Algebraic and Enumerative Combinatorics in Okayama

Date :	February 19, 2018 (Mon) – February 23, 2018 (Fri)
Place :	Graduate School of Natural Science and Technology, Okayama University (Tsushima, Okayama 700-8530, Japan)

Abstracts of Talks (alphabetical order)

1. Roger Behrend (Cardiff University)

   The combinatorics of alternating sign matrices

   In these talks, I will discuss various topics related to the combinatorics of alternating sign matrices. These will include alternating sign matrix symmetry classes, alternating sign matrix statistics, descending plane partitions, totally symmetric self-complementary plane partitions, alternating sign triangles, fully packed loop configurations, the alternating sign matrix polytope, and the alternating sign matrix poset.

2. Mihai Ciucu (Indiana University)

   Lozenge tilings with gaps in a 90 degree wedge domain with mixed boundary conditions

   We consider a triangular gap of side two in a 90 degree angle on the triangular lattice with mixed boundary conditions: a constrained, zig-zag boundary along one side, and a free lattice line boundary along the other. We study the interaction of the gap with the corner as the rest of the angle is completely filled with lozenges. We show that the resulting correlation is governed by the product of the distances between the gap and its three images in the sides of the angle. This provides evidence for a unified way of understanding the interaction of gaps with the boundary under mixed boundary conditions, which we present as a conjecture. Our conjecture is phrased in terms of the steady state heat flow problem in a uniform block of material in which there are a finite number of heat sources and sinks. This new physical analogy is equivalent in the bulk to the electrostatic analogy we developed in previous work, but arises as the correct one for the correlation with the boundary.

   The starting point for our analysis is an exact formula we prove for the number of lozenge tilings of certain trapezoidal regions with mixed boundary conditions, which is equivalent to a new, multi-parameter generalization of a classical plane partition enumeration problem (that of enumerating symmetric, self-complementary plane partitions).

3. Ayumu Hoshino (Hiroshima Institute of Technology)

   Tableau Formulas for One-Row Macdonald Polynomials of Types C_n

We present explicit formulas for the Macdonald polynomials of types C_n in the one-row case. In view of the combinatorial structure, we call them "tableau formulas". For the construction of the tableau formulas, we apply some transformation formulas for the basic hypergeometric series. We remark that the correlation functions of the deformed W algebra generators automatically give rise to the tableau formulas when we principally specialize the coordinate variables. This talk is based on our paper, Feigin, Hoshino, Noumi, Shibahara and Shiraishi, Tableau Formulas for One-Row Macdonald Polynomials of Types C_n and D_n.

4. Masao Ishikawa (Okayama University)

   (q,t)-hook formula for Tailed Insets and a Macdonald polynomial identity

   Okada presented a conjecture on (q,t)-hook formula for general d-complete posets in the paper, Soichi Okada, (q,t)-Deformations of multivariate hook product formulae, J. Algebr. Comb. (2010) 32, 399-416. We consider the Tailed Inset case, and reduce the conjectured identity to an indentity of the Macodonald polynomials rephrasing Okada's (q,t)-weights via Pieri coefficients of the Macodonald polynomials. Joint work with Frederic Jouhet (University of Lyon I).

5. Syuhei Kamioka (Kyoto University)

   Nice formulas for plane partitions from an integrable system

   Plane partitions are so nice that there are nice generating functions for plane partitions which can be factored, such as MacMahon' s formula and its variants. In this talk a close connection between plane partitions and an integrable system, the discrete two-dimensional (2D) Toda molecule, is clarified. The main theorem is: each non-vanishing solution to the discrete 2D Toda molecule gives a nice (weighted) generating function for boxed (reverse) plane partitions. As an example a new weighted generating function for boxed reverse plane partitions of arbitrary shape which generalizes MacMahon's formula and a trace generating function of Gansner type is shown.

