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EVEN-DIMENSIONAL MANIFOLDS STRUCTURED BY A CONSTANT $\mathcal{T}$-PARALLEL CONNECTION

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Abstract. Geometrical and structural properties are proved for even-dimensional manifolds which are equiped with a constant $\mathcal{T}$-parallel connection.

1. Introduction

Manifolds structured by a $\mathcal{T}$-parallel connection have been defined in [17] and have also been studied in [13]. The present paper continues the study of the structural properties of manifolds endowed with a $\mathcal{T}$-parallel connection in the presence of additional geometric structures; as such the present investigation can be situated in the prolongation of the recent publications [3] [4] [5]. A general discussion of the geometrical structures which appear here and in the sequel can be found in the standard references [16] and [26] which also contain more background information and additional references (see also [1] [7] [20] for further reading).

Let now M be a $2m$-dimensional C^∞-manifold and $e_a (a \in \{1, \ldots, 2m\})$ an orthonormal vector basis. We recall that if M carries a globally defined vector field $\mathcal{T}$ and the connection forms satisfy

$$\theta^a_b = \langle \mathcal{T}, e_b \wedge e_a \rangle,$$

where $\wedge$ denotes the wedge product of vector fields, then one says that M is structured by a $\mathcal{T}$-parallel connection. In the present paper we assume in addition that $\mathcal{T}$ is constant. Introducing the notation $\beta = \mathcal{T}^\flat$, β will be called the structural pfaffian. Defining $2t = \|\mathcal{T}\|^2$, we consequently see that this quantity is also constant.

For the above mentioned structure, we prove the following properties:

- **(i):** M is a hyperbolic space-form, i.e. for the curvature forms Θ^a_b one has that
$$\Theta^a_b = -2t\ \omega^a \wedge \omega^b,$$
where $\{\omega^a\}$ denotes the cobasis of the vector basis $\{e_a\}$;
- **(ii):** M carries a locally conformal symplectic form Ω having $\beta (= \mathcal{T}^\flat)$ as covector of Lee [9], i.e.
$$d\Omega = 2\beta \wedge \Omega,$$

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and $\mathcal{T}$ defines a relative conformal transformation [19] [12] of Ω, i.e.

$$d(\mathcal{L}_{\mathcal{T}}\Omega) = 8t\beta \wedge \Omega;$$

(iii): $\mathcal{T}$ is torse forming [23] (see also [12] [19] [21]); moreover, with $\mathcal{T}$ there is associated a second vector field X which defines an infinitesimal automorphism [10] (see also [11]) of Ω, i.e.

$$\mathcal{L}_X\Omega = 0;$$

(iv): both vector fields $\mathcal{T}$ and X turn out to be biconcircular (in the sense of Okumura [14], see also [24]) and exterior concurrent [18]. In addition, $\mathcal{T}$ has also the property to be an affine vector field [16], i.e.

$$\mathcal{L}_{\mathcal{T}}\nabla\mathcal{T} = 0.$$

Finally, if we define the function s by $s = \langle \mathcal{T}, X\rangle$, one also finds that

$$ds = -s\beta,$$

and one further derives that

$$\begin{aligned} \operatorname{grad} s &= 2ts^2, \\ \operatorname{div}\operatorname{grad} s &= 2t(2 - tm)s, \end{aligned}$$

which shows that s is an isoparametric function [22].

