# 保型表現と保型 L-関数の数論的研究

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講演概要

**Paul Gunnells**, "On the cohomology of congruence subgroups of  $SL(4, \mathbf{Z})$ " In this talk we describe the latest results of our computational investigation of the cohomology of subgroups of  $SL(4, \mathbf{Z})$ . We discuss the methods used as well as connections with Eisenstein cohomology and Siegel modular forms. This is joint work with Avner Ash and Mark McConnell.

安田 貴徳, "Non-tempered automorphic representations of inner forms of Sp(4)"
The discrete spectrum of  $L^2$ -space for an inner form of  $Sp_4$  is decomposed into the tempered and non-tempered subspaces. The non-tempered subspace should be divided into A-packets for A-parameters of CAP type by Arthur's conjecture. I will explain the construction of these A-packets and automorphic representations for conjectural multiplicities.

### 岡崎 武生, "Paramodular forms on $GSp_2(\mathbf{A})$ "

By the theta lift from GO(2,2), we give a construction of generic automorphic forms on  $GSp_2(\mathbf{A})$  which are fixed by paramodular groups. We can say our construction is canonical, by the reason why they are not vanishing and why they have acceptable local L-factors and  $\varepsilon$ -factors at bad primes.

#### Neil Dummigan, "Yoshida lifts and Selmer groups"

Given a pair of newforms of weights  $k' > k \ge 2$ , we consider a near-central critical value of the tensor product L-function. Given a large prime dividing the algebraic part of the L-value, there should exist an element of that order in a Bloch-Kato Selmer group. I will outline how, under favourable hypotheses, one could construct such an element using the 4-dimensional Galois representation attached to a suitable genus-two cusp form congruent to the Yoshida lift of f and g. I will speculate about what is needed to deal with the other critical points.

桂田 英典, 水野 義紀, "An explicit formula for the twisted Koecher-Maass series of the Saito-Kurokawa lift and its applications"

Let f be a cuspidal Hecke eigenform for  $SL(2, \mathbf{Z})$ , and g the half-integral weight cusp form corresponding to f via the Shimura correspondence. In this talk, we express the twisted Koecher-Maass series of the Saito-Kurokawa lift of f as a sum of the convolution products of g and the twisted Cohen Eisenstein series of weight 3/2. This is a generalization of Boecherer's result. Furthermore, as an application, we show a linear dependency among the special values of these convolution products at integers.

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#### Bernhard Heim, "Applications of an arithmetic trace formula"

Recently we have found a trace formula comparing periods and special values of automorphic forms and L-functions. In the first part of the talk we recall the formula, which depends on comparing two different spectral decompositions of an Eisenstein series of Siegel type. In the second part we talk about applications. One is obtained by deforming the formula via differential operators. This is a joint project with Böcherer. An other application is given by extracting congruences out of the formula. For example we show that certain periods can be expressed by special values of L-functions mod p without assuming any result related to the Gross-Prasad conjecture. Surprisingly the class number of the imaginary quadratic field  $\mathbf{Q}(\sqrt{-p})$  plays a decisive role.

#### 桂田 英典,河村 尚明, "Ikeda's conjecture on the period of the Ikeda lift"

We shall discuss the period (Petersson norm squared) of a Siegel cusp form of higher genus, namely cuspidal automorphic form on the symplectic group, which is connected to a cusp form of genus one via Ikeda's lifting procedure. Previously, Ikeda provided a certain conjecture on the periods of such forms in terms of some arithmetic invariants attached to associated cusp forms of genus one. In this talk, we would like to explain a proof of the conjecture.

加塩 朋和、"Stark units, CM-periods and multiple gamma functions" 総実体上の Stark unit とは、その総虚でないアーベル拡大体の(予想上の)単数のことである.一方で、CM-period は CM 体の代数的 Hecke 指標に付随する L 関数の critical value の超越数部分を表す.実は吉田氏の定義した class invariant を用いると (そして CM-period に関する吉田予想を仮定すると) Stark unit と CM-period を"関連付ける"ことができる.より正確には、Stark unit の代数性と CM-period に関する吉田予想を含んだ形に予想式を定式化できる、というのが主結果である.

## 都築 正男、"実双曲空間上の特殊サイクルに対する極限周期公式について"

We consider the harmonic Poincare dual forms for special cycles on compact arithmetic quotients  $\Gamma \backslash \mathbf{H}^d$  of real hyperbolic space  $\mathbf{H}^d = SO_0(d,1)/SO(d)$ . We study the behavior of  $L^2$ -norm of the Poincare dual form when  $\Gamma$  shrinks to the identity by means of a version of relative trace formula.

#### 野田 エ,"アイゼンシュタイン級数の漸近展開"

We report one asymptotic formula of the non-holomorphic Eisenstein series for  $SL_2(\mathbf{Z})$  on the critical line by using the uniform expansions of the Bessel function due to Olver. We also report the t-aspect of the Eisenstein series and other asymptotic expansions.