6. Jang Soo Kim (Sungkyunkwan University)

   Combinatorics of the Selberg integral

   In 1944, Selberg evaluated a multivariate integral, which generalizes Euler's beta integral. In 1980, Askey conjectured a q-integral version of the the Selberg integral, which was proved independently by Habsieger and Kadell in 1988. In this talk, we focus on the combinatorial aspects of the Selberg integral. First, we review the following fact observed by Igor Pak: evaluating the Selberg integral is essentially the same as counting the linear extensions of a certain poset. Considering q-integrals over order polytopes, we give a combinatorial interpretation for Askey's q-Selberg integral. We also find a connection between the Selberg integral and Young tableaux. As applications we enumerate Young tableaux of various shapes.

7. Jang Soo Kim (Sungkyunkwan University)

   Hook length property of d-complete posets via q-integrals

   The hook length formula for d-complete posets states that the P-partition generating function for them is given by a product in terms of hook lengths. We give a new proof of the hook length formula using q-integrals. The proof is done by a case-by-case analysis consisting of two steps. First, we express the P-partition generating function for each case as a q-integral and then we evaluate the q-integrals. Several q-integrals are evaluated using partial fraction expansion identities and others are verified by computer. This is joint work with Meesue Yoo.

8. Fumihiko Nakano (Gakushuin University)

   Generalized carries process and riffle shuffles

   Carries process is a Markov chain of carries in adding n numbers. We consider a generalization of that, studied the transition probability matrix, and its relation to combinatorics. The results include :

   (1) the stationary distribution is proportional to the decent statistics of colored permutation group
   (2) left eigenvector matrix is equal to the Foulkes character table of $G(p, n)$
   (3) Stirling-Frobenius number appears in the right eigenvector matrix.
   (4) Discussion on the generalized riffle shuffles whose descent process is equally distributed to the carries process.

   This is a joint work with Taizo Sadahiro (Tsuda College).

9. Hiroshi Naruse (Yamanashi University)

   Equivariant K-theory and hook formula for skew shape on d-complete poset

   We use equivariant K-theory of flag variety to represent a hook formula for generating function of reverse plane partitions on skew shape d-complete poset in terms of excited diagrams. This is a joint work with Soichi Okada.

10. Soichi Okada (Nagoya University)

    d-Complete posets and hook formulas

    d-Complete posets are a class of posets introduced by Proctor as a generalization of shapes (Young diagrams), shifted shapes (shifted Young diagrams) and rooted trees, and Peterson and Proctor obtained a hook product formula for d-complete posets. In the first half of this talk, I will give an introduction to d-complete posets and hook product formulas for them. In the second half, which is based on a joint work with H. Naruse, I present a skew hook formula for d-complete posets. This formula generalizes both Naruse's skew hook formula and a q-hook formula for skew shapes given by Morales-Pak-Panova.

11. Greta Panova (University of Pennsylvania)

Hook formulas for skew shapes – combinatorics, asymptotics and beyond

We will show several combinatorial and aglebraic proofs of this formula, leading to a bijection between SSYTs or reverse plane partitions of skew shape and certain integer arrays that gives two q-analogues of the formula. These formulas can also be proven via non-intersecting lattice paths interpretations, for example connecting Dyck paths and alternating permutations.

We will also show how excited diagrams give asymptotic results for the number of skew Standard Young Tableaux in various regimes of convergence for both partitions. Multivariate versions of the hook formula with consequences to exact product formulas for certain skew SYTs and lozenge tilings with multivariate weights, which also appear to have interesting behavior in the limit. Joint work with A. Morales and I. Pak.

12. Tom Roby (University of Connecticut)

Paths to understanding birational rowmotion on a product of two chains

Birational rowmotion is an action on the space of assignments of rational functions to the elements of a finite partially-ordered set (poset). It is lifted from the well-studied rowmotion map on order ideals (equivariantly on antichains) of a poset P, which when iterated on special posets, has unexpectedly nice properties in terms of periodicity, cyclic sieving, and homomesy (statistics whose averages over each orbit are constant). In this context, rowmotion appears to be related to Auslander-Reiten translation on certain quivers, and birational rowmotion to Y-systems of type $A_m \times A_n$ described in Zamolodchikov periodicity.