In Section 4 we consider some properties of the tangent bundle manifold TM having the manifold M, studied in Section 3, as basis. On TM the canonical vector field $V(V^a)$ $(a = 1, \ldots, 2m)$ is called the Liouville vector field [6]. We will denote the adapted cobasis in TM by $\mathcal{B}^* = \{\omega^a, dV^a\}$. Then, the complete lift Ω^C [25] of the 2-form Ω is given by

$$\Omega^C = \sum_{a=1}^{m}(dV^a \wedge \omega^{a^*} + \omega^a \wedge dV^{a^*}), \qquad a^* = a + m.$$

One can deduce that

$$d\Omega^C = \beta \wedge \Omega^C,$$

which shows that the 2-form Ω^C is, just as Ω, also a conformal symplectic form. Next, since the Liouville vector field V is given by

$$V = \sum_{a=1}^{2m} V^a \frac{\partial}{\partial V^a},$$

the basic 1-form μ (also called the Liouville form) associated with the canonical vector field V (i.e. $\mu = V^\flat$) can be written as [8]

$$\mu = \sum_{a=1}^{2m} V^a \omega^a.$$

Taking the Lie differential of Ω^C, one finds that

$$\mathcal{L}_V \Omega^C = \Omega^C,$$

which expresses that the 2-form Ω^C is a homogeneous 2-form of class 1 [8] on TM. Some further properties of the tangent bundle manifold TM are also discussed.

2. Preliminaries

Let (M, g) be a Riemannian C^∞-manifold and let ∇ be the Levi-Civita operator with respect to the metric tensor g. Let $\Gamma TM = \Xi(M)$ be the set of sections of the tangent bundle, and

$$\flat : TM \xrightarrow{\flat} T^*M \quad \text{and} \quad \sharp : TM \xleftarrow{\sharp} T^*M$$

the classical isomorphisms defined by g (i.e. $^\flat$ is the index lowering operator, and $^\sharp$ is the index raising operator).

Following [16], we denote by

$$A^q(M, TM) = \Gamma \mathrm{Hom}(\Lambda^q TM, TM),$$

the set of vector valued q-forms ($q \langle \dim M$), and we write for the covariant derivative operator with respect to ∇

$$d^\nabla : A^q(M, TM) \to A^{q+1}(M, TM).$$

It should be noticed that in general $d^{\nabla^2} = d^\nabla \circ d^\nabla \neq 0$, unlike $d^2 = d \circ d = 0$. We denote by $dp \in A^1(M, TM)$ the canonical vector valued 1-form of M, which is also called the soldering form of M [2]. Since ∇ is symmetric one has that $d^\nabla(dp) = 0$.

A vector field $Z \in \Xi(M)$ which satisfies

$$d^\nabla(\nabla Z) = \nabla^2 Z = \pi \wedge dp \in A^2(M, TM); \quad \pi \in \Lambda^1 M \tag{1}$$

is defined to be an exterior concurrent vector field [17] (see also [13]). The 1-form π in (4) is called the concurrence form and is defined by

$$\pi = \lambda Z^\flat, \quad \lambda \in \Lambda^0 M. \tag{2}$$

Let $\mathcal{O} = \mathrm{vect}\{e_a | a = 1, \ldots, 2m\}$ be a local field of adapted vectorial frames over M and let $\mathcal{O}^* = \mathrm{covect}\{\omega^a\}$ be its associated coframe. Then the soldering form dp is expressed by

$$dp = \sum_{a=1}^{2m} \omega^a \otimes e_a, \tag{3}$$

and E. Cartan's structure equations can be written in indexless manner are

$$\nabla e = \theta \otimes e, \tag{4}$$

$$d\omega = -\theta \wedge \omega, \tag{5}$$

$$d\theta = -\theta \wedge \theta + \Theta. \tag{6}$$

In the above equations θ (respectively Θ) are the local connection forms in the tangent bundle TM (respectively the curvature 2-forms on M).

