Centralizers and related objects in Coxeter groups

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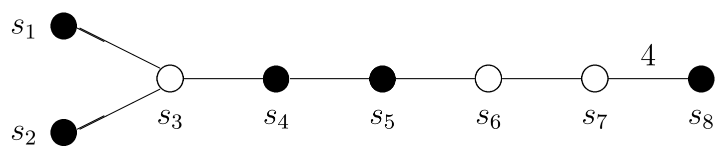
Let (W,S) be a Coxeter system, $W_I = \langle I \rangle$ $(I \subset S)$ be a parabolic subgroup of W. Until recently, the structure of the centralizer $Z_W(W_I)$ of W_I have been examined only for few classes of W_I or of (W,S) ($I = S$, $|I| = 1$ or (W,S) 'even'). My new result determines the structure of $Z_W(W_I)$ for all cases, by using a graph constructed from the information about finite parabolic subgroups of (W,S) only. By the result, we can compute $Z_W(W_I)$ explicitly if S is not too large (even if (W,S) is neither finite nor affine). Moreover, unified description of $Z_W(W_I)$ for some other classes of W_I are obtained.

Example 1. *Let (W,S) be an affine Coxeter system of type $\widetilde{B_7}$ (as below), $I = \{s_1, s_2, s_4, s_5, s_8\}$. Then $Z_W(W_I) \simeq (\mathbb{Z}/2\mathbb{Z})^3 \times (\widetilde{A_1} \rtimes \widetilde{A_1})$, where one of the Coxeter generators of the right side $\widetilde{A_1}$ acts trivially on the left side $\widetilde{A_1}$, and the another one exchanges the two generators of the left side $\widetilde{A_1}$.*

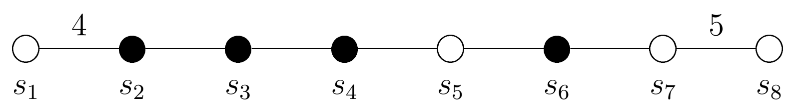


Example 2. *Let (W,S) be a Coxeter system as below, $I = \{s_2, s_3, s_4, s_6\}$. Then $Z_W(W_I) \simeq \mathbb{Z}/2\mathbb{Z} \times (W' \rtimes \langle \sigma, \tau \rangle)$, where W' is a Coxeter group with Coxeter generators $\widetilde{s}_{1,k}, \widetilde{s}_{2,k}$ $(k \in \mathbb{Z})$, $\widetilde{s}_3, \widetilde{s}_4$ and relations*

$$(\widetilde{s}_{1,k}\widetilde{s}_{2,k})^2 = (\widetilde{s}_{1,k}\widetilde{s}_3)^2 = (\widetilde{s}_{2,k}\widetilde{s}_4)^4 = (\widetilde{s}_3\widetilde{s}_4)^2 = 1 \text{ for any } k \in \mathbb{Z},$$

and σ, τ are automorphisms of W' such that

$$\sigma(\widetilde{s}_{1,k}) = \widetilde{s}_{1,k+1}, \sigma(\widetilde{s}_{2,k}) = \widetilde{s}_{2,k+1} \text{ (for any } k \in \mathbb{Z}), \sigma(\widetilde{s}_3) = \widetilde{s}_3, \sigma(\widetilde{s}_4) = \widetilde{s}_4,$$
$$\tau(\widetilde{s}_{1,k}) = \widetilde{s}_{1,-k}, \tau(\widetilde{s}_{2,k}) = \widetilde{s}_{2,-k} \text{ (for any } k \in \mathbb{Z}), \tau(\widetilde{s}_3) = \widetilde{s}_3, \tau(\widetilde{s}_4) = \widetilde{s}_4.$$



Example 3. *Let W_I be irreducible and suppose that any irreducible parabolic subgroup $W_J \supsetneq W_I$ is an infinite group. Then $Z_W(W_I) = Z(W_I) \times W_{I'}$, where $Z(W_I)$ is the center of W_I and I' is the set of all $s \in S \smallsetminus I$ not adjacent to I in the Coxeter graph of (W,S).*

Since $Z_W(W_I)$ contains the fixing subgroup in W of simple roots corresponding to I in the reflection representation of W, we can obtain the explicit structure of this group as a consequence. For example, it is shown that the fixing reflection subgroup of a simple root is not finitely generated in 'almost all' cases (even if W is finitely generated).