13. Ryo Tabata (National Institute of Technology, Ariake College)

Limiting behavior of immanants and Littlewood-Richardson's correspondence

The Littlewood-Richardson rule is one of the most important properties in the representation theory of the symmetric group, which can describe the products of Schur functions using Young diagrams in a combinatorial way. The Littlewood-Richardson rule is also applied to the expansion of immanants, generalizations of the determinant and the permanent, by principal minors. In this talk, we discuss limiting behavior of immanants and some results obtained by applying this rule.

14. Motoki Takigiku (University of Tokyo)

A factorization formula for K-k-Schur functions

We give a Pieri-type formula for the sum of K-k-Schur functions $\sum_{\mu \leq \lambda} g_{\mu}^{(k)}$ over a principal order ideal of the poset of k-bounded partitions under the strong Bruhat order, which sum we denote by $\widetilde{g}_{\lambda}^{(k)}$. As an application of this, we also give a k-rectangle factorization formula $\widetilde{g}_{R_t \cup \lambda}^{(k)} = \widetilde{g}_{R_t}^{(k)} \widetilde{g}_{\lambda}^{(k)}$ where $R_t = (t^{k+1-t})$, analogous to that of k-Schur functions $s_{R_t \cup \lambda}^{(k)} = s_{R_t}^{(k)} s_{\lambda}^{(k)}$.

15. Itaru Terada (University of Tokyo)

On an involution on the set of Littlewood-Richardson tableaux

We interpret a bijection, constructed by O. Azenhas, between the Littlewood–Richardson (LR) tableaux of shape λ/μ and weight ν on one hand and those of shape λ/ν and weight μ on the other, as a correspondence between the irreducible components of algebraic varieties $\mathcal{G}_{\mu\nu}^{\lambda}$ and $\mathcal{G}_{\nu\mu}^{\lambda}$, which are actually isomorphic, but the irreducible components of $\mathcal{G}_{\mu\nu}^{\lambda}$ are naturally labelled by the LR tableaux of shape λ/μ and weight ν while those of $\mathcal{G}_{\nu\mu}^{\lambda}$ by those of shape λ/ν and weight μ. These varieties are closely related with the Hall polynomials indexed by the same triples of partitions.

To be more explicit, Azenhas' bijection consists of iterating a certain deletion operation starting from a given LR tableau T of shape λ/μ and ν to produce a sequence of LR tableaux $T = T^{(n)}, T^{(n-1)}, \ldots, T^{(1)}, T^{(0)} = \varnothing$ in which each $T^{(r)}$ has outer shape with exactly r rows, and while doing so recording how the inner shape shrinks in the form of another LR tableau with μ and ν swiched. Our interpretation is obtained by revealing what is represented by these intermediate LR tableaux, in particular $T^{(n-1)}$ obtained in the first step.

16. Hiro-Fumi Yamada (Kumamoto University)

On Sato's "On Hirota's Bilinear Equations"

In 1980 Mikio Sato published a hand-written note, which contains some tables on the KdV equations. I have been trying to read out information from these tables for a long time. And recently I finally found what Sato wanted to tell us on the KdV equations. I will report the meaning of Sato's tables, and view the KdV equations as the orthogonality relations of the symmetric group characters.

Organized by : Masao Ishikawa (Okayama University), Kento Nakada (Okayama University), Yasuhide Numata (Shinshu University), Soichi Okada (Nagoya University), Takeshi Suzuki (Okayama University), Hiroyuki Tagawa (Wakayama University), Itaru Terada (University of Tokyo), Hiro-Fumi Yamada (Kumamoto University).