3. Manifolds with constant $\mathcal{T}$-parallel connection

Let (M, g) be a $2m$-dimensional C^∞-manifold and

$$\mathcal{T} = \mathcal{T}^a e_a,$$

be a globally defined vector field. Let θ^a_b $(a, b \in \{1, \dots, 2m\})$ be the local connection forms in the tangent bundle TM. Then, by reference to [17] [13], (M, g) is said to be structured by a $\mathcal{T}$-parallel connection if the connection forms θ satisfy

$$\theta^a_b = \langle \mathcal{T}, e_b \wedge e_a \rangle, \tag{7}$$

where $\wedge$ means the wedge product of vector fields. Making use of Cartan's structure equations (4), we can see that

$$\theta^a_b = \mathcal{T}^b \omega^a - \mathcal{T}^a \omega^b. \tag{8}$$

In consequence of (8), the equations (4) take the form

$$\nabla e_a = \mathcal{T}^a dp - \omega^a \otimes \mathcal{T}. \tag{9}$$

In the sequel we assume in addition that $\mathcal{T}^a$ are the components of a constant vector field $\mathcal{T}$, called the structure vector field of M.

Let

$$\mathcal{T}^\flat = \beta = \sum_{a=1}^{2m} \mathcal{T}^a \omega^a \tag{10}$$

be the dual form of $\mathcal{T}$, then by E. Cartan's structure equations (5) one derives that

$$d\omega^a = \beta \wedge \omega^a. \tag{11}$$

Hence, by (11) it follows that all the elements ω^a of the covector basis $\mathcal{O}^*$ are exterior recurrent forms [2]. Consequently, the pfaffian β can be seen to be in fact a closed form, i.e.

$$d\beta = d\mathcal{T}^\flat = 0. \tag{12}$$

Under the present conditions, by (8) and (11) one finds that

$$d\theta^a_b = \beta \wedge \theta^a_b, \tag{13}$$

which expresses that all the connection forms θ^a_b are exterior recurrent [2] with β as recurrence form. Under these conditions, the structure equations (6) involving the curvature forms Θ^a_b are expressed by

$$\Theta^a_b = -2t\ \omega^a \wedge \omega^b, \tag{14}$$

where we have set

$$2t = \|\mathcal{T}\|^2 = \text{const.}. \tag{15}$$

It is well known that the equation (14) thus shows that the manifold M under consideration is a space form of hyperbolic type. We remark that in view of (11), one derives that

$$d\Theta^a_b = 2\beta \wedge \Theta^a_b, \tag{16}$$

which means that all curvature forms are exterior recurrent; we therefore agree to call β the basic pfaffian on M.

In another perspective, we consider on M the local almost symplectic form Ω given by

$$\Omega = \sum_{a=1}^{m} \omega^a \wedge \omega^{a^*}, \qquad a^* = a + m. \tag{17}$$

Taking the exterior derivative of Ω, and in view of (11), one finds that

$$d\Omega = 2\beta \wedge \Omega, \tag{18}$$

which shows that Ω is a locally conformal symplectic form having β as covector of Lee [9].

Taking first the Lie derivative of Ω with respect to the vector field $\mathcal{T}$, we get

$$\mathcal{L}_{\mathcal{T}}\Omega = \sum_{a=1}^{m} \mathcal{L}_{\mathcal{T}}\omega^a \wedge \omega^{a^*} + \sum_{a=1}^{m} \omega^a \wedge \mathcal{L}_{\mathcal{T}}\omega^{a^*},$$

where $\mathcal{L}_{\mathcal{T}}\omega^a$ can be calculated as follows.

$$\mathcal{L}_{\mathcal{T}}\omega^a = (i(\mathcal{T}) \circ d + d \circ i(\mathcal{T}))\, \omega^a \qquad (a = 1, \cdots, 2m)$$

Taking into account equation (11) for $d\omega^a$ and the definition (15) of $2t$, it follows that

$$\mathcal{L}_{\mathcal{T}}\omega^a = 2t\omega^a - 2T^a\beta, \qquad (a = 1, \cdots, 2m).$$

Continuing now the calculation of $\mathcal{L}_{\mathcal{T}}\Omega$ leads to

$$\mathcal{L}_{\mathcal{T}}\Omega = 4t\Omega + 2\beta \wedge {}^{\flat}\mathcal{T},$$

where

$${}^{\flat}\mathcal{T} = -i_{\mathcal{T}}\Omega = \sum_{a=1}^{m} \left(\mathcal{T}^{a^*}\omega^a - \mathcal{T}^a\omega^{a^*} \right).$$

