

# Automorphic representations, automorphic $L$ -functions and arithmetic

January 19-23, 2009, at RIMS Hall 420

Abstract

**Paul Gunnells**, "On the cohomology of congruence subgroups of  $SL(4, \mathbf{Z})$ "

In this talk we describe the latest results of our computational investigation of the cohomology of subgroups of  $SL(4, \mathbf{Z})$ . We discuss the methods used as well as connections with Eisenstein cohomology and Siegel modular forms. This is joint work with Avner Ash and Mark McConnell.

**Takanori Yasuda**, "Non-tempered automorphic representations of inner forms of  $Sp(4)$ "

The discrete spectrum of  $L^2$ -space for an inner form of  $Sp_4$  is decomposed into the tempered and non-tempered subspaces. The non-tempered subspace should be divided into A-packets for A-parameters of CAP type by Arthur's conjecture. I will explain the construction of these A-packets and automorphic representations for conjectural multiplicities.

**Takeo Okazaki**, "Paramodular forms on  $GSp_2(\mathbf{A})$ "

By the theta lift from  $GO(2, 2)$ , we give a construction of generic automorphic forms on  $GSp_2(\mathbf{A})$  which are fixed by paramodular groups. We can say our construction is canonical, by the reason why they are not vanishing and why they have acceptable local  $L$ -factors and  $\varepsilon$ -factors at bad primes.

**Neil Dummigan**, "Yoshida lifts and Selmer groups"

Given a pair of newforms of weights  $k' > k \geq 2$ , we consider a near-central critical value of the tensor product  $L$ -function. Given a large prime dividing the algebraic part of the  $L$ -value, there should exist an element of that order in a Bloch-Kato Selmer group. I will outline how, under favourable hypotheses, one could construct such an element using the 4-dimensional Galois representation attached to a suitable genus-two cusp form congruent to the Yoshida lift of  $f$  and  $g$ . I will speculate about what is needed to deal with the other critical points.

**Hidenori Katsurada, Yoshinori Mizuno**, "An explicit formula for the twisted Koecher-Maass series of the Saito-Kurokawa lift and its applications"

Let  $f$  be a cuspidal Hecke eigenform for  $SL(2, \mathbf{Z})$ , and  $g$  the half-integral weight cusp form corresponding to  $f$  via the Shimura correspondence. In this talk, we express the twisted Koecher-Maass series of the Saito-Kurokawa lift of  $f$  as a sum of the convolution products of  $g$  and the twisted Cohen Eisenstein series of weight  $3/2$ . This is a generalization of Boecherer's result. Furthermore, as an application, we show a linear dependency among the special values of these convolution products at integers.

**Bernhard Heim**, "Applications of an arithmetic trace formula"

Recently we have found a trace formula comparing periods and special values of automorphic forms and  $L$ -functions. In the first part of the talk we recall the formula, which depends on comparing two different spectral decompositions of an Eisenstein series of Siegel type. In the second part we talk about applications. One is obtained by deforming the formula via differential operators. This is a joint project with Böcherer. An other application is given by extracting congruences out of the formula. For example we show that certain periods can be expressed by special values of  $L$ -functions mod  $p$  without assuming any result related to the Gross-Prasad conjecture. Surprisingly the class number of the imaginary quadratic field  $\mathbf{Q}(\sqrt{-p})$  plays a decisive role.

**Hidenori Katsurada, Hisa-aki Kawamura**, "Ikeda's conjecture on the period of the Ikeda lift"

We shall discuss the period (Petersson norm squared) of a Siegel cusp form of higher genus, namely cuspidal automorphic form on the symplectic group, which is connected to a cusp form of genus one via Ikeda's lifting procedure. Previously, Ikeda provided a certain conjecture on the periods of such forms in terms of some arithmetic invariants attached to associated cusp forms of genus one. In this talk, we would like to explain a proof of the conjecture.

**Tomokazu Kashio**, "Stark units, CM-periods and multiple gamma functions"

Stark units over a totally real field are (conjectural) units of its abelian extensions which is not totally imaginary. On the other hand, CM-periods are the transcendental parts of critical values of  $L$ -functions associated with algebraic Hecke characters of CM-fields. By using Yoshida's class invariant (and assuming his conjecture on CM-periods), we can see a "relation" between Stark units and CM-periods. More precisely, in this talk we will formulate a conjecture which contains Yoshida's conjecture on CM-periods and the algebraicity of Stark units.

**Masao Tsuzuki**, "Limit period formulas for special cycles on real hyperbolic spaces"

We consider the harmonic Poincare dual forms for special cycles on compact arithmetic quotients  $\Gamma \backslash \mathbf{H}^d$  of real hyperbolic space  $\mathbf{H}^d = SO_0(d, 1)/SO(d)$ . We study the behavior of  $L^2$ -norm of the Poincare dual form when  $\Gamma$  shrinks to the identity by means of a version of relative trace formula.

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**Takumi Noda**, "Some asymptotic expansions of the Eisenstein series"

We report one asymptotic formula of the non-holomorphic Eisenstein series for  $SL_2(\mathbf{Z})$  on the critical line by using the uniform expansions of the Bessel function due to Olver. We also report the  $t$ -aspect of the Eisenstein series and other asymptotic expansions.

