfies the Jordan-Holder property or not in a purely combinatorial way.

3. Takuma Hayashi (University of Tokyo)

   Half-integrality of the KGB decomposition for SL_3

   For a real reductive Lie group $G_{\mathbb{R}}$ and the complexification K of its maximal compact subgroup, the decomposition of the complex flag variety of G into

K-orbits plays an important role in representation theory of $G_{\mathbb{R}}$. The combinatorial classification of the K-orbits is known by Matsuki.
In this talk, we will prove that the moduli scheme of Borel subgroups of SL_3 over $\mathbb{Z}[1/2]$ is set theoretically decomposed into four SO(3)-invariant subschemes. This result is a half-integral analog of the KGB decomposition of SL_3 over $\mathbb{C}$. This talk is partly based on a joint work with Fabian Januszewski.

4. Toya Hiroshima (Osaka City University)

Queer Supercrystal Structure for Increasing Factorizations of Fixed-Point-Free Involution Words

Edelman-Greene の挿入アルゴリズムの symplectic Hecke word 版において、Edelman-Greene の定理と同様のことが成り立つことを示し、この結果を用いて、symplectic Hecke word の reduced word (FPF-involution word) の増大列分解が queer Lie superalgebra のクリスタルの構造を持つことを示したものです。arXiv:math/1907.10836

5. Ayumu Hoshino (Hiroshima Institute of Technology)

Brunching Rules for Koornwinder polynomials with One Column Diagrams and Matrix Inversions

We present an explicit formula for the transition matrix $\mathcal{C}$ from the type BC_n Koornwinder polynomials $P_{(1^r)}(x|a,b,c,d|q,t)$ with one column diagrams, to the type BC_n monomial symmetric polynomials $m_{(1^r)}(x)$. The entries of the matrix $\mathcal{C}$ enjoy a set of four terms recursion relations. These recursions provide us with the branching rules for the Koornwinder polynomials with one column diagrams, namely the restriction rules from BC_n to BC_{n-1}. To have a good description of the transition matrices involved, we introduce the following degeneration scheme of the Koornwinder polynomials:

$$\begin{aligned} &P_{(1^r)}(x|a,b,c,d|q,t) \leftrightarrow P_{(1^r)}(x|a,-a,c,d|q,t) \leftrightarrow P_{(1^r)}(x|a,-a,c,-c|q,t) \\ &\qquad \leftrightarrow P_{(1^r)}(x|t^{1/2}c,-t^{1/2}c,c,-c|q,t) \leftrightarrow P_{(1^r)}(x|t^{1/2},-t^{1/2},1,-1|q,t). \end{aligned}$$

We prove that the transition matrices associated with each of these degeneration steps are given in terms of the matrix inversion formulas of Bressoud or Krattenthaler. As an application, we give an explicit formula for the Kostka polynomials of type B_n, namely the transition matrix from the Schur polynomials $P^{(B_n,B_n)}_{(1^r)}(x|q;q,q)$ to the Hall–Littlewood polynomials $P^{(B_n,B_n)}_{(1^r)}(x|t;0,t)$.

6. Motohiro Ishii (Gunma University)

Semi-infinite Young tableaux and its applications

We introduce a new tableau model for crystal bases of extremal weight modules over quantum affine algebras of untwisted type A. As an application, we give a tableau criterion for semi-infinite Bruhat order on affine Weyl groups of type A.

7. Masao Ishikawa (Okayama University)

   Distributive Lattice of Half-Turn Symmetric Alternating Sign Matrices

   Striker and Williams defined promotion and rowmotion for the toggle group of an rc-poset. They investigated the distributive lattice of the alternating sign matrices which is the lattice of order ideals of the poset $\mathbf{A}_n$, which is obtained by gluing the positive root posets $\Phi^+(A_k)$. We consider the set of the half-tuen symmetric alternating sign matrices as a distributive lattice by the height function and investigate the underlyng poset, which is obtained by gluing the positive root posets $\Phi^+(B_k)$.

8. Noboru Ito (University of Tokyo)

   Khovanov homology has a property that these homology groups

   Khovanov homology has a property that these homology groups give a graded Euler characteristic is a coefficient of the Jones polynomial. It is known that a two-dimensional representation corresponds to the Jones polynomial and that of N gives a colored Jones polynomial. Since the (standard) colored Jones polynomial is obtained from a decomposition of the tensor product representations, one has desired that a colored Khovanov homology would be obtained in a similar way. Khovanov considered this problem and left some problems (2003).

   However, two coboundary operators do not commute, where one is a coboundary operator, induced by the decomposition of a representation, and the other is the coboundary operator of the original Khovanov homology of the Jones polynomial. Beliakova and Wehrli answered some problems of Khovanov, but formulated the problem of this commutative more concretely, i.e., they left the problem of the existence of such Khovanov-type bicomplex (2005). In this talk, we treat the problem of the existence of bicomplex.