Exterior differentiation of $\mathcal{L}_{\mathcal{T}}\Omega$ gives

$$d(\mathcal{L}_{\mathcal{T}}\Omega) = 4td\Omega + 2d\beta \wedge {}^{\flat}\mathcal{T} - 2\beta \wedge d({}^{\flat}\mathcal{T}).$$

One can verify directly that $d({}^{\flat}\mathcal{T}) = 0$, and recalling that the 1-form $\beta = \mathcal{T}^{\flat}$ is closed, the above expression reduces to

$$d(\mathcal{L}_{\mathcal{T}}\Omega) = 4td\Omega.$$

Replacing $d\Omega$ through equation (18), finally yields

$$d(\mathcal{L}_{\mathcal{T}}\Omega) = 8t\beta \wedge \Omega. \tag{19}$$

Hence, following a known definition [19] (see also [12]), the above equation means that $\mathcal{T}$ defines a relative conformal transformation of Ω.

Further, consider the vector field

$$X = \sum_{a=1}^{2m} X^a e_a. \tag{20}$$

Taking the Lie differential of Ω w.r.t. X, yields

$$\mathcal{L}_X\Omega = -\sum_{a=1}^{m}(dX^a + \beta X^a) \wedge \omega^{a^*} + \sum_{a=1}^{m}(dX^{a^*} + \beta X^{a^*}) \wedge \omega^a. \tag{21}$$

Therefore, the necessary and sufficient condition for X to define an infinitesimal automorphism [10] (see also [11]) of Ω, namely

$$\mathcal{L}_X\Omega = 0, \tag{22}$$

can be seen to be

$$dX^a + \beta X^a = 0. \tag{23}$$

We now introduce the notation

$$\alpha = X^{\flat} = \sum_{a=1}^{2m} X^a \omega^a \tag{24}$$

for the dual form of X.

Taking the exterior derivative of (24) gives

$$d\alpha = \sum_{a=1}^{2m} dX^a \wedge \omega^a + \sum_{a=1}^{2m} X^a d\omega^a.$$

Replacing in the above formula dX^a using (23), and $d\omega^a$ using (11), yields

$$d\alpha = -\sum_{a=1}^{2m} \beta X^a \wedge \omega^a + \sum_{a=1}^{2m} X^a \beta \wedge \omega^a.$$

From this it follows that

$$d\alpha = 0, \tag{25}$$

which shows that X is also a closed vector field.

Further, calculating the covariant differentials of the vector fields $\mathcal{T}$ and X under consideration and invoking (15), one obtains that

$$\nabla\mathcal{T} = 2tdp - 2\beta \otimes \mathcal{T}, \tag{26}$$

and

$$\nabla X = sdp - \alpha \otimes \mathcal{T} - \beta \otimes X, \tag{27}$$

where we have put

$$s = g(X, \mathcal{T}). \tag{28}$$

Equation (26) expresses that the structure vector field $\mathcal{T}$ is torse forming [23] (see also [12] [19] [21]); in this context we will call X an almost torse forming vector field, and by standard terminology [21] $2t = \|\mathcal{T}\|^2$ is the energy of the torse forming vector field $\mathcal{T}$.