**Hirofumi Nagoshi**, "Functional independence and randomness of  $L$ -functions"

This talk will start with an overview of Hilbert's statement in 1900 concerning algebraic-differential independence of the Riemann zeta-function, and its developments. Then we will discuss some random behavior of values of Dirichlet series in a certain class and deduce functional independence of these series. In particular, we will find that under the generalized Ramanujan conjecture, cuspidal automorphic  $L$ -functions for  $GL$  over  $\mathbf{Q}$  and their derivatives are algebraically independent and, more strongly, functionally independent, by virtue of the Rankin-Selberg theory.

**Keiichi Gunji**, "On Siegel Eisenstein series of degree 2 for low weights"

In this talk, we consider the Fourier coefficients of Siegel Eisenstein series for low weights, in the case of level  $p$ . In particular the calculation of the Euler  $p$ -factor of the Siegel series are given. As an application, we will give the dimension of the space of Siegel Eisenstein series with respect to the principal congruence subgroup of level  $p$ , in weight 2 case.

**Yumiko Hironaka**, "Spherical functions on  $U(n, n)/(U(n) \times U(n))$  and hermitian Siegel series"

We give a space  $X$  over a  $p$ -adic field which is isomorphic to  $U(n, n)/(U(n) \times U(n))$  over its algebraic closure, and study spherical functions on  $X$ , those functional equations, locations of possible poles and zeros, explicit formulas for some special points, etc. As an application, we give a functional equation of  $p$ -adic local hermitian Siegel series.

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**Gombodorj Bayarmagnai**, "On the principal series representation of  $SU(2, 2)$ "

In this talk the basic object will be the principal series representation  $\pi$  of  $SU(2, 2)$ , parabolically induced by the minimal parabolic subgroup. More specifically, we discuss about the  $(g, K)$ -module structure and Knapp-Stein intertwiner on  $\pi$ , also the Whittaker functions corresponding to some kind of  $\pi$ .

**Kazuki Hiroe**, "A characterization of Whittaker models of degenerate principal series representations"

Let  $G$  be a semisimple Lie group split over  $\mathbf{R}$ . For degenerate principal series representations of  $G$ , the notion of strongly-spherical  $K$ -types will be introduced. I will give a characterization of generalized Whittaker functions of these strongly-spherical  $K$ -types as kernels of some explicit differential operators.

**Moshe Baruch**, "The classical Hankel transform in the Kirillov model of the discrete series"

We give a short and elementary proof of the Hankel inversion formula using representation theoretic methods. We use the proof to give a complete description of the smooth space of the Kirillov model of discrete series representations of  $SL(2, \mathbf{R})$ .

**Michitaka Miyauchi**, "Formal degrees of supercuspidal representations of ramified  $U(3)$ "

Formal degrees of supercuspidal representations of  $p$ -adic unramified  $U(3)$  are obtained as a part of the explicit Plancherel formula by Jabon-Keys-Moy. In this talk, we compute those of ramified  $U(3)$  in terms of supercuspidal types. As a corollary, we give a new proof of stability of very cuspidal representations of  $U(3)$ .

**Kazutoshi Kariyama**, "Self-dual Bushnell-Kutzko type and discrete series of  $p$ -adic classical groups"

Let  $G$  be a classical group over a non-archimedean local field of odd residual characteristic. We produce a semisimple stratum, and construct a non-supercuspidal simple type in  $G$  in the sense of Bushnell-Kutzko. We define a self-dual simple type, and see that it parameterizes discrete series for  $G$  whose factors are inertially equivalent to a single irreducible self-dual supercuspidal representation of a general linear group, and whose partial cuspidal support is not always trivial.

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**Siegfried Böcherer**, "On Siegel modular Forms mod  $p$ "

We report on attempts by S.Nagaoka and myself to generalize some parts of Serre's work on modular forms mod  $p$  and on  $p$ -adic modular forms to the case of Siegel modular forms. To obtain such results we need some new constructions of modular forms for  $\Gamma_0(p)$  with specific  $p$ -adic properties of their Fourier expansions in all cusps.

**Shunsuke Yamana**, "How many Fourier coefficients determine a holomorphic modular form?"

We prove that Siegel modular forms of degree greater than one, integral weight and level  $N$ , with respect to a Dirichlet character  $\chi$  of conductor  $f_\chi$  are uniquely determined by their Fourier coefficients indexed by matrices whose contents run over all divisors of  $N/f_\chi$ . The case of half-integer weight forms will be included.

**Takashi Taniguchi**, "Extra functional equations of zeta functions of binary cubic forms"

We give some analogous formulas of Ohno and Nakagawa's "extra functional equations" of zeta functions for the space of binary cubic forms. This is a joint work with Yasuo Ohno and Satoshi Wakatsuki. I also would like to give some remarks on zeta functions of other "exceptional type" prehomogeneous vector spaces.

**Masaru Ueda**, "Representation of finite Metaplectic group and Newforms of half-integral weight"

We can characterize the plus spaces of half-integral weight by using a certain representation of Metaplectic group over  $\mathbf{Z}/4\mathbf{Z}$ . In this talk, we generalize this characterization to the case of general level  $N$ . We obtain Newform theory of half-integral weight and level  $8M$  or  $16M$  as an application of such characterization. Here,  $M$  is a squarefree odd positive integer.

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