9. Yuki Kanakubo (Sophia University)

   Adapted Sequence for Polyhedral Realization of Crystal Bases

   T.Nakashima and A.Zelevinsky invented 'polyhedral realization', which is a kind of description of crystal bases $B(\infty)$ as lattice points in some polyhedral convex cone. After that, Nakashima found a polyhedral realization for crystal bases of integrable highest weight representations of quantum groups.

   To construct the polyhedral realization, we need an infinite sequence ι of indices. In the case ι satisfies a 'positivity condition' (resp. 'ample condition'), there is a method to obtain an explicit form of the polyhedral

realization associated with ι. However, it seems to be difficult to confirm whether ι satisfies the positivity (resp. ample) condition or not.
In this talk, I will give a sufficient condition of ι for the positivity (resp. ample) condition in the case the associated Lie algebra is classical type. I will also give explicit forms of the polyhedral realizations in terms of column tableaux for sequences which satisfy the sufficient condition. This is a joint work with Toshiki Nakashima in Sophia University.

10. Ryotarou Kawai (Okayama University of Science)

Multiplicities of Schubert varieties in the flag varieties of classical types

We consider the Schubert variety of flag varieties. Combinatorial formula for multiplicities of points on Schubert variety in Grassmannian of type A,B,C,D (Kodiyalam-Raghavan, Ghorpade-Raghavan, Ikeda-Naruse, Raghavan-Upadhyay). We want to extend this formula to flag varieties. Combinatorial formula for multiplicities of points on Schubert variety in flag variety of type A (Li-Yong). We were able to obtain a combinatorial formula of multiplicities of points on Schubert varieties in flag variety. This formula represents the multiplicity by Young diagrams determined from point on Schubert variety.

11. Koei Kawamura (Kyoto university)

Representations of wreath products on a local field and multivariate Krawtchouk and Hahn polynomials

Dunkl(1976) induced an additional theorem for Krawtchouk polynomials, that are zonal spherical functions of Gelfand pair $(\mathbb{G}, G)$ according to a wreath produnt of symmetric groups $\mathbb{G} = (\mathfrak{S}_{k+1})^N \rtimes \mathfrak{S}_N$ and its subgroup $G = (\mathfrak{S}_k)^N \rtimes \mathfrak{S}_N$.
His method was to decompose spherical representations of $(\mathbb{G}, G)$ into irreducible representations of G.
We apply this method to multivariate case: let $\mathfrak{o}$ be the ring of integers of a non-Archimedean local field, $\mathfrak{p}$ its maximal ideal, and $A = (\mathfrak{o}/\mathfrak{p}^\ell)^N$, $G = (\mathfrak{o}^\times)^N \rtimes \mathfrak{S}_N$.
Then we have an additional theorem for multivariate Krawtchouk polynomials which include multivariate Hahn polynomials as coefficients.

12. Dongsu Kim (Korea Advanced Institute of Science and Technology)

A combinatorial bijection on k-noncrossing partitions

For any integer $k \geq 2$, we prove combinatorially the following Euler (binomial) transformation identity

$$\mathrm{NC}^{(k)}_{n+1}(t) = t \sum_{i=0}^{n} \binom{n}{i} \mathrm{NW}^{(k)}_i(t),$$

where $\mathrm{NC}_m^{(k)}(t)$ (resp. $\mathrm{NW}_m^{(k)}(t)$) is the enumerative polynomial on partitions of $\{1,\ldots,m\}$ avoiding k-crossings (resp. enhanced k-crossings) by number of blocks. The special $k=2$ and $t=1$ case, asserting the Euler transformation of Motzkin numbers are Catalan numbers, was discovered by Donaghey 1977. The result for $k=3$ and $t=1$, arising naturally in a recent study of pattern avoidance in ascent sequences and inversion sequences, was proved only analytically.
It is based on the preprint (arXiv:1905.10526) with Zhicong Lin.

13. Takafumi Kouno (Tokyo Institute of Technology)

A generalization of Lakshmibai-Seshadri paths and Chevalley formula for arbitrary weights

Lakshmibai-Seshadri paths are originally associated to dominant weights, and have a lot of applications to the representation theory of semi-simple Lie algebras. One of those is the Chevalley formula in the torus-equivariant K-theory of flag manifolds, which expresses a product of the class of a line bundle and the class of the structure sheaf of a Schubert variety as a linear combination of the classes of the structure sheaves of Schubert varieties. It is known that the Chevalley formula for line bundles associated to dominant/anti-dominant weights is written in terms of Lakshmibai-Seshadri paths. In this talk, we define Lakshmibai-Seshadri paths associated to arbitrary weights, not necessarily dominant weights. As an application, we give a description of the Chevalley formula for line bundles associated to arbitrary weights in terms of Lakshmibai-Seshadri paths.