Moreover, we notice that any 2 vector fields $Z, Z' \in \Xi(M)$ satisfy

$$\begin{aligned} \langle \nabla_Z \mathcal{T}, Z' \rangle &= \langle \nabla_{Z'} \mathcal{T}, Z \rangle, \\ \langle \nabla_Z X, Z' \rangle &= \langle \nabla_{Z'} \mathcal{T}, Z \rangle. \end{aligned} \tag{29}$$

According to Okumura [14] (see also [24]), the relations (29) show that $\mathcal{T}$ and X are gradient vector fields. On the other hand, since ∇ acts inductively one also derives that

$$d^{\nabla}(\nabla\mathcal{T}) = 2t\mathcal{T}^{\flat} \wedge dp, \quad (\mathcal{T}^{\flat} =: \beta) \tag{30}$$

$$d^{\nabla}(\nabla X) = 2tX^{\flat} \wedge dp. \quad (X^{\flat} =: \alpha) \tag{31}$$

The above equations mean that both $\mathcal{T}$ and X are exterior concurrent vector fields [18]. Therefore, if $\mathcal{R}$ denotes the Ricci curvature, it follows from (30), (31) and [15] that

$$\begin{aligned} \mathcal{R}(\mathcal{T}, Z) &= -(2m-1)2tg(\mathcal{T}, Z), \\ \mathcal{R}(X, Z) &= -(2m-1)2tg(X, Z). \end{aligned} \tag{32}$$

We remark that calculating the Lie differential of $\nabla\mathcal{T}$ with respect to $\mathcal{T}$ reveals that

$$\mathcal{L}_{\mathcal{T}}\nabla\mathcal{T} = 0, \tag{33}$$

which shows that $\mathcal{T}$ is an affine vector field [16]. We recall that with respect to an orthonormal vector basis $\{e_a\}$ the divergence of a vector field Z is calculated according to the formula

$$\operatorname{div} Z = \sum_{a=1}^{2m} \langle \nabla_{e_a} Z, e_a \rangle; \tag{34}$$

when applied to the case under consideration, this gives

$$\operatorname{div} \mathcal{T} = (2m-1)2t = \text{const.}. \tag{35}$$

Furthermore, since the components $\mathcal{T}^a$ are constant, one finds by differentiation of the equality $s = g(\mathcal{T}, X)$ that

$$ds = -s\beta. \tag{36}$$

Consequently one may write that

$$\operatorname{grad} s = -s\mathcal{T} \quad \Longrightarrow \quad \|\operatorname{grad} s\|^2 = 2ts^2, \tag{37}$$

from which one also derives that

$$\operatorname{div}(\operatorname{grad} s) = 2t(2-tm)s. \tag{38}$$

We remind that a function $f : \mathbb{R}^n \to \mathbb{R}$ is called isoparametric [22] if both $\|\operatorname{grad} f\|^2$ and $\operatorname{div}(\operatorname{grad} f)$ are functions of f. We may therefore conclude that s is an isoparametric function.

Summing up, we state the following

Theorem 3.1. *Let $M(\Omega, \mathcal{T}, g)$ be a $2m$-dimensional manifold with almost symplectic form Ω, and structure constant vector field $\mathcal{T}$, such that the connection forms satisfy*

$$\theta^a_b = \langle \mathcal{T}, e_b \wedge e_a \rangle.$$

Then the following properties hold:

- **(i):** *M is a hyperbolic space-form*;
- **(ii):** *Ω is a conformally symplectic form and has $\beta(= \mathcal{T}^\flat)$ as covector of Lee*;
- **(iii):** *the differential of the Lie derivative with respect to $\mathcal{T}$ defines a relative conformal transformation of Ω, i.e.*

$$d(\mathcal{L}_{\mathcal{T}}\Omega) = 8t\beta \wedge \Omega, \qquad 2t = \|\mathcal{T}\|^2;$$

- **(iv):** *a vector field X which satisfies*

$$dX^a + \beta X^a = 0$$

defines an infinitesimal automorphism of Ω, i.e.

$$\mathcal{L}_X\Omega = 0;$$

- **(v):** *$\mathcal{T}$ is a torse forming vector field, as well as an exterior concurrent vector field*;

(vi): *the vector field X is also an exterior concurrent vector field, and both $\mathcal{T}$ and X are gradient vector fields*;

(vii): *the scalar $s = \langle \mathcal{T}, X \rangle$ is an isoparametric function.*