# 名越 弘文, "Functional independence and randomness of L-functions"

This talk will start with an overview of Hilbert's statement in 1900 concerning algebraic-differential independence of the Riemann zeta-function, and its developments. Then we will discuss some random behavior of values of Dirichlet series in a certain class and deduce functional independence of these series. In particular, we will find that under

the generalized Ramanujan conjecture, cuspidal automorphic L-functions for GL over  $\mathbf{Q}$  and their derivatives are algebraically independent and, more strongly, functionally independent, by virtue of the Rankin-Selberg theory.

軍司 圭一, "On Siegel Eisenstein series of degree 2 for low weights"

In this talk, we consider the Fourier coefficients of Siegel Eisenstein series for low weights, in the case of level p. In particular the calculation of the Euler p-factor of the Siegel series are given. As an application, we will give the dimension of the space of Siegel Eisenstein series with respect to the principal congruence subgroup of level p, in weight 2 case.

広中 由美子, "Spherical functions on  $U(n,n)/(U(n)\times U(n))$  and hermitian Siegel series"

p 進体 k 上定義された,代数的閉体上では  $U(2n)/(U(n)\times U(n))$  と同型な空間を,エルミート行列を用いて実現し,その上の球関数を考える.球関数の関数等式,極と零点に関する情報,特別な点での明示式などを与えることを目標とする.応用として,hermitian Siegel series の関数等式を与える.

Gombodorj Bayarmagnai, "On the principal series representation of SU(2,2)" In this talk the basic object will be the principal series representation  $\pi$  of SU(2,2), parabolically induced by the minimal parabolic subgroup. More specifically, we discuss about the (g, K)-module structure and Knapp-Stein intertwiner on  $\pi$ , also the Whittaker functions corresponding to some kind of  $\pi$ .

廣惠 一希, "退化主系列表現の一般 Whittaker 模型"

R上分裂した半単純 Lie 群の退化主系列表現に対して,strongly-spherical K-type と呼ばれる K-type を定義し,その一般 Whittaker 関数を具体的な微分作用素の解空間として特徴付ける.

Moshe Baruch, "The classical Hankel transform in the Kirillov model of the discrete series"

We give a short and elementary proof of the Hankel inversion formula using representation theoretic methods. We use the proof to give a complete description of the smooth space of the Kirillov model of discrete series representations of  $SL(2, \mathbf{R})$ .

宮内 通孝, "分岐 U(3) の supercuspidal 表現の形式次数"

Jabon-Keys-Moy が与えた p-進体上の U(3) の明示的 Plancherel 公式では、supercuspidal 表現の部分は U(3) が不分岐である場合しか扱っていない。本講演では残された場合、すなわち分岐 U(3) の supercuspidal 表現の形式次数の明示式を与える。

刈山 和俊, "p-進古典群の self-dual Bushnell-Kutzko type と離散系列" Bushnell-Kutzko-Stevens により定義された p-進古典群 G の (simple) type を具体的に構成し、そして self-dual type を定義する。この self-dual type が Moeglin, Moeglin-Tadic により分類された G の離散系列の一部を特徴付けること、そして その離散系列を完全に決

Siegfried Böcherer, "On Siegel modular Forms mod p"

We report on attempts by S.Nagaoka and myself to generalize some parts of Serre's work on modular forms mod p and on p-adic modular forms to the case of Siegel modular forms. To obtain such results we need some new constructions of modular forms for  $\Gamma_0(p)$  with specific p-adic properties of their Fourier expansions in all cusps.

山名 俊介, "How many Fourier coefficients determine a holomorphic modular form?" We prove that Siegel modular forms of degree greater than one, integral weight and level N, with respect to a Dirichlet character  $\chi$  of conductor  $f_{\chi}$  are uniquely determined by their Fourier coefficients indexed by matrices whose contents run over all divisors of  $N/f_{\chi}$ . The case of half-integer weight forms will be included.

谷口 隆, "Extra functional equations of zeta functions of binary cubic forms" We give some analogous formulas of Ohno and Nakagawa's "extra functional equations" of zeta functions for the space of binary cubic forms. This is a joint work with Yasuo Ohno and Satoshi Wakatsuki. I also would like to give some remarks on zeta functions of other "exceptional type" prehomogeneous vector spaces.

上田 勝, "Representation of finite Metaplectic group and Newforms of half-integral weight"

半整数ウェイトのプラス空間は,保型形式の空間に実現される  $\mathbf{Z}/4\mathbf{Z}$  上のメタプレクティック群の表現で特徴付けることが可能である.(Skoruppa の学位論文の結果だが,筆者も独立に示した.) これらの結果を一般のレベル N に拡張することで半整数ウェイトのニューフォーム理論を得ることができる.今回は特に,レベル  $8\times M$ ,  $16\times M$  (M:squarefree odd) の場合に解説する.

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