14. Sho Matsumoto (Kagoshima University)

Stanley character formulas for spin representations of symmetric groups

Normalizations of irreducible characters for symmetric groups can be explicitly expressed in terms of certain coloring functions on cycles, or in terms of multi-rectangular coordinates for Young diagrams. The expressions are called Stanley character formulas. The rectangular-shape case of them was obtained by Stanly (2004), and general cases were conjectured by Stanley (2006) and proved by Féray (2010). In this talk, we present an analogue of the formula for spin symmetric groups. This is a joint work with Piotr Śniady. [arXiv1811.10434]

15. Yuta Nishiyama (Kumamoto University)

A generalized class equation of the symmetric group and its bijective proof

By calculating the inner product of the Macdonald symmetric polynomials $P_\lambda(q;t)$ corresponding to the partition $\lambda=(1^n)$, we obtain an equation which can be regarded as a generalization of the class equation of the symmetric group. In this talk, I give a bijective proof of the equation, using certain transformations of the Young diagrams.

16. Ryo Ohkawa (Wasewda University)

(−2) **blow-up formula**

We consider Nekrasov partition function defined from A_1 singularity. These are integrations over moduli of framed sheaves on resolutions of the singularity. We consider two resolutions, the minimal resolution, and the quotient stack of the projective plane by the cyclic group of order 2. We show functional equations among Nekrasov functions defined from these two resolutions. This is analogy of the blowup formula by Nakajima-Yoshioka.

17. Soichi Okada (Nagoya University)

Explicit formula for birational rowmotion on shifted staircases

Birational rowmotion is a discrete dynamical system acting on the space of assignments of rational functions to the elements of a finite poset. It provides a birational lift of combinatorial rowmotion (also called Fon-der-Flaass action and other names) acting on order ideals. Musiker–Roby gave an explicit formula in terms of non-intersecting lattice paths for the iterations of the birational rowmotion map on a a product of two chains, which is a minuscule poset associated to the Grassmannian. This formula plays a key role in the proof of the periodicity and the birational file homomesy. In this talk, we give a similar formula for the iterations of the birational rowmotion map on a shifted staircase, which is a minuscule poset associated to the even orthogonal Grassmannian. And we use this formula to prove the birational homomesy along the main diagonal.

18. Shoma Sugimoto (RIMS, Kyoto University)

On the Feigin-Tipunin conjecture

One of the most well-studied examples of C_2-cofinite and irrational VOAs is the triplet VOA, the kernel of the narrow screening operator on the rescaled root lattice of A_1 type. However, there are not much known about the logarithmic W-algebras $W(p)_Q$, the ADE type generalizations of the triplet VOA. In this talk, using a geometric method introduced in [Feigin-Tipunin], we will give

(a) the geometric realizations,

(b) the $W^k(g)$-module structures,

(c) the character formulas of $W(p)_Q$ that conjectured in [Feigin-Tipunin].

19. Motoki Takigiku (University of Tokyo)

A proof of Nandi's conjecture

We will give a proof of partition theorems conjectured through vertex operator theoretic consideration for level 4 standard modules of the affine Lie algebra of type $A_2^{(2)}$ in D.Nandi's PhD thesis (2014). This is a joint work with Shunsuke Tsuchioka.

20. Mamoru Ueda (RIMS, Kyoto University)

    Coproduct for the Yangian of type $A_2^{(2)}$

    The Yangian is one kind of quantum group. Guay, Nakajima, and Wendland defined the coproduct for the Yangian of affine Lie algebras except of types $A_1^{(1)}$ and $A_2^{(2)}$. After recalling the result of Guay, Nakajima, and Wendland, we will explain how to define the coproduct for the Yangian of type $A_2^{(2)}$. This solves the problem of a construction of coproduct for the affine Yangian.

21. Hideya Watanabe (RIMS, Kyoto University)

    Alcove paths and Gelfand-Tsetlin patterns

    In their study of the equivariant K-theory of the generalized flag varieties G/P, Lenart and Postnikov introduced a combinatorial tool, called the alcove paths model. It provides a new model for the highest weight crystals with dominant integral highest weights. In this talk, I introduce a simple and explicit formula describing the crystal isomorphism between the alcove paths model of type A and the Gelfand-Tsetlin patterns model. This talk is based on a joint work with Keita Yamamura.

Symposia (open), "Representation Theory and its Combinatorial Aspects"
Organized by Masao Ishikawa (Okayama University)