4. Geometry of the tangent bundle

In this section we will discuss some properties of the tangent bundle manifold TM having as basis the manifold M studied in Section 3. Denote by $V(V^a)$ $(A = 1, \ldots, 2m)$ the Liouville vector field (or the canonical vector field on TM [8]). Accordingly, one may consider the set

$$B^* = \{\omega^a, dV^a | a = 1, \ldots, 2m\}$$

as an adapted cobasis in TM (see also [13]). Following [25] the complete lift Ω^C of the conformal symplectic form Ω of M is the 2-form of rank $4m$ on TM given by

$$\Omega^C = \sum_{a=1}^{m} (dV^a \wedge \omega^{a^*} + \omega^a \wedge dV^{a^*}), \qquad a^* = a + m. \tag{39}$$

On the other hand, the Liouville vector field V is expressed by

$$V = \sum_{a=1}^{2m} V^a \frac{\partial}{\partial V^a}. \tag{40}$$

It is also known that the associated basic 1-form

$$\mu = \sum_{a=1}^{2m} V^a \omega^a \tag{41}$$

is called the Liouville form (see also [8]). (Alternatively, one can also write that $\mu = V^\flat$.) Then, on behalf of (11), the exterior differential of Ω^C is given by

$$d\Omega^C = \beta \wedge \Omega^C. \tag{42}$$

Hence, the complete lift Ω^C of Ω defines on TM a conformal symplectic structure, as Ω does on M; this result is meaningful, since it should be stressed that conformal properties are not preserved by complete lifts in general. On behalf of (40) one has that

$$i_V \Omega^C = \sum_{a=1}^{m} (V^a \omega^{a^*} - V^{a^*} \omega^a), \tag{43}$$

and in view of (42) and (43) one gets

$$\mathcal{L}_V \Omega^C = \Omega^C. \tag{44}$$

Equation (44) shows that Ω^C is a homogeneous 2-form of class 1 [8] on TM.

Further, taking the exterior differential of the Liouville form μ, one derives by (41) that

$$d\mu = \beta \wedge \mu + \psi, \tag{45}$$

where we have introduced the notation

$$\psi = \sum_{a=1}^{2m} dV^a \wedge \omega^a. \tag{46}$$

By reference to (46) and (11), it follows that

$$d\psi = \beta \wedge \psi, \tag{47}$$

which shows that ψ is an exterior recurrent form with β as recurrence form. Since the 2-form ψ is of maximal rank, we will refer to ψ as the canonical conformal symplectic form of M. One finally gets that

$$\mathcal{L}_V \psi = \psi, \tag{48}$$

which shows that, as Ω^C, the form ψ is also a homogeneous 2-form of class 1.

We remind that the vertical operator i_V in the sense of [6] possesses by definition the following properties (see also [8]):

$$i_V \lambda = 0, \quad i_V \omega^a = 0, \quad i_V dV^a = \omega^a, \tag{49}$$

from which one calculates by (46) that

$$i_V \psi = 0. \tag{50}$$

Together with (47) we conclude from this that ψ is a Finslerian form [6].

Theorem 4.1. *Let TM be the tangent bundle manifold having as basis the manifold $M(\Omega, \mathcal{T}, \beta)$ considered in Section 3. Let V, and μ, be the Liouville vector field, and the Liouville form of TM respectively. One has the following properties*:

- **(i):** *the complete lift Ω^C on TM is a conformally symplectic form, and is a homogeneous 2-form of class 1, i.e.*

$$\mathcal{L}_V \Omega^C = \Omega^C;$$

- **(ii):** *μ satisfies*

$$d\mu = \beta \wedge \mu + \sum_{a=1}^{2m} dV^a \wedge \omega^a,$$

where

$$\psi = \sum_{a=1}^{2m} dV^a \wedge \omega^a,$$

is the canonical conformal symplectic form and turns out to be a Finslerian form.

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